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COUPLED ELECTRO-ELASTICITY AT FINITE STRAINS

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Abstract. In this paper we propose a multiplicative split of the deformation gradient into a part related to the elastic behaviour of the material and further one which describes the deformation induced by the electric field. Already available and well tested functions of elastic free energy functions can be immediately deployed without any modifications provided the argument of the function is considered to be the deformation tensor which is defined by the elastic part of the deformation gradient only. An appropriate constitutive relation is formulated for the electrically induced part of the deformation gradient. The approach is elegant, straightforward and above all, provides clear physical insight. A numerical example of highly non-linear coupled deformations demonstrate the potential and strength of the theory.

1 INTRODUCTION

Electroactive polymers (EAP) have been discovered to be very useful, because, in contrast to piezoelectric ceramics, they are less critical with regard to deformability and formability. The field of application as actuators, sensors and energy harvesting devices shows a broad versatility ranging from bio- and micro-manipulation, biomimetic robotics, prosthetics, and smart structures [1, 2].

A model was developed in [3] for an application to electric-sensitive hydrogels which is called the refined multi-effect-coupling electric-stimulus (rMECe) model, wherein the

fixed charge density and finite deformation were considered. An overview about Ionic polymer-metal composites (IPMCs) was given by [4]. An experimental study on dielectric elastomers undertaken by [5] demonstrated that the homogeneous deformation of a layer of a dielectric elastomer subjected to a voltage could be unstable, giving way to an inhomogeneous deformation, such that two regions coexisted within the layer. In [6] an electrical measurement technique was presented for obtaining information on the transient strain in the actuator and analyze the behavior of the actuator in safe and failure operation regimes. Similarly, another electromechanical loading experiments were conducted that captured the large deformation dynamic behavior of axisymmetric dielectric elastomer membranes subjected to dynamic electrical loading and dynamic mechanical loading experiments [7]. A model for a biaxially pre-strained circular actuator was proposed to characterize the electromechanical behavior of a dielectric elastomers [8]. In this approach, the deformation of the actuator for a given activation voltage depended on the three-dimensional mechanical behavior of the film.

The general non-linear electroelasticity equations were developed by [9] and [10]. Constitutive relations of non-linear electroelastic material were formulated by [11] as well as [12]. [13] formulated the variational approach for realistic modelling of EAP, and provided the finite element implementation details. An extension of the theories of electroactive polymers into the realm of generalised continua was recently achieved in [14]. Such continua naturally provide scale effects.

By the very fact that large elastic deformations can be sustained, electroactive polymers lends themselves to frameworks based on the formulation of a free energy function. One of the pressing issues of many of such formulations relates to the adequate consideration of coupling terms, where the deformation tensor, usually considered to be the right *Cauchy-Green* tensor $\mathbf{C}$, and the electric field $\mathbf{e}$ become intertwined. Existing formulations, however, lack transparency. The coupling terms are chosen to be very simple and no real guidance is provided as to the appropriate choice of such terms. To overcome these difficulties the paper embarks on a new formulation based on the idea of splitting the deformation gradient $\mathbf{F}$ in a multiplicative fashion into an elastic (mechanically induced) part and a part induced by the electric field: $\mathbf{F} = \mathbf{F}_{mech}\mathbf{F}_{elec}$. The advantages of such a decomposition become immediately apparent: 1) Existing well established and tested formulations of free energy functions for elastic polymers can be directly adopted provided the function is formulated in terms of deformation tensors only based on $\mathbf{F}_{mech}$. 2) The coupling is directly given by an appropriate constitutive law for $\mathbf{F}_{elec}$ in terms of the electric field. Such formulation becomes transparent, straightforward and can be extended to arbitrary non-linearities. 3) Physical insight regarding the nature of the constitutive law is immediately provided. We note that such a decomposition has been extensively considered in the theory of plastic deformations; see e.g. [15, 16, 17]. The nature of the constitutive law is, however, completely different since plasticity is a dissipative process where the inelastic part of the deformation gradient is defined by means of an integration process.

The paper is organized as follows: In Sec. 2 the basics of electrostatics is presented followed by introducing the multiplicative electromechanical coupling theory. Finally, a numerical example illustrates the capabilities of the formulation.

