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PARAMETRIC IDENTIFICATION OF MATHEMATICAL MODELS OF COUPLED CONDUCTIVE-RADIATIVE HEAT TRANSFER

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Abstract. In many practical situations it is impossible to measure directly such characteristics of analyzed materials as thermal and radiation properties. The only way, which can often be used to overcome these difficulties, is indirect measurements. This type of measurements is usually formulated as the solution of inverse heat transfer problems. Such problems are ill-posed in mathematical sense and their main feature shows itself in the solution instabilities. That is why special regularizing methods are needed to solve them. The experimental methods of identification of the mathematical models of heat transfer based on solving of the inverse problems are one of the modern effective solving manners.

The goal of this paper is to estimate thermal and radiation properties of advanced materials using the approach based on inverse methods (as example: thermal conductivity $\lambda(T)$, heat capacity $C(T)$ and emissivity $\varepsilon(T)$). New metrology under development is the combination of accurate enough measurements of thermal quantities, which can be experimentally observable under real conditions and accurate data processing, which are based on the solutions of inverse heat transfer problems. In this paper, the development of methods for estimating thermal and radiation characteristics is carried out for thermally stable high temperature materials. Such problems are of great practical importance in the study of properties of materials used as non-destructive surface coating in objects of space engineering, power engineering etc.

Also the corresponded optimal experiment design problem is considered. The algorithm is based on the theory of Fisher information matrix.

1 INTRODUCTION

In the modern engineering systems we deal with structures operating in the conditions of intensive, often extreme thermal effects. The general tendency in the development of technology is connected with the increase of the number of responsible, thermally loaded engineering objects. For such systems the support of thermal conditions is one of the most important aspects of design, determining the main design solutions. The modern approaches to the design of structures assume broad application of mathematical and physical simulation methods. But mathematical simulation is impossible if there is no true information available on the characteristics (properties) of objects analyzed. In the majority of cases in practice the direct measurement of materials' thermophysical properties, especially of complex composition, is impossible. There is only one way which permits to overcome these complexities - the indirect measurement. Mathematically, such an approach is usually formulated as a solution of the inverse problem: through direct measurements of system's state (temperature, component concentration, etc.) define the properties of a system analyzed, for example, the thermophysical properties. Violation of cause-and-effect relations in the statement of these problems results in their correctness in mathematical sense (i.e., the absence of existence and/or uniqueness and/or stability of the solution). Hence to solve such problems we develop special methods usually called regularized.

In estimating properties of modern structural, thermal-protective and thermal-insulating materials - as temperature-dependent - the most effective are methods based on solution of the coefficient inverse heat conduction problems. The most promising direction in further development of research methods for non-destructive composite materials using the procedure of inverse problems is the simultaneous determination of a combination of material's thermophysical and radiation properties (thermal conductivity $k(T)$, heat capacity $C(T)$ and integral emissivity $\varepsilon(T)$). Such problems are of great practical importance in the study of properties of composite materials used as non-destructive surface coating in objects of space technology, power engineering etc. [1], [2], [3], [4], [5], [6], [7]. The experimental equipment and the method developed could be applied for determination of material's three characteristics; the availability of two specimens of the material allows us to provide uniqueness of the solution. The mathematical model of heat transfer in specimen is

$$C(T)\frac{\partial T}{\partial \tau} = \frac{\partial}{\partial x}\left(k(T)\frac{\partial T}{\partial x}\right), \; x \in (X_0, X_1), \; \tau \in (\tau_{\min}, \tau_{\max}] \tag{1}$$

$$T(x, \tau_{\min}) = \mathrm{T}_0(x), \;\; \mathrm{x} \in [X_0, X_1] \tag{2}$$

$$-\beta_1 k(T)\frac{\partial T(X_0, \tau)}{\partial x} + \alpha_1 T(X_0, \tau) = q_1(\tau), \; \tau \in (\tau_{\min}, \tau_{\max}] \tag{3}$$

$$-k(T)\frac{\partial T(X_1, \tau)}{\partial x} = \varepsilon_{eff}(T)\sigma\left(T_h^4(\tau) - T^4(X_1, \tau)\right), \; \tau \in (\tau_{\min}, \tau_{\max}] \tag{4}$$

