

FINITE ELEMENT MODELING OF THERMO-HYDRO-MECHANICALLY (THM) COUPLED PROBLEMS IN FROZEN GROUND ENGINEERING: STATE-OF-THE-ART

YARED W. BEKELE*

*Sustainable Arctic Marine and Coastal Technology (SAMCoT),
Centre for Research-based Innovation - CRI
Norwegian University of Science and Technology, Trondheim, Norway
E-mail: yared.bekele@ntnu.no, www.ntnu.no

Key words: Thermo-Hydro-Mechanical, Coupled Problems, Finite Element, Frozen Ground Engineering.

Abstract. Fully coupled Thermo-Hydro-Mechanical (THM) modeling has been widely studied in various areas of geomechanics, owing to the multiphase nature of geomaterials. Several researches have dealt with THM coupled modeling of geomaterials in high temperature regimes, but a limited work is available for geomaterials in low temperature regimes. A review and summary of existing work in the literature on THM coupled modeling of frozen soils is presented here. THM coupled modeling in general and its applications are pointed out. The basic governing equations of a coupled THM model in general form, namely mass, momentum and energy balance equations, are discussed. A review of fully coupled models is made and the numerical aspects of THM modeling are briefly discussed. A mechanical constitutive model makes up an important component of a fully coupled THM model and a brief review of existing constitutive models for frozen soils is presented. The models reviewed range from elastoplastic models to viscoplastic or creep and damage coupled models. Some models that consider different approaches from the plasticity framework are briefly reviewed. The state-of-the-art is summarized by pointing out the main aspects of THM coupled modeling and directions for future work.

1 INTRODUCTION

Coupled Thermo-Hydro-Mechanical (THM) modeling is essential in several areas of geomechanics where the multiphysics nature and response of the porous medium needs to be well understood. It has been mostly applied in geomechanics of the high temperature regime environment as in [1], [2], [3], [4], [5], [6], [7], [8] and [9]. Some application areas of THM coupled modeling in high temperature regime geomechanics are geothermal energy extraction, safety assessment of nuclear waste repositories, oil and gas reservoir engineering, underground energy storage and CO₂ sequestration. Nowadays, application of THM coupled modeling is gaining popularity in low temperature regime geomechanics, or frozen ground engineering, and has been studied, for instance, by [10], [11], [12], [13] and [14]. Some

specific application areas in frozen ground engineering include frost heave prediction, pipelines buried in permafrost, artificial ground freezing and foundations in cold regions.

In both the high and low temperature regime geomechanics of THM coupled modeling, the main components of the coupled model are the main governing or balance equations and the constitutive equations. The basic governing equations of a coupled model are the mass balance equation, the energy balance equation and the momentum (linear or angular) balance equation. The specific forms of these governing equations strongly depend on the assumptions made for the porous medium in question. The constitutive equations include some known laws for materials, such as Darcy's law for hydraulics and Fourier's law for heat transfer, and the equations of the mechanical constitutive model for deformation. The numerical implementation considers the solutions of both the governing and constitutive equations using, for example, the finite element method.

The state-of-the-art in coupled THM modeling of frozen soils is presented here. The governing equations of a coupled model are first presented in general forms. A summary of the literature review from coupled models for frozen soils is discussed. A brief discussion on the numerical aspects of THM modeling, including solution methods and difficulties, is made referring to the studies by [15], [16], [17], [18], [19], [20], and [21].

The mechanical part of a coupled THM model is completed by the constitutive model. The frozen soil models in the literature vary from simple thermo-elastic models to advanced elastoplastic, viscoplastic/creep and damage coupled models. Some of the elastoplastic models that are reviewed here include [10], [22] and [23]. Other researchers studied viscoplastic/creep modeling of frozen soils such as [24], [25], [26], [27], [28], [29], [30] and [31]. Damage coupled modeling is proposed by a number of studies such as by [32], [33], [34], [35], [36] and [37]. Other models for frozen soils in different frameworks, such as hypoplasticity and fracture, have also been proposed by some studies such as [26], [38], [39], [40], [41] and [42].

The coupled THM models and some of the selected mechanical constitutive models are reviewed and discussed in the following sections to show the state-of-the-art status in coupled THM modeling of frozen soils. A summary and conclusion regarding the state-of-the-art status and the observations made are presented at the end.

