Error Analysis of Programmable Metasurfaces for Beam Steering

Hamidreza Taghvaee, Albert Cabellos-Aparicio, Julius Georgiou, and Sergi Abadal

Abstract—Recent years have seen the emergence of programmable metasurfaces, where the user can modify the electromagnetic (EM) response of the device via software. Adding reconfigurability to the already powerful EM capabilities of metasurfaces opens the door to novel cyber-physical systems with exciting applications in domains such as holography, cloaking, or wireless communications. This paradigm shift, however, comes with a non-trivial increase of the complexity of the metasurfaces that will pose new reliability challenges stemming from the need to integrate tuning, control, and communication resources to implement the programmability. While metasurfaces will become prone to failures, little is known about their tolerance to errors. To bridge this gap, this paper examines the reliability problem in programmable metamaterials by proposing an error model and a general methodology for error analysis. To derive the error model, the causes and potential impact of faults are identified and discussed qualitatively. The methodology is presented and exemplified for beam steering, which constitutes a relevant case for programmable metasurfaces. Results show that performance degradation depends on the type of error and its spatial distribution and that, in beam steering, error rates over 20% can still be considered acceptable.

Index Terms—Error analysis, Programmable Metasurface, millimeter-wave

I. INTRODUCTION

Metamaterials have garnered significant attention in the last decade as they enable unprecedented levels of electromagnetic (EM) control [1] and have opened the door to disruptive advances across domains such as imaging, integrated optics, or wireless communications [2]–[5]. Metasurfaces, the thin-film analog of metamaterials, are generally comprised of a planar array of subwavelength elements over a substrate, i.e., the unit cells, and inherit the unique properties of their 3D counterparts while minimizing bulkiness, losses, and cost. Functionalities such as beam steering, focusing, vorticity control, or RCS reduction have been demonstrated across the spectrum, from microwaves [6]–[8] to terahertz [9]–[13], or optical frequencies [14], [15].

Early works in the field of metamaterials had two main drawbacks, namely, non-adaptivity and non-reconfigurability. This is because, due to their highly resonant nature, unit cells are generally designed for a particular EM function and scope. To address this issue, tunable elements or materials have been introduced in the unit cell design loop in order to provide global or local reconfigurability [16]. Further, recent years have seen the emergence of programmable metasurfaces, this is, metasurfaces that incorporate local tunability and digital logic to easily reconfigure the EM behavior from the outside. Two main approaches have been proposed for the implementation of programmable metasurfaces, namely, (i) by interfacing the tunable elements through an external Field-Programmable Gate Array (FPGA) [17], [18], or (ii) by integrating the tunable elements on their own to control, actuate, and sense the environment, namely, Reconfigurable Intelligent Surfaces (RISs), leading to the interconnection of metasurfaces, the use of machine learning and, eventually, the implementation of software-driven distributed intelligence on EM control [19], [23]–[25].

This has potential to exert a disruptive impact in a plethora of application domains, including but not limited to holographic displays [7], stealth technology [26], active control of surface plasmons [27] or wireless communications [21], [28]. In the latter case, the metasurfaces allow modifying the wireless channel by controlling the direction and phase of reflection. Figure 1 illustrates a plausible scenario where service would normally be disrupted due to blockage, but that metasurfaces are capable of redress by directing the reflections to the user. This is a true paradigm shift for wireless communications, as the recent explosion of works can attest [21], [25], [28]–[32], because the wireless channel has traditionally been an inescapable limiting factor.

Alas, the transition from static to intelligent programmable metasurfaces has come at the cost of added design, fabrication, and embedding complexity. Programmable metasurfaces need to integrate tuning and control elements on a per-cell basis, electronic circuits in order to provide global or local reconfigurability [16]. Further, recent years have seen the emergence of programmable metasurfaces, this is, metasurfaces that incorporate local tunability and digital logic to easily reconfigure the EM behavior from the outside. Two main approaches have been proposed for the implementation of programmable metasurfaces, namely, (i) by interfacing the tunable elements through an external Field-Programmable Gate Array (FPGA) [17], [18], or (ii) by integrating the tunable elements on their own to control, actuate, and sense the environment, namely, Reconfigurable Intelligent Surfaces (RISs), leading to the interconnection of metasurfaces, the use of machine learning and, eventually, the implementation of software-driven distributed intelligence on EM control [19], [23]–[25].