2 ELECTROMECHANICAL COUPLING

A summary of the standard equations of electromechanical coupling is given in this section. For further details the reader is also referred to [18, 19].

Let$\mathbf{F}$ be the deformation gradient. As a deformation measure we make use of the right *Cauchy-Green* deformation tensor $\mathbf{C}$ defined by

$$\mathbf{C} = \mathbf{F}^T \mathbf{F} \,. \tag{1}$$

In the current configuration the electric field is given by

$$\mathbf{e}_t = -\text{grad}\, \phi \,, \tag{2}$$

where ϕ denotes the electric potential. For dielectric material the electric displacement is expressed as

$$\mathbf{d}_t = \epsilon_0 \, \mathbf{e}_t + \mathbf{p}_t \,, \tag{3}$$

where ϵ_0 denotes the *vacuum electric permittivity* and $\mathbf{p}_t$ the dielectric polarization or simply the polarization. The polarization represents a derived quantity, as it depends on the electric field and on the material and its state which includes density, temperature and strain. The latter essentially means that not only does a dielectric material deform when subjected to an externally induced electric field, but also in turn does the deformation influence the electric field. This is clear, as the polarization generates its own electric field which contributes to the total electric field $\mathbf{e}_t$. In case of linear dielectrics, i.e. the material exhibits a linear response to an applied electric field, the electric displacement (Eq. 3) can be expressed as

$$\mathbf{d}_t = \epsilon_0 \, (1 + \chi_e) \, \mathbf{e}_t = \epsilon \, \mathbf{e}_t \,, \tag{4}$$

where the constant χ_e denotes the *electric susceptibility* and ϵ the material's permittivity. *In vacuo* the electric displacement is reduced to $\mathbf{d}_t = \epsilon_0 \, \mathbf{e}_t$.

In electrodynamics, electric and magnetic fields are governed by *Maxwell's* equations. Here, however, we want to focus on electrostatics and do not consider motion and changes in time. Then, if magnetic fields, free electric currents and charges are not taken into account, only *Gauss's* law for electricity and *Faraday's* law of induction need to be considered:

$$\text{div}\, \mathbf{d}_t = 0 \quad \text{in } \mathcal{B}_t \qquad \text{and} \qquad \text{curl}\, \mathbf{e}_t = \mathbf{0} \quad \text{in } \mathcal{B}_t \,, \tag{5a,b}$$

respectively, where the div and curl denote the divergence and curl operator in the current configuration respectively. The governing equations are supplemented with corresponding Neumann and Dirichlet boundary conditions which read as follows:

$$\mathbf{d}_t \cdot \mathbf{n}_t = -q_t^s \quad \text{on } \partial\mathcal{B}_{t,N}^q \,, \tag{6}$$

and

$$\phi = h_\phi \quad \text{on } \partial\mathcal{B}_{t,D}^\phi \,, \tag{7}$$

respectively, q_t^s denotes the electric surface charge density in the deformed configuration, $\mathbf{n}$ the normal vector on $\partial\mathcal{B}$ with $\mathbf{n}_t$ as its equivalent in the deformed configuration and $\partial\mathcal{B}_D^\phi \subset \partial\mathcal{B}$ and $\partial\mathcal{B}_N^q = \partial\mathcal{B} \setminus \partial\mathcal{B}_D^\phi$.

For later use we also want to provide all needed equations and definitions in the Lagrangian description. For further details the reader is referred to [14]. The electrical field in the reference configuration $\mathbf{e}$ is related to the corresponding quantity in the current configuration $\mathbf{e}_t$ by

$$\mathbf{e} = \mathbf{F}^T \mathbf{e}_t \tag{8}$$

and the electric displacement by

$$\mathbf{d} = J\,\mathbf{F}^{-1}\,\mathbf{d}_t \,, \tag{9}$$

where $J = \det \mathbf{F}$ denotes the Jacobian with det being the determinant. In accordance with Eq. (3) the polarization in the undeformed configuration may be related to its equivalent in the deformed configuration via

$$\mathbf{p} = J\,\mathbf{F}^{-1}\,\mathbf{p}_t \,, \tag{10}$$

assuming that $\mathbf{p}_t$ is given by a constitutive equation. Note that these equations represent "pull back" operations. With Eqs. (3), (8) and (9) the electric displacement in the reference configuration takes

$$\mathbf{d} = \epsilon_0\, J\, \mathbf{C}^{-1}\mathbf{e} + \mathbf{p} \,. \tag{11}$$