Where

$$\varepsilon_{eff}(T) = \frac{\varepsilon(T)\varepsilon_h(T_h)}{\varepsilon(T) + \varepsilon_h(T_h) - \varepsilon_h(T_h)\varepsilon_h(T_h)}$$

In model (1)-(4) the quantities $C(T)$, $k(T)$and $\varepsilon(T)$ are unknown. If emissivity of the heater material (ε_h) is known a-priori, and its temperature is measured, the heat flux from the heater can be calculated as irradiation with known (measured by thermocouple) temperature of the heater $T_h(\tau)$ and a-priory known (theoretically) emissivity of heater $\varepsilon_h(T)$. In presented paper a case is considered, when $\varepsilon_h(T)$ is not known a-priori and should be estimated. At this paper the emissivity of the heater is considered as additional (forth) estimating functions. Therefore the accuracy of the inverse problems, considered bellow, will not depend to the a-priory information about the radiation properties of the heater's material.

The results of temperature measurements inside the specimen are assigned as necessary additional information to solve an inverse problem

$$T^{exp}(x_m, \tau) = f_m(\tau), \quad \mathrm{m} = \overline{1, \mathrm{M}} \tag{5}$$

With the presented statement of inverse problem, the data gained in one experiment are not sufficient for simultaneous recovery of three thermal and radiative characteristics (thermal conductivity, heat capacity and emissivity), because data by values of the heat flux applied to a specimen are also needed.

The execution of the single experiment is not enough to provide the conditions of uniqueness of the inverse problem solving by simultaneous estimating of thermal conduction, heat capacity and emissivity of the testing material. To solve this problem the data of several N (in partial three) similar experiments with equal material specimen and different heating regimes were processed simultaneously.

The experimental equipment and the method described below could be applied for estimating of material's three characteristics; the availability of a few specimens of the material allows us to provide uniqueness of the solution. This paper is concerned with modification of the approach, presented at [8].

2. INVERSE PROBLEM ALGORITHM

In the inverse problem Eqn.(1)- Eqn.(5) it is necessary first of all to indicate as a temperature range $[T_{\min}, T_{\max}]$ of the unknown functions, which is general for all experiments, and for which the inverse problem analysis has a unique solution. For $T_{\min}$ the minimum value of initial temperature is used. Of much greater importance is a correct sampling of value $T_{\max}$. Proceeding from the necessity to provide uniqueness of solution, it seems possible to sample, for $T_{\max}$, a minimum among maximum temperature values gained on the thermocouple positioned on the heated surface at every testing specimen. The same should be done for the heater temperature range $[T_{h,\min}, T_{h,\max}]$.

Suppose then that the unknown characteristics are given in their parametric form. With this purpose introduce in the interval $[T_{\min}, T_{\max}]$, three uniform difference grids with the number of nodes $N_i,\ i = 1,2,3$.

$$\omega_i = \{T_k = T_{\min} + (k-1)\Delta T, \ \ \mathrm{k} = \overline{1, \mathrm{N_i} - 1}\}, \ \ \mathrm{i} = \overline{1,3} \tag{6}$$

The function

$$B^{(j-1)}=B^{(j-1)}\left(T_k, T_{k+1}, \ldots, T_{k+j}, \tau\right)=\sum_{s=k}^{k+j}\frac{\left(T_s-T\right)_+^{j-1}}{\omega_k^{'}\left(T_s\right)} \tag{6a}$$

where $\omega_k^{'}=\left(T-T_k\right)\left(T-T_{k+1}\right)\ldots\left(T-T_{k+j}\right)$ and $\left(T_s-T\right)_+^{j-1}=\max\left\{0\,,\left(T_s-T\right)^{j-1}\right\}$ is called B-spline of the (j-1) degree relatively with nodes $T_k, T_{k+1}, \ldots, T_{k+j}$.