2 THM COUPLED MODELING

The present section focuses on the general formulations of THM coupled modeling in geomechanics. Several researches have dealt with THM coupled modeling of geomaterials in high temperature regimes in the different application of THM modeling, but a limited work has been done for low temperature regimes. For both temperature regimes in general, thermo-hydraulic, thermo—mechanical and hydro-mechanical interactions are involved, as depicted in Figure 1. The specific types of interaction depend on the assumptions made for the porous medium under consideration

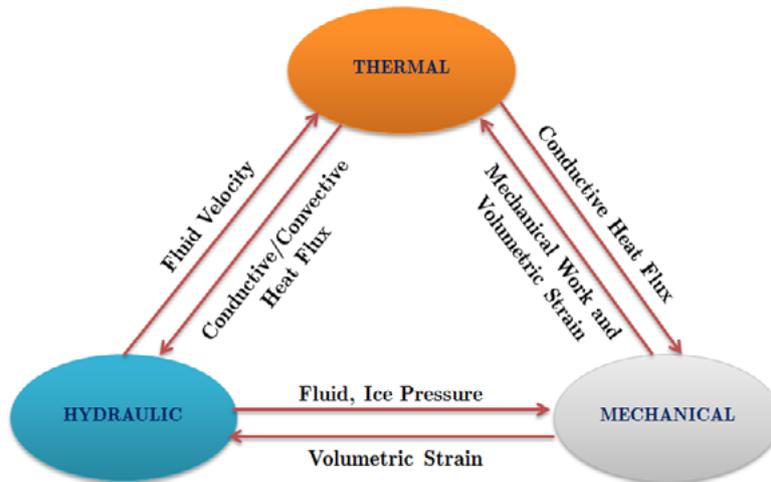


Figure 1: Thermo-Hydro-Mechanical interactions (after [43])

The volume fraction concept and formulation of the governing equations play an important role in the coupled modeling of a porous medium as frozen soil. These are discussed in the next section, in their general forms.

2.1 Volume Fraction Concept

The volume fraction concept is used to create a homogenized media for a multiphase porous medium. In the volume fraction concept, it is assumed that the porous solid always models a control space and that only the liquids and/or gases contained in the pores can leave the control space, [44]. The basis of the description of porous media, using elements of the theory of mixtures restricted by the volume fraction concept, is the model of a macroscopic body, where neither a geometrical interpretation of the pore-structure nor the exact location of the individual components of the body or constituents are considered.

The formulation of the volume fraction concept from [44] is presented here. The porous medium in a representative volume element is assumed to consist of constituents φ^α , with real volumes v^α , where index α denotes κ individual constituents. The concept of volume fractions can be formulated as follows:

$$n^\alpha(\mathbf{x}, t) = \frac{dv^\alpha}{dv}, \tag{1}$$

where \mathbf{x} is the position vector of the actual placement and t the time. The volume elements of the real materials and the bulk volume are denoted by dv and dv^α . The volume fractions n^α in Eq. (1) satisfy the volume fraction condition for κ constituents φ^α ,

$$\sum_{\alpha=1}^{\kappa} n^\alpha = 1 \tag{2}$$

The volume fraction concept assumes that each constituent occupies the whole representative element. Let the real density and the partial density of the constituent materials

be denoted as:

$$\rho^{\alpha R} = \rho^{\alpha R}(\mathbf{x}, t), \rho^\alpha = \rho^\alpha(\mathbf{x}, t) \quad (3)$$

The above equation indicates that:

$$\rho^\alpha(\mathbf{x}, t) = n^\alpha(\mathbf{x}, t)\rho^{\alpha R}(\mathbf{x}, t) \quad (4)$$

The volume fraction concept is important in formulating the governing equations of a THM coupled model. Due to this concept, all geometric and physical quantities, such as motion, deformation, and stress, are defined in the total representative element, and thus, they can be interpreted as the statistical average values of the real quantities.

2.2 Governing Equations in General Form

For THM coupled modeling of a porous medium like a frozen soil, balance equations – balance of mass, balance of linear momentum and moment of momentum, as well as balance of energy – have to be established for each constituent φ^α in consideration of all interaction and external agencies. The interaction effects in the sum have to be equal to zero. Such governing equations in general form are included here as given in [44].

Balance of Mass

The principle of mass balance can be formulated in two ways: for the bulk mixture body as a whole and for each individual constituent in such a way that the superposition of the mass balance can be applied. The balance of mass for the individual constituents φ^α requires that the rate of mass φ^α equal a mass term caused by other constituents:

$$\frac{\partial M^\alpha}{\partial t} = \frac{\partial}{\partial t} \left(\int_{B_\alpha} \rho^\alpha dv \right) = \int_{B_\alpha} \hat{\rho}^\alpha dv \quad (5)$$

where $\hat{\rho}^\alpha$ is the mass supply per volume element. The integration in Eq. (5) covers the domain B_α of each individual constituent. Using the transport theorem with \mathbf{v}_α as the velocity of phase α , we can write:

$$\frac{\partial \rho^\alpha}{\partial t} + \nabla \cdot (\rho^\alpha \mathbf{v}_\alpha) = \hat{\rho}^\alpha \quad \text{and} \quad \sum_{\alpha=1}^{\kappa} \left[\frac{\partial \rho^\alpha}{\partial t} + \nabla \cdot (\rho^\alpha \mathbf{v}_\alpha) \right] = \sum_{\alpha=1}^{\kappa} \hat{\rho}^\alpha = 0 \quad (6)$$