Programmable metasurfaces have opened the door to disruptive paradigms such as Software-Defined Metamaterials (SDMs) and Reconfigurable Intelligent Surfaces (RISs), leading to the interconnection of metasurfaces, the use of machine learning and, eventually, the implementation of software-driven distributed intelligence on EM control [19], [23]–[25]. This has potential to exert a disruptive impact in a plethora of application domains, including but not limited to holographic displays [7], stealth technology [26], active control of surface plasmons [27] or wireless communications [21], [28]. In the latter case, the metasurfaces allow modifying the wireless channel by controlling the direction and phase of reflection. Figure 1 illustrates a plausible scenario where service would normally be disrupted due to blockage, but that metasurfaces are capable of redress by directing the reflections to the user. This is a true paradigm shift for wireless communications, as the recent explosion of works can attest [21], [25], [28]–[32], because the wireless channel has traditionally been an inescapable limiting factor.

Alas, the transition from static to intelligent programmable metasurfaces has come at the cost of added design, fabrication, and embedding complexity. Programmable metasurfaces need to integrate tuning and control elements on a per-cell basis, electronic circuits to implement intelligence within the device, as well as mechanisms to interface the surface with the world. This poses important challenges at fabrication, calibration, deployment, and run time that, among others, affect reliability. In other words, metasurfaces will become prone to failure as they continue integrating sophisticated tuning, control and sensing circuits. However, the impact of faults on the performance of individual metasurfaces is not well understood yet.

We claim that error analysis is a necessary first step to understand the impact of transient or permanent failures on the performance of both a single metasurface and, crucially, a complete system. In the scenario of Figure 1, for instance, faults might lead to inaccuracies in the steering of the reflection and cause a drop in quality of service. Further, an error
Section IV-B analyzes possible causes of errors and derives an error model. Section V-B presents the evaluation methodology, instantiated for the beam steering case. Section VI-F shows the results of the analysis and Section VII concludes the paper.

II. BACKGROUND: PROGRAMMABLE METASURFACES

Metasurfaces are structures generally composed of an array of subwavelength resonators referred to as unit cells. The characteristics of these building blocks determine the metasurface response, this is, its absorption, reflection, and transmission characteristic [4], [39], [40]. With the introduction of tunable or switchable elements, metasurfaces can adapt to different environments or take multiple functionalities [12], [16], [41], [42]. Tuning has been demonstrated in several forms, including thermal [43], electrical [10], [44], and optical tuning [45]; or the use pin diodes [46], varactors [47], memristors [48], and microelectromechanical systems (MEMS) [49].

Determining the characteristics of the unit cell that will lead to the desired behavior is typically performed analytically, through methods such as impedance matching [22]. A large subset of designs has been based on the generalization of the Snell’s laws of reflection and refraction [6], [50]–[52], which provides fundamental understanding on how to achieve certain functionalities through the drawing of specific phase gradients. When analytical methods are not practical, computational optimization methods [53], [54] or even machine learning approaches can be employed [55]–[57]. Another method worth remarking is that of coding metasurfaces, where the metasurface is encoded using a discrete set of unit cell options or states [17]. This allows drawing clear parallelism with information theory, enabling new ways to compose advanced metasurfaces by, for instance, adding or convoluting the codes of two desired EM functionalities [58], [59]. For instance, the design process is shown in Section III-C follows the principles of generalized Snell’s law and coding metasurfaces.

The addition of control methods, together with the tunability on a per-cell basis, enables the programming of coding metasurfaces [17], [60], [61]. The example in [46] uses a Field-Programmable Gate Array (FPGA) to drive the pin diodes of a reconfigurable metasurface, where each pin diode determines the binary state of an individual unit cell. Such a device, however, exemplifies the potential reliability issues of programmable metasurfaces. In this particular case, the pin diodes enabling the reconfigurability may generate a considerable current density in the device, which could lead to electromigration. Also, the FPGA is a potential single point of

![Fig. 1. Graphical illustration of an environment augmented with potentially faulty programmable metasurfaces. Since Line-of-Sight (LoS) propagation is not possible, the metasurfaces attempt to maximize the non-LoS power at the receiver by directing the reflections and altering the phase for coherent detection. A faulty metasurface may lead to service disruption by not pointing the reflected beam accurately.](image)

![Fig. 2. Progression of metasurface design, from static to programmable with fully integrated tuning and control electronics.](image)
failure, rendering the metasurface useless if the FPGA breaks down.