The electric field $\mathbf{e}$ with respect to the undeformed configuration is defined via Eqs. (2) and (8) by

$$\mathbf{e} = -\mathrm{Grad}\,\phi \,. \tag{12}$$

Finally, from the governing equations of the electric field and the electric displacement in the Eulerian description, Eqs. (5a,b), and by making use of Eqs. (8) and (9) and

resorting to standard algebraic manipulations, their Lagrangian counterparts are obtained as follows:

$$\operatorname{Curl}\mathbf{e} = \mathbf{0} \quad \text{in } \mathcal{B} \qquad \text{and} \qquad \operatorname{Div}\mathbf{d} = 0 \quad \text{in } \mathcal{B}\,, \tag{13a,b}$$

where the Div and Curl denote the divergence and curl operator in the reference configuration respectively.

The electric Neumann and Dirichlet boundary conditions (Eq. 6) and (Eq. 7) are expressed in the undeformed configuration as

$$\mathbf{d}\cdot\mathbf{n} = -q^s \quad \text{on } \partial\mathcal{B}_N^q \tag{14}$$

and

$$\phi = h_\phi \quad \text{on } \partial\mathcal{B}_D^\phi\,, \tag{15}$$

respectively, where q^s denotes the electric surface charge density in the reference configuration.

To complete the theory, the equilibrium equation in its Lagrangian form is written as

$$\operatorname{Div}(\mathbf{FS}) + \mathbf{b} = \mathbf{0} \quad \text{on } \mathcal{B}\,, \tag{16}$$

and the corresponding Neumann boundary condition is given by

$$\mathbf{FSn} = \mathbf{t}^{(\mathbf{n})} \quad \text{on } \partial\mathcal{B}_N^\sigma\,, \tag{17}$$

where $\mathbf{S}$ is the symmetric second *Piola-Kirchhoff* stress tensor, $\mathbf{b}$ denotes the mechanical body force and $\mathbf{t}^{(\mathbf{n})}$ the external traction. Note that generally the external traction consists of a mechanical and an electric contribution. The electric part is due to the fact that associated with the electric field outside of body $\mathcal{B}$ we find a Maxwell stress field which causes an electric traction force acting on the charges lying on $\partial\mathcal{B}$. For simplicity however, we do not consider an electric field outside of $\mathcal{B}$, here and in the following. Consequently, the traction forces reduce to purely mechanical ones.

3 MULTIPLICATIVE DECOMPOSITION OF THE DEFORMATION GRADIENT

3.1 A modified framework and a multiplicative decomposition

As a starting point we consider purely hyperelastic deformations together with deformations induced by the electric field suggesting the existing of a corresponding coupled free energy function $\Psi_{coupled}(\mathbf{C},\mathbf{e})$ which depends on the deformation tensor $\mathbf{C}$ and the electric field $\mathbf{e}$. The free energy is considered per unit volume. Note that this contribution relates solely to the deformation of the body. Further contributions must be considered to account for the electric field in a non-deforming medium which will be considered at

a later stage. Hence, in a completely uncoupled formulation the dependency on $\mathbf{e}$ will vanish and $\Psi_{coupled}$ will depend on $\mathbf{C}$ alone. The question now arises of how to define $\Psi_{coupled}\,(\mathbf{C}, \mathbf{e})$. The material constants involved are to be determined by experiments. Very often, however, the nature of these coupling terms is not clear from the outset and one is motivated to resort to very simple forms of them. While it is true that one can resort to representation theorems of tensor-valued functions, such representations are by far not adequate as they result in general in a large number of material constants and entails complicated issues regarding convexity and existence of solutions.