Balance of Linear Momentum

The balance principle of momentum states that the material time derivative of the momentum is equal to the sum of external forces. Thus,

$$\frac{\partial I^\alpha}{\partial t} = \frac{\partial}{\partial t} \left(\int_{B_\alpha} \rho^\alpha \mathbf{v}_\alpha dv \right) = \mathbf{f}^\alpha \quad (7)$$

The external force vector can be written as the sum of body forces $\rho^\alpha \mathbf{b}^\alpha$ and surface tractions \mathbf{t}^α , as well as the interaction forces $\hat{\mathbf{p}}^\alpha$ which belong to the volume forces. Using Cauchy's theorem, the divergence theorem and the mass balance principle, the momentum

balance equation for φ^α with velocity \mathbf{v}^α and acceleration \mathbf{a}_α and a summation for the mixture body can be written as:

$$\begin{aligned} \nabla \cdot \boldsymbol{\sigma}^\alpha + \rho^\alpha \mathbf{b}^\alpha + \hat{\mathbf{p}}^\alpha &= \rho^\alpha \mathbf{a}_\alpha + \hat{\rho}^\alpha \mathbf{v}^\alpha \\ \sum_{\alpha=1}^{\kappa} (\nabla \cdot \boldsymbol{\sigma}^\alpha + \rho^\alpha \mathbf{b}^\alpha + \hat{\mathbf{p}}^\alpha) &= \sum_{\alpha=1}^{\kappa} (\rho^\alpha \mathbf{a}_\alpha + \hat{\rho}^\alpha \mathbf{v}^\alpha) \end{aligned} \quad (8)$$

Balance of Energy

The first law of thermodynamics (balance of energy) is the most fundamental relation in the thermodynamics of one-component materials. It states that the sum of the material time derivatives of the internal and kinetic energies equals the rates of the mechanical work and the heat. This balance principle is transferred to the individual constituents. Applying the above statement to the constituents, the following balance principle is obtained:

$$\frac{\partial E^\alpha}{\partial t} + \frac{\partial K^\alpha}{\partial t} = W^\alpha + Q^\alpha + \int_{B_\alpha} \hat{e}^\alpha dv \quad (9)$$

where E^α , K^α , W^α , Q^α and \hat{e}^α are the internal energy, the kinetic energy, the rate of the mechanical energy, the rate of the heat of the constituent φ^α and the energy supply to φ^α caused by all other constituents, respectively. The general expressions for E^α , K^α , W^α and Q^α are given by:

$$\begin{aligned} E^\alpha &= \int_{B_\alpha} \rho^\alpha e^\alpha dv \\ K^\alpha &= \int_{B_\alpha} \frac{1}{2} \rho^\alpha \mathbf{v}_\alpha \cdot \mathbf{v}_\alpha dv \\ W^\alpha &= \int_{B_\alpha} \mathbf{v}_\alpha \cdot \rho^\alpha \mathbf{b}^\alpha dv + \int_{\partial B_\alpha} \mathbf{v}_\alpha \cdot \mathbf{t}^\alpha da \\ Q^\alpha &= \int_{B_\alpha} \rho^\alpha r^\alpha dv - \int_{\partial B_\alpha} \mathbf{q}^\alpha \cdot da \end{aligned} \quad (10)$$

Here, $e^\alpha = e^\alpha(\mathbf{x}, t)$ is the specific internal energy, $r^\alpha = r^\alpha(\mathbf{x}, t)$ is the partial energy source and $\mathbf{q}^\alpha = \mathbf{q}^\alpha(\mathbf{x}, t)$ is the partial heat flux vector.

3 THM COUPLED MODELS FOR FROZEN SOILS

Coupled THM models for frozen soils have been proposed by some researchers such as [10], [11], [12], [13] and [14]. The basic framework almost in all cases is to define the governing balance equations based on varying assumptions and to propose a mechanical constitutive model. Selection of the coupling parameters also makes an important aspect of the fully-coupled model. A full review of some of the fully coupled THM models is presented below.