Beyond that, several authors have proposed to integrate a network of communicating chips within the metasurface containing actuators, control circuits, and even sensors [19], [21], [22], [62]. This concept, referred to as HyperSurface (HSF), opens new opportunities in the design of autonomous self-adaptive programmable metasurfaces but, at the same time, poses further challenges in the implementation, co-integration, and testing of the electronics within and around the metasurface.

Figure 3 shows a graphical representation of the HSF structure, illustrating the logic planes of the device. Essentially, the HSF receives external programmatic commands from a gateway controller that are disseminated to the internal control logic at the controller chips via chip-to-chip interconnects and routing logic [63], [64]. These commands contain the state (within the discrete set of possible states) that should be applied to each unit cell. The control logic translates the state into an analog value to be applied to the tuning element, e.g. the voltage applied to a varactor to achieve a target capacitance. Additionally, embedded sensors can pick up data from the environment and send it to the control logic or external devices again via the communications plane.

From the above, it seems clear that the integration of the controller chips, the chip-to-chip network, as well as the sensors and tuning elements, complicate the HSF design from the reliability perspective. As we will see in Section IV-B, each of these components may fail in several ways leading to different consequences.

III. METASURFACE MODEL FOR ANOMALOUS REFLECTION

In this paper, we evaluate the impact of faults in the beam steering capability of a programmable metasurface. To that end, the metasurface needs to implement anomalous reflection with the incidence and reflection angles as potential inputs. Here, we first describe the model used in this work to obtain the EM response of each individual unit cell (Section III-A) and of the complete metasurface in the far field (Section III-B). Finally, we outline the methods to determine the coding of the metasurface, this is the states that need to be applied to each unit cell to direct the beam to a given target direction (Section III-C).

A. UNIT CELL MODEL

Unit cells are generally designed with a certain function in mind. For instance, the anomalous reflection will require unit cells to exhibit a reflection coefficient with high amplitude plus reconfigurable phase to change the angle of reflection [50]. As shown in several works, providing phase reconfigurability can be achieved via several tuning mechanisms [22]. Since one of the aims of this paper is to capture the possible loss of performance arising from component faults, here we provide a particular unit cell design.

The case shown here revolves around the promising application of programmable metasurfaces in millimeter-wave communications for 5G (Fig. 1). We assume a square unit cell (c = 2 mm) with a metallic backplane for operation around 25 GHz, aimed at giving service to one of the available 5G bands according to new recommendations by the International Telecommunication Union (ITU) [66]. A square metallic patch (b = 1.85 mm) is stacked on top of a substrate (Rogers RO4003C) with permittivity ɛr = 3.5 and thickness a = 0.81 mm. It is possible to modify the phase response of the unit cell by adding capacitance to the square metallic patch. For phase tunability, this capacitance is given by varactors, which are embedded within the controllers and hidden under the backplane, but connected to the top patch with vertical vias [20].

We implement the proposed unit cell in a full-wave solver, CST MWS [67], and evaluate the reflection coefficient when the unit cell is illuminated by a normal incident plane wave and for a set of capacitance values. Assuming that our design implements four coding states, it is standard practice in anomalous reflection metasurfaces that the 2π phase range is divided into evenly spaced states with π/2 separation with high reflection amplitude [22]. As shown in Figure 5, the unit cell achieves these objectives around the target frequency, 25 GHz, with a reflection amplitude Φ of 0.9 and phases Φ at {45, 135, 225, 315} degrees. The figure plots the capacitances that have achieved such separation: 0.01 pF, 0.04 pF, 0.06 pF, and 0.9 pF. We will see that, if the capacitances deviate from such values, the unit cell may inaccurately point to different amplitude and phase.
B. Metasurface Model