When solving practical problems, B-splines are used with so-called "natural" boundary conditions

$$u^{''}\left(T_{\min}\right)=u''\left(T_{\max}\right)=0 \tag{6b}$$

where u is desired function.

Then, in case of cubic B-splines (j-1=3), the unknown function is presented as

$$u_i\left(\tau\right)=\sum_{k=1}^{N_i}u_k\varphi_{i,k}\left(T\right) \tag{6c}$$

$$\varphi_1\left(T\right)=2B_0\left(\overline{T}+\Delta T\right)+B_0\left(\overline{T}\right) \tag{6d}$$

$$\varphi_2\left(T\right)=-B_0\left(\overline{T}+\Delta T\right)+B_0\left(\overline{T}-\Delta T\right) \tag{6e}$$

$$\varphi_k\left(T\right)=B_{k-1}\left(\overline{T}\right),\ k=3,\ldots,N_i-2 \tag{6f}$$

$$\varphi_{N_i-1}\left(T\right)=B_0\left(\overline{T}-(N_i-2)\Delta T\right)-B_0\left(\overline{T}-N_i\Delta T\right) \tag{6g}$$

$$\varphi_{N_i}\left(T\right)=B_0\left(\overline{T}-(N_i-1)\Delta T\right)+2B_0\left(\overline{T}-N_i\Delta T\right) \tag{6h}$$

where

$$B_k\left(T\right)=B_0\left(\overline{T}-k\Delta T\right) \tag{6i}$$

$$\overline{T}=T-T_{\min}\ ;$$

$$B_0\left(T\right)=\left\{\left(T+2\Delta T\right)_+^3-4\left(T+\Delta T\right)_+^3+6\left(T\right)_+^3-4\left(T-\Delta T\right)_+^3+\left(T-2\Delta T\right)_+^3\right\}/\left(6\Delta T^3\right) \tag{6j}$$

The function $B_0\left(T\right)$ has the property

$$B_0\left(T\right)=\begin{cases}>0, & if\ \ -2\Delta T<T<2\Delta T\ ;\\ =0, & if\ \ \left|T\right|\geq 2\Delta T\ .\end{cases} \tag{6k}$$

This property makes the computational algorithm simpler.

Approximating the unknown functions on grids Eqn. (6) using the cubic B-splines

$$C(T)=\sum_{k=1}^{N_1} C_k \varphi_k^1(T)$$

$$k(T)=\sum_{k=1}^{N_2} k_k \varphi_k^2(T)$$

$$\varepsilon(T)=\sum_{k=1}^{N_3} \varepsilon_k \varphi_k^3(T) \tag{7}$$

where C_k, $\mathrm{k}=1,\ldots,N_1$, k_k, $\mathrm{k}=1,\ldots,N_2$, ε_k, $\mathrm{k}=1,\ldots,N_3$ - parameters.

Let's introduce in the interval $\left[T_{h\min}, T_{h\max}\right]$ uniform difference grids with the number of nodes N_4

$$\omega_4=\left\{T_k=T_{h\min}+(k-1)\Delta T_h,\quad \mathrm{k}=\overline{1, \mathrm{N}_4-1}\right\} \tag{6l}$$

and approximate the unknown function $\varepsilon_h(T)$ on grids (6a) using the cubic B-splines

$$\varepsilon_h(T)=\sum_{k=1}^{N_4} \varepsilon_{hk} \varphi_k^4(T) \tag{7a}$$

where ε_{hk}, $\mathrm{k}=1,..,N_4$ - parameters.