One of the earliest studies in the fully coupled THM modeling of frozen soils was presented by [13]. The most important, or simplifying, assumptions they made include: 1) the volume of soil particles remains constant in the freezing process, 2) both unfrozen and frozen soil are isotropic, 3) unfrozen soil is an elastic body with a constant Young's modulus and 4) Young's modulus and yield point are independent of the strain rate and confining pressure. The basic moisture transport, heat transport and phase change equations make up the theoretical framework. The stresses and deformations to complete the mechanical part of the fully coupled model are presented as functions of several state variables. Specifically, the creep strain was described according to Prandtl-Reuss law, and Mises equivalent stress was used. The Young's modulus was expressed as a function of temperature. The finite difference method was used to solve the heat and moisture equations and the finite element method for the mechanical equations. Experimental frost penetration and heave were simulated by the proposed model.

[10] presented the formulation and application of the THM coupled finite element analysis of frozen soil. The basic governing equations of mass, momentum and energy balance make up the theoretical formulation. The thermodynamic equilibrium of freezing soil was described by the Clausius-Clapeyron equation for phase change and the freezing characteristic function by the van Genuchten model with saturation. Darcy's law was used for fluid flow in the porous medium. An elastoplastic hardening constitutive model, based on the Barcelona Basic model for unsaturated soils, was proposed, which is briefly discussed in the next section. The proposed fully coupled model was applied to the analysis of frost heave prediction and reasonably good agreements were obtained. CODE_BRIGHT was used for the simulation.

Another fully coupled THM model for frozen soil was proposed by [11]. The effective stress equation for frozen soil was presented as a function of thermal stress, ice swelling and pore pressure. The continuity equation was derived based on the conservation of mass and Darcy's law. The energy conservation equation considered the deformation energy, adsorbed heat energy of soil skeleton and ice, pore pressure energy, adsorbed heat energy of pore water and phase-changing energy. No advanced mechanical constitutive model was included in the fully coupled model but rather simple thermo-elasticity was considered. The numerical implementation was performed by solving the governing equations in a finite element program. A simple thermo-elastic consolidation and numerical modeling of a pile foundation were studied by the proposed model.

A relatively recent attempt in fully coupled THM modeling of frozen soil is made by [14]. The theoretical formulation comprises description of the thermal, hydraulic and stress-strain fields. The thermal field uses the modified Fourier equation with both conduction and convection and empirical equation for the overall thermal conductivity. The hydraulic field uses the mixed type Richard's equation with modification for ice term to describe fluid motion. Analogy of the soil water characteristic curve from unsaturated soils was applied to describe the freezing process. In the stress-strain field, the total strain was defined as the sum of elastic, thermal, phase change of water, change of matric potential and initial strains. COMSOL Multiphysics was used for the numerical simulation and problems where benchmark data are available were analyzed. The proposed model fails to consider the

dependence of viscosity on temperature and there was no proper mechanical constitutive model.

[12] also proposed a fully coupled THM model for a frozen medium. The continuity flow equation was derived from fluid mass balance where the kinematic flow was defined by Darcy's law. To define the equilibrium equation, the total stress was decomposed into effective and hydrostatic components by using the Bishop parameter. The energy conservation equation was obtained by combining energy conservation equations for solid and fluid constituents. For the mechanical part of the model, the Mohr-Coulomb criterion was used to define the yield locus and plastic potential, with associated flow. The governing equations were solved by using the finite element method based on Galerkin's formulation. Temperature distribution and deformation close to a heat source are investigated as an evaluation problem, in addition to simulation of a freeze-thaw experiment.

Fully coupled THM modeling is a relatively new approach for frozen soils compared to other areas of geomechanics, such as unsaturated soils. A few other researchers have also attempted the coupled modeling of frozen soils. [45] discussed the numerical simulation of water-heat coupled movements in a seasonally frozen soil. [43] presented the simulation of a fully coupled THM system in freezing and thawing rock. [46] proposed a model for coupled heat, moisture and stress-field of saturated soil during freezing. Perhaps fully coupled THM modeling is more extensively studied in the geomechanics of high temperature regime geomaterials. The experience and findings from these studies are believed to provide an important background for modeling in low temperature regimes.

4 MECHANICAL CONSTITUTIVE MODELS

A review of mechanical constitutive models for frozen soil is presented in this section. The reviewed models are categorized in to different groups based on the complexity of the model and the plasticity framework used. The models reviewed range from simple thermoelastic models, advanced elastoplastic and viscoplastic models to damage coupled models. These models are discussed separately in the following sections.

4.1 Elastoplastic Models

Elastoplastic modeling of frozen soils has been a subject for some researchers including [10], [22] and [23]. [10] adopt an effective stress definition based on Bishop's parameter where the weighted pore water and ice pressures are subtracted from the total stresses. The proposed elastoplastic constitutive model was developed based on the Barcelona Basic Model (BBM) for unfrozen-unsaturated soils, [47].