Beam steering is a particular case of wavefront manipulation that occurs in the far field. As such, the metasurface can be accurately modeled as a compact array following the Huygens principle [68]. This method has been validated in several works via extensive simulations [52]. Considering each unit cell as an element of the array, the far field of the metasurface can be obtained as

$$ F(\theta, \phi) = f_E(\theta, \phi) \cdot f_A(\theta, \phi), \quad (1) $$

where $\theta$ is the elevation angle, $\phi$ is the azimuth angle of an arbitrary direction, $f_E(\theta, \phi)$ is the element factor (pattern function of unit cell) and $f_A(\theta, \phi)$ is the array factor (pattern function of unit cell arrangement). With the widespread assumption of a planar wave covering the entire metasurface, the scattering pattern will depend only on the array factor. For the metasurface with $N \times M$ unit cells, the far field pattern becomes

$$ F(\theta, \phi) = \sum_{m=1}^{M} \sum_{n=1}^{N} A_{mn} e^{j\alpha_{mn}} f_{mn}(\theta_i, \phi_i) $$

$$ \Gamma_{mn} e^{j\Phi_{mn}} f_{mn}(\theta, \phi) e^{j k_0 \zeta_{mn}(\theta, \phi)} \quad (2) $$

where $A_{mn}$ and $\alpha_{mn}$ are the amplitude and phase of the wave incident to the $mn$-th unit cell; $\Gamma_{mn}$ and $\Phi_{mn}$ are the amplitude and phase of the response of the $mn$-th unit cell; $f_{mn}(\theta, \phi)$ denotes the scattering diagram of the $mn$-th unit cell towards an arbitrary direction of reflection, whereas $f_{mn}(\theta_i, \phi_i)$ denotes the response of the $mn$-th unit cell at the direction of incidence determined by $\theta_i, \phi_i$ and $k_0 = 2\pi / \lambda_0$ is the wave number (air is assumed). Finally, $\zeta_{mn}(\theta, \phi)$ is the relative phase shift of the unit cells with respect to the radiation pattern coordinates, given by

$$ \zeta_{mn}(\theta, \phi) = D_u \sin \theta [(m - \frac{1}{2}) \cos \phi + (n - \frac{1}{2}) \sin \phi] \quad (3) $$

We further make the plausible assumption of plane wave incidence, so that factors $A_{mn}$, $\alpha_{mn}$, and $f_{mn}(\theta_i, \phi_i)$ are constants for all $m, n$. Further, we model the scattering pattern of the unit cell over the positive semisphere with the function $\cos(\theta)$, which is a widespread assumption [46]. Finally, and without loss of generality, we consider the normal incidence ($\theta_i = \phi_i = 0$). Then, Eq. (2) becomes

$$ E(\theta, \phi) = K \cos \theta \sum_{m=1}^{M} \sum_{n=1}^{N} \Gamma_{mn} e^{j[\Phi_{mn} + k_0 \zeta_{mn}(\theta, \phi)]} \quad (4) $$

where $K$ is a constant. By controlling the phase shift of the unit cells $\Phi_{mn}$, we can implement anomalous reflection as described next.

C. Metasurface Coding

The coding of the cells of a programmable metasurface allows obtaining the desired functionality. In other words, we need to derive the amplitude $\Gamma_{mn}$ and phase $\Phi_{mn}$ of each unit cell so that the collective response matches with the required functionality. Then, we map the required $\Gamma$ and $\Phi$ to the closest available unit cell states.

In the case of anomalous reflection for beam steering, analytical methods provide high accuracy. Moreover, the error analysis performed in this work requires numerous iterations accounting for diverse combinations of faults—hence, the low computation times of analytical methods are desirable. In this work, we follow the well-known principles of wavefront manipulation whereby a phase gradient is used to determine the direction of reflection [50]. Assuming that the metasurface
imposes the phase profile $\Phi(x, y)$, we assign the virtual wave vector $k_\Phi = \nabla \Phi_x \hat{x} + \nabla \Phi_y \hat{y}$ to the metasurface. In this context, the momentum conservation law for wave vectors can be expressed as

$$
k_\ell \sin \theta_\ell \cos \phi_\ell + \frac{d\Phi_x}{d\phi_\ell} = k_r \sin \theta_r \cos \phi_r
$$

$$
k_\ell \sin \theta_\ell \sin \phi_\ell + \frac{d\Phi_y}{d\phi_\ell} = k_r \sin \theta_r \sin \phi_r
$$

(5)

where $\frac{d\Phi_x}{dx}$ and $\frac{d\Phi_y}{dy}$ describe the gradients in the $x$ and $y$ directions, respectively.