As a result of approximation, the inverse problem is reduced to the search of a vector of unknown parameters $\overline{p}=\{p_k\}$, $\mathrm{k}=1,\ldots,N_P$, with dimensions $N_P=N_1+N_2+N_3+N_4$. Writing down a leas-square discrepancy of the calculated and experimental temperature values in points of thermal sensors positioning, than the residual functional will depend to four functions

$$J(\overline{p})=J\left(C(T), k(T), \varepsilon(T), \varepsilon_h(T)\right)=\sum_{n=1}^{N} \sum_{m=1}^{M_n} \int_{\tau_{\min}^m}^{\tau_{\max}^m}\left(T^n\left(x_m^n, \tau\right)-f_m^n(\tau)\right)^2 d\tau \tag{8}$$

where $T^n(x,\tau)$ is determined from a solution of the boundary-value problem Eqn. (1)- Eqn. (4) for n-th experiment using the approximations of Eqn. (7). It is assumed here that the conditions of uniqueness of the inverse problem solving are satisfied. Bellow to simplify the notation of equations index n will be excluded.

So, proceeding from the principle of iterative regularization [8], [9], [10], the unknown vector can be determined through minimization of functional Eqn. (8) by gradient methods of the first order prior to a fulfilment of the condition

$$J(\overline{p})\leq \delta_f \tag{9}$$

where $\delta_f=\sum_{m=1}^{M} \int_{\tau_{\min}}^{\tau_{\max}} \sigma_m(\tau)\, d\tau$ - integral error of temperature measurements $f_m(\tau)$, $m=1,M,$ and σ_m - measurement variance.

To construct an iterative algorithm of the inverse problem solving a conjugate gradient method was used.

The greatest difficulties in realizing the gradient methods are connected with calculation of the minimized functional gradient. In the approach being developed the methods of calculus of variations are used.

$$J'_{C_k} = -\int_{\tau_{\max}}^{\tau_{\max}} \int_{X_0}^{X_1} \psi(x,\tau)\cdot\varphi_k^1(T)\frac{\partial T}{\partial \tau}\, dx d\tau \qquad k=\overline{1,N_1} \tag{10}$$

$$\begin{aligned}J'_{k_k} = &-\int_{\tau_{\max}}^{\tau_{\max}} \int_{X_0}^{X_1} \psi(x,\tau)\left(\frac{\partial^2 T}{\partial x^2}\cdot\varphi_k^2(T)+\left(\frac{\partial T}{\partial x}\right)^2\cdot\frac{\partial \varphi_k^2}{\partial T}\right)dx d\tau -\\ &-\beta_1\int_{\tau_{\max}}^{\tau_{\max}} \psi(X_0,\tau)\frac{\partial T}{\partial x}\cdot(X_0,\tau)\varphi_k^2(T(X_0,\tau))\,d\tau +\\ &+\int_{\tau_{\max}}^{\tau_{\max}} \psi(X_1,\tau)\frac{\partial T}{\partial x}(X_1,\tau)\cdot\varphi_k^2(T(X_1,\tau))\,d\tau,\end{aligned}$$

$$k=\overline{1,N_2} \tag{11}$$

$$J'_{\varepsilon_k} = -\int_{\tau_{\max}}^{\tau_{\max}} \frac{\psi_{M+1}(X_1,\tau)}{\left(\varepsilon(T)+\varepsilon_h(T_h)-\varepsilon(T)\varepsilon_h(T_h)\right)}\left((1-\varepsilon_h(T_h))K\frac{\partial T}{\partial x}+\sigma\varepsilon_h\left(T_h^4-T^4\right)\right)\varphi_k^3(T)d\tau$$

$$k=\overline{1,N_3} \tag{12}$$

$$J'_{\varepsilon_{hk}} = -\int_{\tau_{\max}}^{\tau_{\max}} \frac{\psi_{M+1}(X_1,\tau)}{\left(\varepsilon(T)+\varepsilon_h(T_h)-\varepsilon(T)\varepsilon_h(T_h)\right)}\left((1-\varepsilon(T))K\frac{\partial T}{\partial x}+\sigma\varepsilon(T)\left(T_h^4-T^4\right)\right)d\tau$$

$$k=\overline{1,N_4} \tag{13}$$

where $\psi(x,\tau)$ - solution of a boundary-value problem adjoint to a linearized form of the initial problem Eqn. (1)- Eqn. (4).