Instead of the above approach, in describing the polarization of the dielectric material and to achieve more clarity about a coupled formulation we resort now to an alternative approach and apply here the idea of multiplicatively splitting the deformation gradient into a mechanical (elastic) part and a part induced by the electric field. That is, we consider the multiplicative decomposition of the deformation gradient

$$\mathbf{F} = \mathbf{F}_{mech}\,\mathbf{F}_{elec}\,. \tag{18}$$

With the help of the above split in Eq. (18) a purely mechanical (elastic) right *Cauchy-Green* deformation tensor $\mathbf{C}_{mech}$ can be defined as

$$\mathbf{C}_{mech} = \mathbf{F}_{mech}^{T}\mathbf{F}_{mech} = \mathbf{F}_{elec}^{-T}\,\mathbf{C}\,\mathbf{F}_{elec}^{-1}\,. \tag{19}$$

Now, $\Psi_{coupled}(\mathbf{C}, \mathbf{e})$ can be defined as purely elastic energy given as $\Psi_{mech}(\mathbf{C}_{mech})$ which is to depend solely on $\mathbf{C}_{mech}$. The dependency on $\mathbf{e}$, and so the coupling, is implicit through the dependency of $\mathbf{F}_{elec}$ on $\mathbf{e}$.

To complete the formulation of the free energy we consider the function

$$\Psi(\mathbf{C}, \mathbf{e}) = \Psi_{mech}(\mathbf{C}_{mech}) - \frac{1}{2}\,c_3\,J\,\mathbf{C}^{-1} : \mathbf{e} \otimes \mathbf{e}\,, \tag{20}$$

where the extra term accompanied by the electro-mechanical material constant c_3 accounts for an energy contribution of the nearly rigid material. It refers to the polarization of linear dielectrics as in Eq. (4) but transferred to the reference configuration via Eq. (10). Consequently, it can be assumed $c_3 = \epsilon_0\,\chi_e$.

Now, from well established fundamental thermodynamical considerations (see e.g. [19]) the relation hold:

$$\mathbf{p} = -\frac{\partial \Psi}{\partial \mathbf{e}}\,. \tag{21}$$

Making use of the following relation

$$\frac{\partial \Psi_{mech}}{\partial \mathbf{C}_{mech}} : \frac{\partial \mathbf{C}_{mech}}{\partial \mathbf{e}} = -4\,\left(\mathbf{C}_{mech}\,\frac{\partial \Psi_{mech}}{\partial \mathbf{C}_{mech}}\,\mathbf{F}_{elec}^{-T} : \overset{\mathbf{4}}{\mathbb{C}}_{(elec)}\right)\mathbf{e}\,, \tag{22}$$

the polarization in the un-deformed configuration is given by

$$\mathbf{p} = -\frac{\partial \Psi}{\partial \mathbf{e}} = 4 \left(\mathbf{C}_{mech} \frac{\partial \Psi_{mech}}{\partial \mathbf{C}_{mech}} \mathbf{F}_{elec}^{-T} : \overset{4}{\mathbb{C}}_{elec} \right) \mathbf{e} + c_3 \, J \, \mathbf{C}^{-1} \, \mathbf{e} \,, \tag{23}$$

where the double contraction operation is here defined as follows

$$\left(\mathbf{C}_{mech} \frac{\partial \Psi_{mech}}{\partial \mathbf{C}_{mech}} \mathbf{F}_{elec}^{-T} : \overset{4}{\mathbb{C}}_{elec} \right)_{yz} = \left(\mathbf{C}_{mech} \frac{\partial \Psi_{mech}}{\partial \mathbf{C}_{mech}} \mathbf{F}_{elec}^{-T} \right)_{kn} \mathbb{C}_{(elec)\, knyz} \,. \tag{24}$$