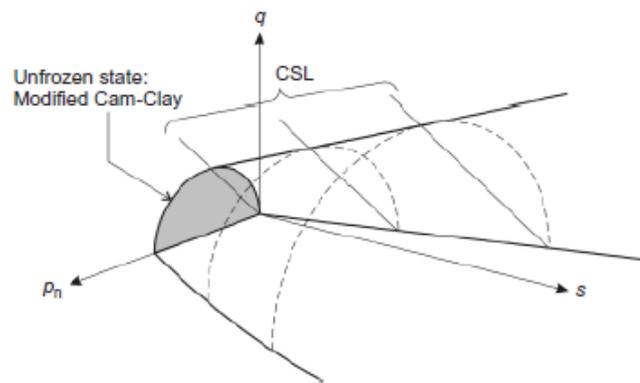


Figure 2: Three-dimensional view of the yield surface proposed by [10]

[22] proposed an elastoplastic frozen soil model based on triaxial compression test results under varying confining pressures and at selected temperatures, implying an isothermal model. They proposed a strength criterion based on a combination of an extended Lade-Duncan strength function in the π -plane and in the p - q plane. Another isothermal elastoplastic model developed based on triaxial compressive test results at a discrete temperature is proposed by [23]. The presented model focuses mainly on frozen soils subjected to high confining pressures and it was observed that the strength of frozen soil increases to a peak value with increasing confining pressure, but with a further increase in confining pressure, the strength decreases due to pressure melting and crushing phenomena.

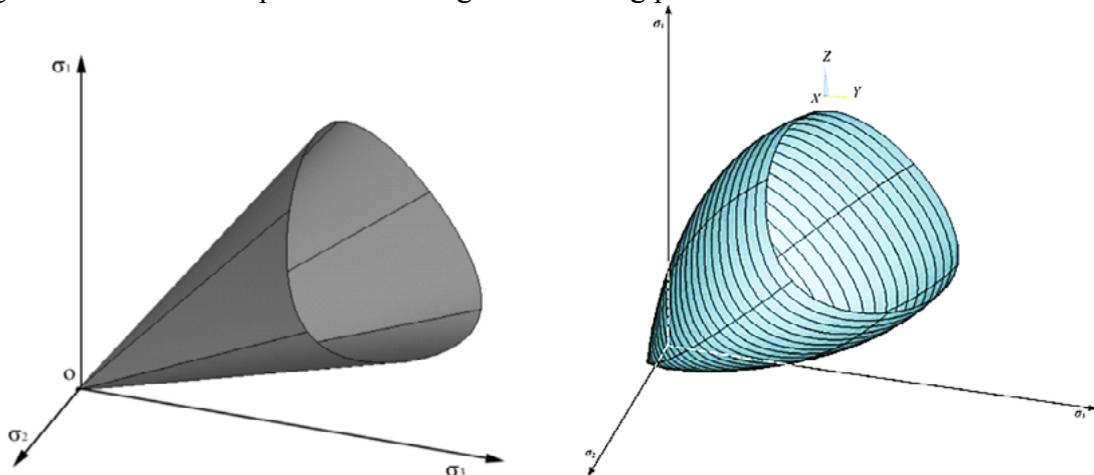


Figure 3: The Lade-Duncan yield surface for unfrozen sand (Left) and yield surface for frozen silt in principal stress space, based on the Lade-Duncan yield surface proposed by [22]

4.2 Viscoplastic/Creep Models

Viscoplastic or creep modeling of frozen soils has been studied by some researchers as found from the literature study. Some of the authors that studied this include [24], [25], [26], [27], [28], [29], [30] and [31].

[24] studied creep of artificially frozen soil by conducting a series of experiments at

discrete temperatures and by proposing an isothermal viscoplastic constitutive model based on the results from the experiments. [25] proposed a combined creep and strength model by a single (unified) constitutive equation. The model was developed for the entire primary, secondary and tertiary creep stages and the long-term strength of frozen soil under multiaxial stress at both constant stresses and constant strain rates. [26] presented a brief review of some frozen soil creep models and proposed a simple hypoplastic constitutive model. [27] discussed the creep behavior of frozen soils starting with uniaxial state of stress and proposed expressions for the effect of temperature on creep rate and strength.

4.3 Damage Coupled Models

A number of researches claim that failure in frozen soils can be associated with damage. Some of the studies that deal with damage coupled modeling of frozen soils include [32], [33], [34], [35], [36] and [37].

[36] proposed a statistical damage constitutive model for warm frozen clay based on the Mohr-Coulomb criterion. They argue that there is a need for stochastic damage modeling of frozen soils due to the microdefects randomly distributed in frozen soils leading to randomness of the mechanical properties. [33] also studied a damage statistical constitutive model and stochastic simulation for warm ice-rich frozen silt. [35] developed a constitutive model of frozen soil with damage and studied a numerical simulation for the coupled problem. One of the earliest studies which tried to couple creep of frozen soil with damage mechanics was presented by [34]. The modeling was based on experimental study and microscopic observation.