$$-c(T)\frac{\partial \psi_m}{\partial \tau} = k\frac{\partial^2 \psi_m}{\partial x^2}, \qquad x\in(x_{m-1},x_m),\ x_0=X_0,\ x_{M+1}=X_1,\ m=\overline{1,M+1},\ \tau\in(\tau_{\min},\tau_{\max}] \tag{14}$$

$$\psi_m(x,\tau_{\max})=0\ ,\quad x\in[x_{m-1},x_m],\quad m=\overline{1,M+1} \tag{15}$$

$$-k\frac{\partial \psi_{M+1}(X_1,\tau)}{\partial x}(\varepsilon+\varepsilon_h-\varepsilon\varepsilon_h)-\frac{d\varepsilon}{dT}\frac{\partial T}{\partial x}(1-\varepsilon_h)k\frac{\partial T}{\partial x}\psi_{M+1}(X_1,\tau)+$$

$$+\frac{d\varepsilon}{dT}\frac{\partial T}{\partial x}\varepsilon_h\sigma\left(T_h^4-T^4\left(X_1,\tau\right)\right)\psi_{M+1}\left(X_1,\tau\right)-4\varepsilon\varepsilon_h T^3(X_1,\tau)_2\psi_{m+1}\left(X_1,\tau\right)=0, \tag{16}$$

$$\psi_m\left(x_m,\tau\right)=\psi_{m+1}\left(x_m,\tau\right),\quad \mathrm{m}=\overline{1,\mathrm{M}} \tag{179}$$

$$k(T)\cdot\left(\frac{\partial\psi_m\left(x_m,\tau\right)}{\partial x}-\frac{\partial\psi_{m+1}\left(x_m,\tau\right)}{\partial x}\right)=2\left(T\left(Y_m,\tau\right)-f_m(\tau)\right),\quad m=\overline{1,M} \tag{18}$$

3. OPTIMAL EXPERIMENT DESIGN

As has been said above, the developed method of determining the $C(T)$, $k(T)$and $\varepsilon(T)$ of a material necessitates the solution of an ill-posed inverse problem. The accuracy of estimating of the desired properties is determined largely by the experimental merits, and one is connected with the problem of optimal experiment design. When the processing of the experimental data is the solution of ill-posed problems, optimal experiment design essentially entails the sampling of merits that will ensure the best conditioning of the computational algorithm [9]. The available a priori information about the experimental data are used to formulate a certain scalar criterion of optimality$\Phi(\xi)$, which depends on the experiment design ξ and characterizes the conditioning of the algorithm for solving the inverse problem. It is reasonable to assume that $\Phi(\xi)$ has a lower bound on the set of possible designs Σ and that optimal design ξ^* exists such that

$$\xi^*=\arg\inf_{\xi\in\Sigma}\Phi(\xi) \tag{19}$$

In order to formulate the experiment design problem, it is necessary to select the design criterion $\Phi(\xi)$, to identify the experimental merits comprised in its actual design, i.e., the merits that significantly influence the criterion $\Phi(\xi)$, and to formulate the domain of possible designs Σ.
On the basis of the simulation presented at [9]the following set of merits is used in the present paper for the experiment design

$$\xi=\left\{M, x_m, \mathrm{m}=\overline{1,\mathrm{M}}\right\} \tag{20}$$

Where M is the namber of thermocouple and $x_m, \mathrm{m}=\overline{1,\mathrm{M}}$ is the coordinate of the thermocouple installation. In order to solve the problem (11), it is necessary to determine the domain of possible designs Σ. The following considerations must be taken into account in forming this set:
1) $x_m\in\left[X_o; X_1\right]$;
2) $M\in\left[1;\ 3\right]$;