Substituting Eq. (23) into Eq. (11), the electric displacement or electric charge potential is expressed as

$$\begin{aligned} \mathbf{d} &= \epsilon_0 \, J \, \mathbf{C}^{-1} \mathbf{e} + 4 \left(\mathbf{C}_{mech} \frac{\partial \Psi_{mech}}{\partial \mathbf{C}_{mech}} \mathbf{F}_{elec}^{-T} : \overset{4}{\mathbb{C}}_{elec} \right) \mathbf{e} + c_3 \, J \, \mathbf{C}^{-1} \, \mathbf{e} \\ &= \epsilon \, J \, \mathbf{C}^{-1} \mathbf{e} + 4 \left(\mathbf{C}_{mech} \frac{\partial \Psi_{mech}}{\partial \mathbf{C}_{mech}} \mathbf{F}_{elec}^{-T} : \overset{4}{\mathbb{C}}_{elec} \right) \mathbf{e} \,. \end{aligned} \tag{25}$$

Proceeding similarly as in [13, 14] an augmented free energy function per unit volume is formulated as

$$\Psi^A \left(\mathbf{C}, \mathbf{e} \right) := \Psi_{mech}(\mathbf{C}_{mech}) - \frac{1}{2} \, \epsilon \, J \, \mathbf{C}^{-1} : \mathbf{e} \otimes \mathbf{e} \,. \tag{26}$$

which additionally includes the purely electric energy contribution we find *in vacuo* with $\epsilon = \epsilon_0 + c_3$. Then, the electric charge potential (Eq. (25)) is given by

$$\mathbf{d} = -\frac{\partial \Psi^A}{\partial \mathbf{e}} \,. \tag{27}$$

Finally, the stress tensor is given by

$$\mathbf{S} = 2 \, \frac{\partial \Psi^A}{\partial \mathbf{C}} \,. \tag{28}$$

With the help of

$$\frac{\partial \Psi_{mech}}{\partial \mathbf{C}_{mech}} : \frac{\partial \mathbf{C}_{mech}}{\partial \mathbf{C}} = \mathbf{F}_{elec}^{-1} \frac{\partial \Psi_{mech}}{\partial \mathbf{C}_{mech}} \mathbf{F}_{elec}^{-T} \,, \tag{29}$$

we arrive at the expression

$$\mathbf{S} = \mathbf{F}^{-1} \mathbf{P} = 2 \, \mathbf{F}_{elec}^{-1} \frac{\partial \Psi_{mech}}{\partial \mathbf{C}_{mech}} \mathbf{F}_{elec}^{-T} + \epsilon \, J \left[\mathbf{C}^{-1} \, \left(\mathbf{e} \otimes \mathbf{e} \right) \, \mathbf{C}^{-1} - \frac{1}{2} \, \left(\mathbf{C}^{-1} : \mathbf{e} \otimes \mathbf{e} \right) \, \mathbf{C}^{-1} \right] . \tag{30}$$

Note that Ψ_{mech} must be formulated in a way such that $\mathbf{S}$ vanishes at the reference configuration. The theory is complete once expressions have been found for $\Psi_{mech}(\mathbf{C}_{mech})$ and a constitutive law is formulated for $\mathbf{F}_{elec}$ as a function of $\mathbf{e}$. In this paper $\Psi_{mech}(\mathbf{C}_{mech})$ is assumed to be the one proposed in [20, 14] which accounts for the incompressible material behaviour of EAP. It features a non-linear statistically based hyperelastic material law originally developed to describe nearly incompressible behaviour of rubber material. As to $\mathbf{F}_{elec}$ the formulation is addressed in the next subsection.

3.2 A constitutive law for $\mathbf{F}_{elec}$

In what follows we present briefly a constitutive law for $\mathbf{F}_{elec}$ for full details the reader is referred to [21]. Since polymers exhibit almost no volume change during deformations, it is meaningful to assume for incompressibility to hold for $\mathbf{F}_{elec}$. That is $\det \mathbf{F}_{elec} = 1$ and so $\mathbf{F}_{elec} \in SL^+(3, R)$, where $SL^+(3, R)$ is the special linear group defined over the real numbers of matrices with determinants equal one. To achieve this requirement we assume the following form for $\mathbf{F}_{elec}$:

$$\mathbf{F}_{elec} := \exp \mathbf{D}, \qquad \mathrm{tr}\mathbf{D} = 0, \tag{31}$$

where exp denotes the exponential map defined by

$$\exp \mathbf{D} \ = \mathbf{1} + \mathbf{D} + \frac{1}{2!}\mathbf{D}^2 + \frac{1}{3!}\mathbf{D}^3 + \dots \,, \tag{32}$$