4.4 Other Models

The most common frozen soil models in the well-known plasticity frameworks of elasticity, elastoplasticity and viscoplasticity have been discussed in the previous sections, including damage coupled models. A few other models that consider different approaches are available in the literature. A model based on the hypoplasticity framework is proposed by [26], where a brief review of existing creep models is also presented. [38] proposed a nonlinear fracture model for frozen soil and presented the corresponding numerical simulation. [39] used the method of temperature-time analogy to determine the long-term strength of frozen soil in triaxial compression. A photo-viscoelastic creep modeling approach for frozen soils was proposed by [40]. [41] discussed the constitutive modeling of saturated frozen silt in torsion. The triaxial creep modeling of frozen soil under dynamic loading was studied by [42].

5 NUMERICAL ASPECTS OF THM COUPLED MODELING

Coupled numerical modeling of thermal, hydraulic and mechanical properties together is observed to exhibit a highly nonlinear behavior. The governing and constitutive equations involved in formulating a fully coupled THM model are usually complex and large in number. Solving such equations using the finite difference or the finite element method has the potential to create several numerical difficulties.

Studying the numerical difficulties and proposing optimized modeling approaches and solution methods for coupled THM models has been the subject of a number of researchers. [15] proposed a parallel finite element scheme for THM coupled problems in porous media. A parallel approach was also presented by [16]. [17] discussed the numerical difficulties and computational procedures for THM coupled problems of saturated porous media. [18] proposed an objected-oriented finite element analysis of THM coupled problems in porous media. [19] and [20] used an algebraic multigrid method for solving coupled THM coupled problems. [21] discussed the application of standard and staggered Newton schemes in THM coupled problems.

6 SUMMARY AND CONCLUSIONS

A state-of-the-art review of coupled THM models for frozen soils has been presented. The applications and components of a coupled THM model were discussed and the governing equations in general form were presented. A review of the related literature was performed in two parts. Firstly, studies which presented coupled THM models for frozen soils were reviewed and discussed. And secondly, separate mechanical constitutive models for frozen soil varying from simple thermoelastic models to advanced elastoplastic, viscoplastic or creep and damage coupled models were reviewed. The numerical aspects of THM coupled modeling and mechanical constitutive models outside the well-known plasticity framework were briefly mentioned.

The following observations have been made from the state-of-the-art review of THM coupled modeling of frozen soils:

- The governing equations of a coupled THM model strongly depend on the assumptions made for the multiphase porous medium such as phase composition, saturated or unsaturated, mechanisms of heat transfer etc.
- THM coupled modeling in geomechanics has been widely studied in the high temperature regime but is gaining popularity for the low temperature regime i.e. frozen ground engineering.
- All the mechanical models reviewed here deal with isothermal conditions i.e. they are based on a discrete temperature, or experiments at a constant temperature. Non-isothermal constitutive modeling remains a challenge.
- Mechanical constitutive modeling of frozen soil has been studied under the various branches of the plasticity framework i.e. elasticity, elastoplasticity, viscoplasticity or creep and damage. Damage coupled modeling was frequently used in recent studies.
- Numerical solution of the coupled governing and constitutive equations shows a strongly nonlinear behavior and usually results in numerical difficulties during implementation.

In general, there remain several challenges both in the theoretical formulation and the numerical implementation of THM coupled models for frozen soils. A further study aims to look at some of the challenges and contribute to a progress in the research topic.

ACKNOWLEDGEMENT

The author wishes to acknowledge the support from the Research Council of Norway through the Centre for Research-based Innovation SAMCoT and the support from all SAMCoT partners, and particularly Knut V. Høyland for reviewing the paper and Thomas Benz for discussions.