Following [9] the determination of the vector $\overline{p}=\{p\}_1^{N_p}$ can be reduced to the solution of the system of linear equations

$$A\overline{p} = d \tag{21}$$

where (without number of experiments)

$$A = \{a_{kn}\},\ k = \overline{1,N_p},\ n = \overline{1,N_p}\ ,$$

$$a_{km} = \sum_{m=1}^{m} \int_{0}^{\tau_m} \vartheta_k(x_m,\tau)\vartheta_n(x_m,\tau)d\tau$$

where $\vartheta_n(x_m,\tau)$ and $\vartheta_k(x_m,\tau)$ are the corresponded sensitivity functions, and A is the Fisher information matrix of the considered system [9]. Following [9], the determinant of A is adopted as the optimality criterion of the experimental design:

$$\Phi(\xi) = -\det A(\xi). \tag{22}$$

Problem (11) then acquires the form

$$\xi^* = \arg \inf_{\xi \in \Sigma} \left(-\det A(\xi)\right) \tag{23}$$

The optimal design problem (23) is solved by the numerical projective gradient method of optimization. The iterative process is formulated as follows in this case:

1) an initial approximation of the experiment design ξ^0 is specified,

2) the value of the gradient of the functional $\Phi'_\xi(\xi)^S$, the descent step α^S, and the experimental design in the next iteration are computed, where

$$\xi^{s+1} = \xi^s + \alpha^s\left(\Phi'_\xi(\xi)\right)^s,\ s = 0,1,2,...,\ \xi \in \Sigma \tag{24}$$

3) the iterative process is stopped when the optimality criterion has the same value in two successive iterations, i.e., when the following condition is satisfied

$$\left(\left|\Phi^{s+1}(\xi) - \Phi^s(\xi)\right| / \left|\Phi^s(\xi)\right|\right) < \varepsilon^* \tag{25}$$

where $\varepsilon^* > 0$ is the a priori specified relative error of exit from the iterative process.

The size of the step α^S is selected on the basis of the condition

$$\min_{\alpha^s \in R^+,\ \xi \in \Sigma} \Phi\left(\xi^s + \alpha^s\left(\Phi'_\xi(\xi)\right)^s\right) \tag{26}$$

by one of the conventional techniques of one-dimensional optimization.

4. EXPERIMENTAL VERIFICATION

An example to how apply the approach suggested is presented bellow. Given are the results of data processing of specimen experimental investigations with modern composite materials. The investigations were carried out on a set of pairs of specially manufactured specimens (the first in the pair for simultaneous estimating material's heat capacity per and thermal conductivity and the second for determining boundary conditions).

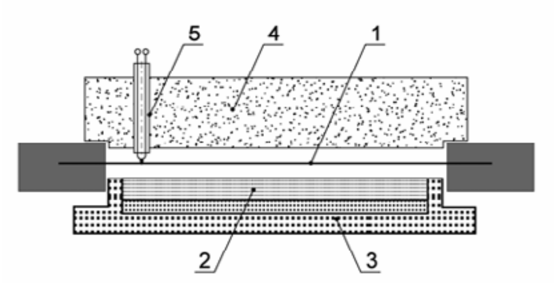

Figure 1: Experimental module: 1 – heater, 2 - test specimen, 3 - insulating basement, 4 - insulating cover, 5 - control thermocouple.

The models of test material are the square plates of 50x50x15 mm (Fig. 2) with four thermocouples installed in the specimen. An installation of thermosensors in specimens was chosen from a solution of the problem of optimal experiment design. The co-ordinates of thermocouple positioning (according optimal experiment design) in the first set of specimens, for estimating the material's thermal characteristics, had the following values: $x_0 = 0\ mm$ (for a boundary condition of the first kind sensor readings on the internal surface were used), $x_1 = 7.5\ mm$, $x_2 = 11.8\ mm$, $x_3 = 15\ mm$ (positioned on the exposed surface). The second set of specimens, for defining the emissivity, has the thermocouples at points $x_0 = 0\ mm$, $x_1 = 12.65\ mm$, $x_2 = 13.65\ mm$, $x_3 = 15\ mm$.