$\mathbf{D}$ is a second order tensor given via a separate electro-mechanical constitutive law, and tr denotes the trace operation of a second order tensor. Accordingly, $\mathbf{D}$ must be trace free. The above assumption makes use of the fact that $\det(\exp \mathbf{D}) = \exp(\mathrm{tr}\mathbf{D})$. Obviously the choice $\mathrm{tr}\mathbf{D} = 0$ results in $\det(\exp \mathbf{D}) = 1$. The advantage of the assumption lies in the very fact that it is much easier to fulfil the linear condition of $\mathrm{tr}\mathbf{D} = 0$ than the highly non-linear condition of $\det\mathbf{F}_{elec} = 1$.

We assume now that $\mathbf{D}$ depends only on the electric field vector in the undeformed configuration. An immediate relation is $\mathbf{D} = \mathbb{H}\mathbf{e}$, with $\mathbb{H}$ being a third order tensor. The relation is linear and must satisfy objectivity requirements which essentially mean that the formulation is to be defined in the local co-ordinates and the results then transformed to the global co-ordinate system. Such requirements can be more easily dealt with if we consider a quadratic form such as:

$$\mathbf{D} = \overset{\mathbf{4}}{\mathbb{C}}_{elec} : \mathbf{e} \otimes \mathbf{e} \,, \tag{33}$$

where $\overset{\mathbf{4}}{\mathbb{C}}_{elec}$ denotes a fourth order constitutive tensor and the : operator represents the double contraction of a fourth order tensor with a second order tensor defined as $D_{ij} = \mathbb{C}_{(elec)\,ijkl}\, e_k\, e_l$. The component matrix $\mathbb{C}_{(elec)\,ijkl}$ is considered to be symmetric with respect to indices k, l. In addition, due to the near incompressibility of the material, the resulting deformation in compression and expansion needs to accommodate the volume preservation. The condition results in

$$\mathrm{tr}\,\mathbf{D} = 0 \qquad \Rightarrow D_{11} + D_{22} + D_{33} = 0 \,. \tag{34}$$

Now, as the electro-mechanical constitutive law (Eq. 33) is assumed to account only for contraction and expansion due to the electric field but no shear, consequently, the

component matrix of the electro-mechanical constitutive tensor is sparsely set as shown below:

$$\mathbb{C}_{(elec)\,1111} = \mathbb{C}_{(elec)\,2222} = \mathbb{C}_{(elec)\,3333} = c_1$$

as well as

$$\mathbb{C}_{(elec)\,1122} = \mathbb{C}_{(elec)\,1133} = \mathbb{C}_{(elec)\,2211} = \mathbb{C}_{(elec)\,2233} = \mathbb{C}_{(elec)\,3311} = \mathbb{C}_{(elec)\,3322} = c_2\,.$$

The additional electro-mechanical material constants c_1 and c_2 are required to be determined by experiments. The remaining entries of the component matrix have zero values. Accordingly, with Eq. (34) we find that the two material constants are related by

$$c_2 = -0.5\,c_1\,, \tag{35}$$

which reduces the material constants to only one.

We close this subsection by noting that it is straightforward to extend the dependency of $\mathbf{F}_{elec}$ on $\mathbf{e}$ to account for higher order terms than quadratic, should any experimental evidence suggests that, which reflects further advantages of the present formulation.

4 NUMERICAL EXAMPLES

In the following an examples is presented to demonstrate the applicability of the above theory. For further details about the theoretical framework and the numerical implementation the reader is referred to [21]. As mentioned before, the mechanical part of the constitutive model resorts to a non-linear hyperelastic material law [20, 14]. It has been originally developed to describe nearly incompressible behaviour of rubber material making use of three constants, the shear modulus C_R, the bulk modulus κ and parameter N which addresses the the limited extensibility of the macromolecular network structure of the material. The electro-mechanical coupling of the dielectric material subjected to an electric field is assumed as to respond with contraction in direction of the electric field vector and corresponding transversal expansion. Consequently, the material constant c_1 in Eq. (33) need to have a negative value and c_2 a positive one to reflect the presumed coupling of electric field and mechanical deformation. Note that the choice of those two parameters as well as the electric susceptibility constant χ_e is purely academic due to lack of experimental data.