Since we can address any given (oblique) wave with a translation formulation [69], let us consider the normal incident wave case ($\theta_i = \phi_i = 0$) without loss of generality. Assuming air as the medium of the incident and reflected wave, we can simplify the formulation above as

$$
d\Phi_x = \frac{2\pi dx \cos \phi_\ell \sin \theta_\ell}{\lambda_0}, \quad d\Phi_y = \frac{2\pi dy \sin \phi_\ell \sin \theta_\ell}{\lambda_0}
$$

(6)

which express the change in phase ($\Phi_x$ and $\Phi_y$) that needs to be performed per unit of distance ($dx$ and $dy$) in the $x$ and $y$ directions. Then, we set the unit cell size ($dx = dy = D_u$) in Equation (6) to obtain the phase required at $mn$-th unit cell as

$$
\Phi_{mn} = \frac{2\pi D_u (m \cos \phi_\ell \sin \theta_\ell + n \sin \phi_\ell \sin \theta_\ell)}{\lambda_0}
$$

(7)

To assign states to each unit cell, the required phase $\Phi_{mn}$ is calculated for all the unit cells. Then, a closest-neighbor mapping is done between the required phase and that provided by the different unit cell states. For instance, in our particular case where the number of states is $N_s = 4$, we have $\{s_0, s_1, s_2, s_3\}$ with the respective phases of $\{45, 135, 225, 315\}$ degrees, then required phases of $53^\circ$ and $188^\circ$ would be mapped to $s_0$ and $s_2$ states, respectively.

**IV. Error Model**

This section presents the model that we propose for the error analysis of programmable metasurfaces. The model describes both the impact of faults on the behavior of individual unit cells and how the faults can be distributed across the metasurface in Sections IV-A and IV-B, respectively. We also reason about the possible sources of each type of fault and attempt to exemplify a few relevant causes.

Generally speaking, faults in electronic systems may occur for a wide variety of reasons. This is also true in the mixed-signal HSF platform, where the metasurface and its associated tuning, control, and communication subsystems are integrated together. In any case, the relevance of different types of failure will eventually depend, among others, on the maturity of the technology, the manufacturing process, or the application environment. Several examples are outlined below.

For instance, it is widely known that chip failure rates and fabrication mismatches increase as the technology nodes go down [70], which may become necessary in HSFs operating at mmWave and THz frequencies. Manufacturing defects could lead to stuck unit cells, similar to dead pixels in displays. When interconnecting the chips that drive the different unit cells, connector constraints or bad fitting can also lead to errors of different typologies. Once deployed, chip connections might fail over time due to thermal cycling or flexing. Metasurfaces could be exposed to physically challenging conditions that could lead to hard faults, such as physical damage in a conflict zone where bullets could impact the metasurface or bit flips due to cosmic radiation in space applications. Last but not least, ultra-low-power HSFs could power-gate a set of controllers in order to save energy in environments where a given performance degradation is tolerable. Here, the error analysis would help to determine which controllers should be powered off and at which state they should be kept. In any case, power gating can be regarded as an intentional transient fault.

**A. Types of Errors**

Here, we describe the impact that faults can have on the performance of individual unit cells. We assume that each unit cell is assigned a valid state $s \in \Sigma$, where $\Sigma$ represents the set with cardinality $N_s$ of valid states for a particular unit cell design. As shown in Section III-C, the state $s$ basically determines the amplitude and phase of the reflection coefficient at an arbitrary unit cell.