The number of approximation parameters N1, N2, N3 and N4 for every characteristic was assumed to be 5. During specimen heating a theoretically preset time dependence of surface temperature (Fig. 3) was provided. The measurement errors were estimated as 5 %. A comparison of experimentally measured and calculated (with the help of thermal characteristics obtained from a solution of the inverse problem) temperature values in points of thermocouple positioning is shown on Fig. 3 (only for one specimen). The results are in agreement, which shows the robustness of the inverse problem algorithm.

The results proper of the inverse problem solving - the composite material thermal characteristics and emissivity are given on Fig. 4 (the results for two sets by two experiments in vacuum and air). The accuracy of these results of the inverse heat conduction problem was ere verified using different (quite distinct from each other) initial approximations for an iterative process. The results show reasonable agreement.

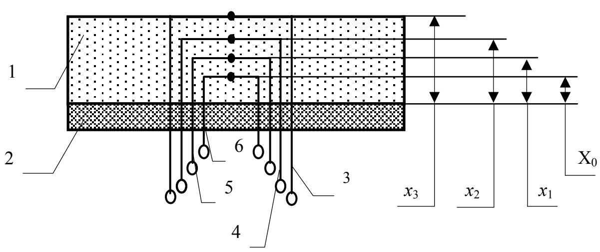

Figure 2: Test specimen: 1 - test materials, 2 - metallic basement, 3-6 - thermocouples.

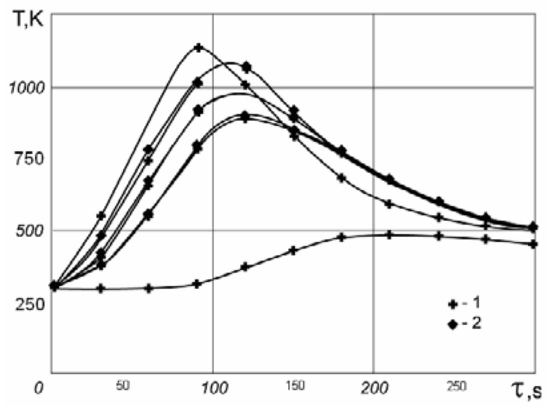

Figure 3: Temperature values in points of thermocouple positioning: 1 - calculated, 2 – experimentally measured