The example displayed in Fig. 1 is a spherical shell with 6 holes subjected to variable electric potential boundary conditions uniformly applied to rim of both the top and the bottom hole with opposite sign each. Four holes are arranged around the z coordinate axis by rotating lines connected to the origin of the coordinate system and enclosing an angle of $\alpha_{side} = 45^o$ with the positive as well as negative x and y coordinate directions, respectively. The hole at the top and the bottom of the sphere are cut out by rotating lines enclosing an angle $\alpha_{pole} = 30^o$ with the positive and negative z coordinate axis, respectively. Due

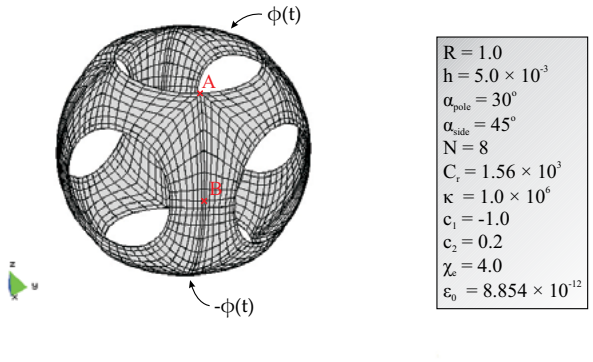

Figure 1: problem definition

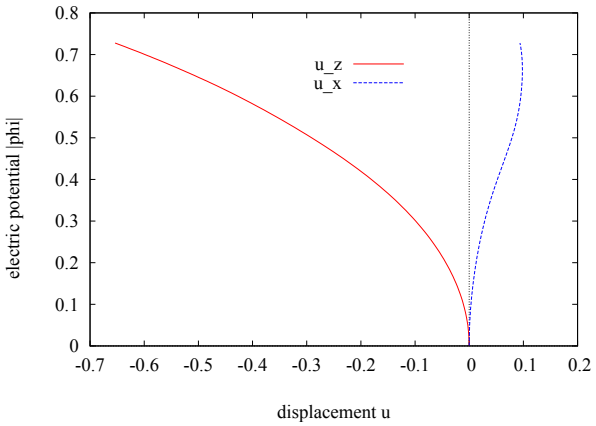

Figure 2: electric potential vs. displacement at point A in z-coordinate direction and point B in x-coordinate direction

to the symmetric configuration of this problem only one eighth of the domain is modelled applying the appropriate symmetry conditions. The problem domain is discretized by 384 particles distributed in 3 equally spaced horizontal layers. Corresponding to the linearly increasing electric potential boundary conditions with opposite sign at the top and bottom hole an we find an electric field which leads to compression of the sphere as depicted in Fig. 3. It can be clearly seen that the at first the compression in z coordinate direction results in expansion in x and y coordinate direction as illustrated in Fig. 2. At the final stage of the simulation the sphere is also contracting in x and y direction.

5 CONCLUSION

In this paper, the electro-mechanically coupled material behaviour is implicitly incorporated via a multiplicative decomposition of the deformation gradient into an elastic

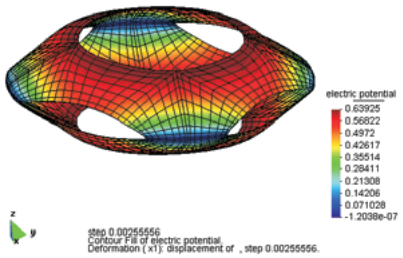

Figure 3: electric potential contour plot of the deformed configuration at $|\phi| = 0.64$ at the rim of the top and bottom hole

and an electrically-induced part. The electric deformation gradient is formulated via a separate constitutive law which provides the flexibility to address different kinds of electromechanical coupling. In fact, in the light of the mentioned decomposition, the formulation of the electromechanical coupling becomes almost straightforward. The strength of the present framework is demonstrated by various numerical examples of large electromechanically coupled deformations based on a meshfree method.

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