In the presence of a fault, we assume that the unit cell will transition to a state $s'$ which may or may not be within the set of valid states of the metasurface. The value of $s'$ and its probability will depend on the type of fault, that we comprehensively classify as follows (see Figure 6):

- **Stuck at state**: the unit cell is stuck at a random, but valid unit cell state $s' \in \Sigma$. This type of error assumes that a failure disconnects the unit cell from the rest of the system, leaving it in an old state. Such a disconnection can occur due to failures in the communication or control planes (e.g. in the router or in the controller) that prevent control signals to reach the tuning element. The value of $s'$ is picked randomly with uniform distribution.

- **Out of state**: the unit cell takes a random invalid state $s' \notin \Sigma$, which essentially means random amplitude and/or phase. Possible causes of this error may be failures that affect the tuning elements and, thus, lead to a wrong capacitance. Via disconnections arising from manufacturing defects or aging, or defects in the DAC circuits that drive the tuning element, could lead to such error.

- **Deterministic**: The unit cell stays in a known fixed, generally invalid state, which is the same across all unit cells with the same type of fault. For instance, a deterministic error could be caused by a physically damaged unit cell, i.e. a bullet making a small hole within the HSF could be approximated as zero phase and full transmittance.

- **Biased**: The unit cell is at a state which is at a fixed given distance $\Delta$ of the actual required state $s' = s + \Delta \in \Sigma$. This may be caused by flip-bit errors at the computing plane, or by external biases, perhaps due to attacks.

It is worth discriminating the mapping sequence from types of errors. In the end of Section III-C, a process is described which maps the desired phase (in the $[0^\circ, 360^\circ]$ range) to the closest available phase for four states $s_0, s_1, s_2, s_3$. Errors occur after the mapping done by design and, thus, may cause
the unit cells states to shift to valid states (e.g. $s_0, s_1, s_2, s_3$; yet not the intended state for that unit cell) or invalid states (outside the phases mapped to $s_0, s_1, s_2, s_3$).

Next, we illustrate how single component faults can affect the performance of an individual unit cell. In particular, we evaluate the impact of biasing a single varactor to the wrong voltage or completely disconnecting it. To that end, we simulate the different combinations using our unit cell design from Section III-A as a baseline and calculate the phase error as the difference between the correct and erroneous reflection phase.

The results of this example are shown in Figure 7. Red bars represent the impact of Via disconnection with respect to the initial states. Exponential growth from state 1 (0.01 pF) to state 4 (0.9 pF) is observed, revealing that the disconnection of large capacitances has a more significant impact. Blue, black, green and yellow bars indicate the phase error resulting from the wrong biasing of a single Via from the state indicated in the X-axis to state 1, 2, 3, and 4 respectively. For instance, the first black bar indicates the phase error of biasing the Via to state 2 instead of state 1. The impact of each change is subtle and does not follow a clear trend. In this particular example, then, we could approximate single Via failures as an out of state error with a random phase.

B. Spatial Distribution

Next, we describe the possible spatial distribution of errors across the metasurface. We base our reasoning on the fact that faults may impact multiple unit cells or have cascading effects, this is, lead to further faults. We distinguish between the following distributions, represented in Figure 8:

- **Independent**: The errors are randomly distributed over the metasurface and can be modeled with a spatial Poisson process. Individual uncorrelated faults, maybe with different origins, could yield such a distribution.
- **Clustered**: The errors appear around a given area. Cascading effects of a fault or faults that affect several unit cells can lead to such behavior. For instance, faults rendering a controller chip useless will impact all its associated unit cells. Another example would relate to the loss of connectivity at the network: faults in a few interconnects can leave an entire region of the metasurface isolated and stuck in an old state [63].
- **Aligned**: The errors are spatially co-located following a line. For instance, let us assume that power or ground signals are distributed through the HSF through a matrix of electrical lines. We speculate that, in such a case, if one line representing a row or column fails, the whole row and column could be affected.
- **State-specific**: Another speculative type of spatial distribution would be that all unit cells that are supposed to be in a specific state, behave incorrectly. This could happen if the actuator uses an external value (e.g. voltage from a centralized regulator) to determine the given state; if that value is incorrect, the state will be erroneous.