REFERENCES

1. Nowak, T., et al., *Coupled 3-D thermo-hydro-mechanical analysis of geotechnological in situ tests*. International Journal of Rock Mechanics and Mining Sciences, 2011. **48**(1): p. 1-15.
2. Laredj, N., et al., *A Coupled Thermo-Hydro-Mechanical Model of Expansive Clays Subjected to Heating and Hydratation*. Acta Geotechnica Slovenica, 2011. **8**(1): p. 29-37.
3. Dumont, M., et al., *A thermo-hydro-mechanical model for unsaturated soils based on the effective stress concept*. International Journal for Numerical and Analytical Methods in Geomechanics, 2011. **35**(12): p. 1299-1317.
4. Tong, F.G., L.R. Jing, and R.W. Zimmerman, *A fully coupled thermo-hydro-mechanical model for simulating multiphase flow, deformation and heat transfer in buffer material and rock masses*. International Journal of Rock Mechanics and Mining Sciences, 2010. **47**(2): p. 205-217.
5. Liu, Z.J., X.K. Li, and L.Q. Tang, *A coupled chemo-thermo-hydro-mechanical constitutive model for porous media*. Computational Science - ICCS 2007, Pt 3, Proceedings, 2007. **4489**: p. 1210-1213.
6. Wu, W.H., et al., *A thermo-hydro-mechanical constitutive model and its numerical modelling for unsaturated soils*. Computers and Geotechnics, 2004. **31**(2): p. 155-167.
7. Khalili, N. and A.P.S. Selvadurai, *A fully coupled constitutive model for thermo-hydro-mechanical analysis in elastic media with double porosity*. Geophysical Research Letters, 2003. **30**(24).
8. Collin, F., et al., *Thermo-hydro-mechanical coupling in clay barriers*. Engineering Geology, 2002. **64**(2-3): p. 179-193.
9. Rutqvist, J., et al., *Thermohydromechanics of partially saturated geological media: governing equations and formulation of four finite element models*. International Journal of Rock Mechanics and Mining Sciences, 2001. **38**(1): p. 105-127.
10. Nishimura, S., et al., *THM-coupled finite element analysis of frozen soil: formulation and application*. Geotechnique, 2009. **59**(3): p. 159-171.
11. Li, N., et al., *The coupled heat-moisture-mechanic model of the frozen soil*. Cold Regions Science and Technology, 2000. **31**(3): p. 199-205.
12. Neaupane, K.M. and T. Yamabe, *A fully coupled thermo-hydro-mechanical nonlinear model for a frozen medium*. Computers and Geotechnics, 2001. **28**(8): p. 613-637.
13. Mu, S. and B. Ladanyi, *Modeling of Coupled Heat, Moisture and Stress-Field in Freezing Soil*. Cold Regions Science and Technology, 1987. **14**(3): p. 237-246.
14. Liu, Z. and X. Yu, *Coupled thermo-hydro-mechanical model for porous materials under frost action: theory and implementation*. Acta Geotechnica, 2011. **6**(2): p. 51-65.
15. Wang, W.Q., G. Kosakowski, and O. Kolditz, *A parallel finite element scheme for thermo-hydro-mechanical (THM) coupled problems in porous media*. Computers &

- Geosciences, 2009. **35**(8): p. 1631-1641.
16. Vardon, P.J., et al., *Coupled Thermo-Hydro-Mechanical Modelling: A New Parallel Approach*. 2009 Ieee International Symposium on Parallel & Distributed Processing, Vols 1-5, 2009: p. 2534-2542.
 17. Simoni, L., S. Secchi, and B.A. Schrefler, *Numerical difficulties and computational procedures for thermo-hydro-mechanical coupled problems of saturated porous media*. Computational Mechanics, 2008. **43**(1): p. 179-189.
 18. Wang, W.Q. and O. Kolditz, *Object-oriented finite element analysis of thermo-hydro-mechanical (THM) problems in porous media*. International Journal for Numerical Methods in Engineering, 2007. **69**(1): p. 162-201.
 19. Wang, X.C. and B.A. Schrefler, *Fully coupled thermo-hydro-mechanical analysis by an algebraic multigrid method*. Engineering Computations, 2003. **20**(1-2): p. 211-229.
 20. Wang, X.C., Z.J. Ge, and H.Y. Wu, *An algebraic multigrid method for coupled thermo-hydro-mechanical problems*. Applied Mathematics and Mechanics-English Edition, 2002. **23**(12): p. 1464-1471.
 21. Schrefler, B.A., L. Simoni, and E. Turska, *Standard staggered and staggered Newton schemes in thermo-hydro-mechanical problems*. Computer Methods in Applied Mechanics and Engineering, 1997. **144**(1-2): p. 93-109.
 22. Lai, Y.M., et al., *Strength criterion and elastoplastic constitutive model of frozen silt in generalized plastic mechanics*. International Journal of Plasticity, 2010. **26**(10): p. 1461-1484.
 23. Yang, Y.G., et al., *The strength criterion and elastoplastic constitutive model of frozen soil under high confining pressures*. Cold Regions Science and Technology, 2010. **60**(2): p. 154-160.
 24. Li, D.W., J.H. Fan, and R.H. Wang, *Research on visco-elastic-plastic creep model of artificially frozen soil under high confining pressures*. Cold Regions Science and Technology, 2011. **65**(2): p. 219-225.
 25. Fish, A.M., *Creep and Yield Model of Frozen Soil under Triaxial Compression*. Proceedings of the Fifth (1995) International Offshore and Polar Engineering Conference, Vol I, 1995: p. 473-481.
 26. Qi, J., S. Wang, and F. Yu, *A Review on Creep of Frozen Soils*, in *Constitutive Modeling of Geomaterials*, Q. Yang, et al., Editors. 2013, Springer Berlin Heidelberg. p. 129-133.
 27. Ladanyi, B., *An Engineering Theory of Creep of Frozen Soils*. Canadian Geotechnical Journal, 1972. **9**(1): p. 63-80.
 28. Ji, Z.L. and L. Domaschuk, *An Effective Stress Creep Model for Consolidation of Frozen Soil*. 46th Annual Canadian Geotechnical Conference - Conference Proceedings, 1993: p. 301-306.
 29. Domaschuk, L., et al., *Creep of Frozen Sand under Isotropic and Deviatoric Components of Stress*. Journal of Energy Resources Technology-Transactions of the Asme, 1985. **107**(2): p. 199-203.
 30. Domaschuk, L., D.H. Shields, and M. Rahman, *A Model for Attenuating Creep of Frozen Sand*. Cold Regions Science and Technology, 1991. **19**(2): p. 145-161.
 31. He, P., Y.L. Zhu, and G.D. Cheng, *Constitutive models of frozen soil*. Canadian Geotechnical Journal, 2000. **37**(4): p. 811-816.
 32. Lai, Y.M., L. Jin, and X.X. Chang, *Yield criterion and elasto-plastic damage constitutive*