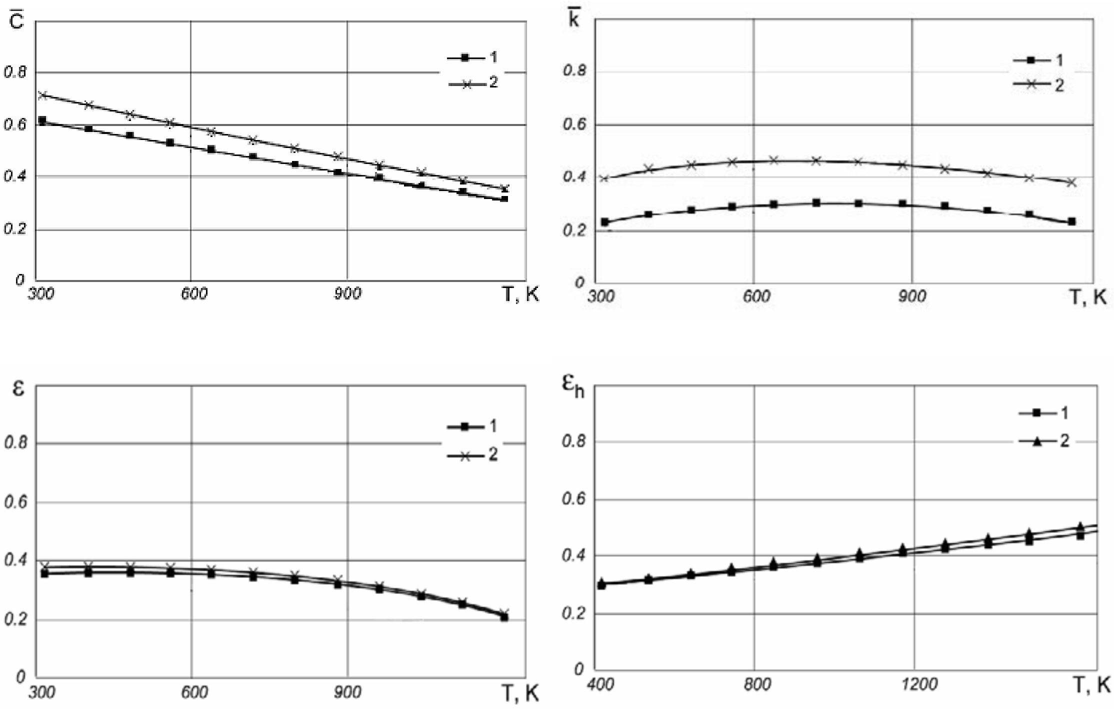

Figure 4: Results of testing: a - heat capacity, b - conductivity, c – emissivity. 1 – vacuum, 2 – air.

CONCLUSIONS

The paper seeks to describe the algorithm developed to process the data of unsteady-state thermal experiments. The algorithm is suggested for determining these unknown on the surface of a slab as a solution of the nonlinear inverse heat conduction problem in an extreme formulation.

The following main factors have an influence on the accuracy of the inverse heat conduction problem (in sequence of significance): the errors in coordinates of thermosensor positions; the errors in values of different characteristics; the errors in estimating the residual level. It was shown that in the cases considered the accuracy of the inverse problems solution is compatible with the errors of the simulated "experimental measurements". Next step in the development of the proposed approach is to consider an estimating interface conductance between periodically contacting surface of specimen and heater foil using the approach similar [11].

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REFERENCES

[1] Artyukhin E.A., Ivanov G.A. and Nenarokomov A.V., "Determination of a complex of materials thermophysical properties through data of nonstationary temperature measurements", High Temperature, Vol. 31, pp.199-202, 1993.
[2] Artyukhin E.A. and Nenarokomov A.V., "Coefficient Inverse heat conduction problem", Journal of Engineering Physics, Vol. 53, pp.1085-1090, 1987.
[3] D.Baillis, M.Raynaud and Sacadura J.F., "Spectral radiative properties of open-cell foam insulation", Journal of Thermophysics and Heat Transfer, Vol. 13, pp.292-299, 1999.
[4] Ho C.-H. and Ozisik M.N., "An inverse radiation problem", Int. Journal Heat and Mass Transfer, Vol. 32, pp.335-342, 1989.
[5] Li H.Y., "Estimation of thermal properties in combined conduction and radiation", Int. Journal Heat and Mass Transfer, Vol. 114, pp.1060-1063, 1999.
[6] Li H.Y. and Ozisik M.N. "Inverse radiation problem for simultaneous estimation of temperature profile and surface reflectivity", Journal of Thermophysics and Heat Transfer, Vol.7, pp.88-93, 1993.
[7] Li H.Y. and Yang C.Y., "A genetic algorithm for inverse radiation problems", Int. Journal Heat and Mass Transfer, Vol. 40, pp.1545-1553, 1997.
[8] Nenarokomov A.V. and Alifanov O.M., "Inverse radiative-conductive problems for estimating material properties", Proc. of the Third International Symposium on Radiative Transfer, Begell House, New York\Wallingford (UK), pp.466-473, 2001.
[9] Alifanov O.M., Artyukhin E.A. and Rumyantsev S.V., Extreme Methods for Solving Ill-Posed Problems with Applications to Inverse Problems, Begell House, New York\Wallinford(UK), 1995.
[10] Alifanov O.M., Inverse Heat Transfer Problems, Springer-Verlag, Berlin \ Heidelberg, 1994.
[11] Orlande H.R.B. and Ozisik M.N., "Inverse problem of estimating interface conductance between periodically contacting surfaces", Journal of Thermophysics and Heat Transfer, Vol. 7, pp.319-325, 1993.
[12] E.A. Artyukhin and A.V.Nenarokomov, "Inverse problem of deriving the integral emissivity of a solid",.High Temperature, Vol.24, pp.725-729, 1986.