V. METHODOLOGY

A general methodology for the analysis of errors in metasurfaces would simply evaluate the metasurface in the presence of an increasing number of errors, and compare it with the performance of a golden reference. Basically, the golden reference is coded according to the EM functionality and evaluated using the methods exemplified in Section III-C for beam steering. Then, faults are introduced using the guidelines discussed in Section V-A. To better understand the impact of errors, a set of representative metrics is used to characterize the performance degradation as discussed in Section V-B.
A. Introducing Errors

The analytical formulation allows to trivially introduce errors by modifying the terms $\Gamma_{mn}$ and $\Phi_{mn}$ of the affected unit cells in Equation (4). The type of error, its spatial distribution, together with the percentage of faulty unit cells, define the error scenario as represented in Figure 9. To apply a particular error scenario, the steps are:

1) To set the number of faulty unit cells according to the input percentage.
2) To set the position of the faulty unit cells according to the spatial distribution, setting the $m$ and $n$ values in $\Gamma_{mn}$ and $\Phi_{mn}$, using spatial Poisson processes if required.
3) To set the $\Gamma$ and $\Phi$ values of each particular unit cell, either within the discrete set of valid states $\Sigma$ or within a continuous range (valid or invalid states), depending to the type of error and using random number generators if required.

B. Performance Metrics

Far field analysis can provide valuable, yet mainly qualitative information about the impact of errors. To quantify the performance degradation and extract behavioral trends, the use of a set of performance metrics is suggested. In this work, we consider the following performance metrics related to beam steering:

- **Target deviation (TD):** The main purpose of beam steering is to reflect the EM wave toward a determined angle. However, there is usually a difference between the targeted angle and the actual reflected angle due to inaccuracies in the phase profile or, in our case, the appearance of errors. We call this factor target deviation.
and calculate it as the Euclidean distance between the angle pointing to the direction of the reflected main lobe \((\theta_a, \phi_a)\) and the desired angle for steering \((\theta_r, \phi_r)\) as

\[
TD = \sqrt{(\theta_r - \theta_a)^2 + (\phi_r - \phi_a)^2}. \tag{8}
\]

- **Directivity** \(D(\theta, \phi)\): As a fundamental antenna parameter, the directivity describes concentration of energy at a given direction. We evaluate it in specific relevant angles such as the target angle of reflection \(D(\theta_r, \phi_r)\) and the actual angle of reflection \(D(\theta_a, \phi_a)\) (see Figure 11).

- **Secondary lobe level** \(SLL\): The \(SLL\) is defined as the ratio (in dB) of the far field strength in the direction of the side-lobe nearest to the main beam to the far field strength of the main beam.

- **Side Lobe Accumulation** \(SLA\): In addition to the secondary beam, a set of minor reflected beams may appear due to the fundamental operation of MS structure. We measure the accumulation of power within these lobes and report it normalized to the power of the main beam.

- **Half power beam width** \(HPBW\): The waist of the main reflected beam defines the resolution of steering. The HPBW, that calculates the beam width at the -3dB of a normalized lobe, is a conventional factor to assess the beam width.

**VI. RESULTS**

This section applies the proposed methodology on a particular case of beam steering metasurface. We take, as the basic building block, the unit cell described in Section III-A with the four states represented in Figure 5. We consider a metasurface of \(15 \times 15\) unit cells and the far field is obtained with equations from Section III-B. The metasurface is coded to point to \(\theta = 0\) words, deterministic errors react as a mirror reflecting the incident wave according to Snell’s law \((\theta_i = \theta_r\) and \(\phi_i = \phi_r)\) irrespective to the erroneous value, as long as it is the same in all erroneous unit cells. In our case (normal incident of plane wave), where we put \(\theta_i = 0\) and arbitrary \(\phi_i\), we obtain \(\theta_r = 0\) and arbitrary \(\phi_r = 0\).

The next sections complement this qualitative analysis with
the evaluation of performance metrics for different combinations of type of error and spatial distribution. For the sake of brevity, we consider the combinations outlined in Table I. The rest of the possible combinations have been evaluated, but are not shown due to their behavioral similarity with the combinations from Table I.