- model for frozen sandy soil*. International Journal of Plasticity, 2009. **25**(6): p. 1177-1205.
33. Lai, Y.M., J.B. Li, and Q.Z. Li, *Study on damage statistical constitutive model and stochastic simulation for warm ice-rich frozen silt*. Cold Regions Science and Technology, 2012. **71**: p. 102-110.
 34. Miao, T.A., X.X. Wei, and C.Q. Zhang, *Creep of Frozen Soil by Damage Mechanics*. Science in China Series B-Chemistry Life Sciences & Earth Sciences, 1995. **38**(8): p. 996-1002.
 35. Zhu, Z.W., J.G. Ning, and W. Ma, *A constitutive model of frozen soil with damage and numerical simulation for the coupled problem*. Science China-Physics Mechanics & Astronomy, 2010. **53**(4): p. 699-711.
 36. Li, S.Y., et al., *An improved statistical damage constitutive model for warm frozen clay based on Mohr-Coulomb criterion*. Cold Regions Science and Technology, 2009. **57**(2-3): p. 154-159.
 37. Lai, Y.M., et al., *Strength distributions of warm frozen clay and its stochastic damage constitutive model*. Cold Regions Science and Technology, 2008. **53**(2): p. 200-215.
 38. Wang, Y.D., et al., *A numerical simulation for nonlinear fracture failure model of frozen soil*. CMESM 2006: Proceedings of the 1st International Conference on Enhancement and Promotion of Computational Methods in Engineering Science and Mechanics, 2006: p. 525-529.
 39. Roman, L.T., et al., *Using Method of Temperature-Time Analogy to Determine Long-Term Strength of Frozen Soil in Triaxial Compression*. Ground Freezing 94, 1994: p. 417-418.
 40. Zhao, X.H., et al., *The photoviscoelastic modeling of the creep of frozen soil*. Progress in Natural Science, 1996. **6**(4): p. 465-471.
 41. Ping, H., et al., *Constitutive relation of saturated frozen silt in torsion*. Ground Freezing 97, 1997: p. 313-316.
 42. Zhu, Y.L., et al., *Triaxial creep model of frozen soil under dynamic loading*. Progress in Natural Science, 1997. **7**(4): p. 465-468.
 43. Neaupane, K.M., T. Yamabe, and R. Yoshinaka, *Simulation of a fully coupled thermo-hydro-mechanical system in freezing and thawing rock*. International Journal of Rock Mechanics and Mining Sciences, 1999. **36**(5): p. 563-580.
 44. Boer, R., *Trends in Continuum Mechanics of Porous Media*. 2005, Dordrecht: Springer. : v.: digital.
 45. Wang, Z.L., et al., *Numerical simulation of water-heat coupled movements in seasonal frozen soil*. Mathematical and Computer Modelling, 2011. **54**(3-4): p. 970-975.
 46. Peng, X.M., X.B. Chen, and Y.Q. Wang, *A Model Coupled Heat, Moisture and Stress-Field of Saturated Soil during Freezing*. Ground Freezing 91, Vol 1, 1991: p. 77-82.
 47. Alonso, E.E., A. Gens, and A. Josa, *A Constitutive Model for Partially Saturated Soils*. Geotechnique, 1990. **40**(3): p. 405-430.