B. Directivity

Figure 14 illustrates the impact of the different types of errors by plotting the Directivity at the desired reflection angle $D(\theta_r, \phi_r)$ over the error percentage. As expected, the most detrimental type of error is deterministic because all wrong values are mapped to the same phase, which has a more detrimental effect in the beam steering case due to its phase-gradient requirements. This reasoning also implies
that different types of errors may have a completely different impact on metasurfaces implementing different functionalities: for instance, absorbers may set the same value to all unit cells and, therefore, deterministic errors may not reduce performance vitally. Assuming a 3 dB threshold as the acceptable performance degradation, we observe that CD is the most degrading option as error rates beyond 20% cannot be tolerated, while Out of state and Biased errors (IO, IB, CO and CB) guarantee good performance beyond 40% of erroneous cells.

C. Target Deviation

In this part, we want to emphasize the importance of the differences between the desired and actual position of the main beam. Figure 17 shows the accuracy of the beam steering metasurface versus different error scenarios. As mentioned above, deterministic errors may make the main beam shift from the target direction to specular reflection. This is the main cause of the jump observed at 33% for TD. Beside CS that forces the main beam to deviate, other kinds of errors are not affecting TD considerably. In fact, TD takes values below 2 degrees, which is acceptable for most applications. This suggests that loss of directivity or increase of side lobe may be more critical in the present scenario.

D. Half-Power Beamwidth

From Figure 18 it can be inferred that, on one hand, the HPBW is not affected by the IS, IO and IB combinations of errors. However, their associated types with cluster distributions, CS, CO, and CB smoothly decrease the HPBW. We can relate this effect to the consolidated spot of errors leading to a united constructive response. On the other hand, for Deterministic cases (i.e., ID and CD), a jump at 33% is observed due to the deterministic values of errors. At this turning point, anomalous reflection (i.e. the desired behaviour) is very weak, while specular reflection (emerged from deterministic errors)
is strong enough to take over as the main beam. This is the reason for the jumps for HPBW in Figure 18. In other words, before 33% we have the HPBW of anomalous reflection and after that of specular reflection.

E. Side-Lobe Level

Figure 19 illustrates the performance degradation in terms of side lobe level. It is quickly observed that SLL is the most sensitive performance metric so far. Fundamentally, SLL grows proportionally to the ratio of errors for all kinds. Among the monotonically increasing cases, clustered distributions seem to have a higher impact in this metric, whereas independent and out of state errors are the least relevant. Since random coding [51] leads to random scattering, we argue that uncorrelated random errors generate scattering that does not accumulate as a large secondary lobe. On the other hand, clustered and deterministic errors, which tend to group unit cells together and to apply a uniform state, lead to large secondary lobes.

Deterministic errors lead to a characteristic behaviour that we elucidated with several simulations at specific points for CD and ID. The dip appearing between 29% and 37% is due to the fact that a secondary lobe starts to disappear while another minor lobe begins to rise. At 32%, this minor lobe takes the place of secondary lobe then at 37% main and secondary beams are exchanging their roles and SLL starts to decline once again.

F. Side-Lobe Accumulated Energy

SLL cannot accurately describe the distribution of energy around the main lobe, so we evaluate the SLA metric in Figure 20 to consider the impact of all the minor lobes. This figure demonstrates that, as expected, increasing fault rates reduce the power at the main beam and distribute it around a set of side-lobes. The main lobe is debilitated very similarly for all error combinations. Likewise, deterministic errors exhibit abnormal behaviour for SLA showing a dip between [29%-32%] and decline after 37%. The reason being of such trend is similar to that discussed in Section VI-E.

VII. CONCLUSION

This paper has proposed a general methodology for the error analysis of programmable metasurfaces, where faults are distinguished by their impact on individual unit cells and their spatial distribution across the metasurface. Albeit applicable to any functionality, the methodology has been used to study a beam-steering metasurface at 26 GHz for 5G applications. This case is particularly robust against spatially uncorrelated errors with random values, since these generate mild scattering that does not form detrimental side lobes. For the metasurface under evaluation, the functionality is preserved with minor degradation even if faults affect 1/4 of the unit cells. On the contrary, clustered errors that set all the unit cells to the same state are very detrimental as they favor the formation of powerful side lobes. Overall, these results illustrate the value of the error analysis to guide the design of future programmable metasurfaces by, for instance, demonstrating that 25% of the controller chips could be powered off without reducing the beam steering performance significantly.

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REFERENCES
