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S. Idelsohn, M. Papadrakakisand B. Schrefler (Eds)

MULTIDISCIPLINARY AND MULTIPHYSICS COMPUTATION OF SUPERSONIC FLOW, VIA HYBRID SOLUTIONS FOR COMPRESSIBLE NAVIER-STOKES LAYER

A. NASTASE*

* Aerodynamics of Flight
RWTH, Aachen University
Templergraben 55, 52062 Aachen, Germany
e-mail: nastase@lafaero.rwth-aachen.de

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Abstract. *The author proposes hybrid meshless numerical solutions for the computation of the three-dimensional compressible full partial-differential equations (PDEs) of the Navier-Stokes layer (NSL) over flying configurations (FCs) and for the performing of the aerodynamical global optimal design of their shapes. These hybrid solutions have important analytical properties and the derivatives of the velocity's components can be exactly computed. By using a logarithmic density and by applying of the collocation method, a splitting of the NSL's PDEs occurs and all the physical entities are expressed only as functions of the spectral coefficients of the velocity's components, which are obtained by iterative solving of the impulse PDEs. The viscous-inviscid interaction does not need interface and a weak interaction aerodynamics-structure is proposed.*

1 INTRODUCTION

The author proposes new hybrid numerical solutions for the three-dimensional, compressible, Navier-Stokes layer (NSL) over a flying configuration (FC), in supersonic flow. The velocity's components of these hybrid NSL's solutions are expressed as products between the corresponding analytical determined potential velocity's components of the same FC and polynomial expansions, with arbitrary coefficients, as in [1–3]. The potential solutions are used also as outer flow, at the NSL's edge.

The absolute temperature and the logarithmic density function are also expressed in form of polynomial expansions with arbitrary coefficients, which are used to satisfy the temperature and, respectively, the continuity PDEs, in some chosen points. By using the logarithmic density instead of the density it was possible to split the NSL's PDEs and to express all the physical entities only as functions of spectral coefficients of the velocity's components. By using the collocation method, these coefficients of velocity's components are determined by iteratively solving of impulse PDEs, which are considered now as a linear algebraic system with variable coefficients and which are iteratively solved.

The determined pressure distribution on FC and the load distribution of its structure produce together the deformation of the structure. By cruising flight the pressure of the lower side produces a compression of the structure and, the depression on its upper side, dilates the structure of FC. The deformation of the structure produces the change of the shape of FC and, consequently, the change of the pressure distribution on FC occurs. This iterative process must have a damped character.

2 HYBRID SOLUTIONS FOR THE THREE-DIMENSIONAL COMPRESSIBLE NAVIER-STOKES PDES

The author proposes hybrid numerical solutions for the three-dimensional compressible NSL's PDEs, as in [1–3]. Let us firstly introduce the following spectral variable η

$$\eta = \frac{x_3 - Z(x_1, x_2)}{\delta(x_1, x_2)} \quad . \qquad (0 < \eta < 1) \tag{1}$$

Hereby $Z(x_1, x_2)$ is the equation of the surface of the flattened FC and $\delta(x_1, x_2)$ is the NSL's thickness distribution. The following spectral forms of the axial, lateral and vertical velocity's components u_δ, v_δ and w_δ, of the density function $R = \ln \rho$ (here introduced) and of the absolute temperature T are here proposed, as in [1–3],

$$u_\delta = u_e \sum_{i=1}^{N} u_i \eta^i, \qquad v_\delta = v_e \sum_{i=1}^{N} v_i \eta^i, \qquad w_\delta = w_e \sum_{i=1}^{N} w_i \eta^i,$$

$$R = R_w + (R_e - R_w) \sum_{i=1}^{N} r_i \eta^i \quad , \qquad T = T_w + (T_e - T_w) \sum_{i=1}^{N} t_i \eta^i \quad . \tag{2a-e}$$

Further, the physical equation of ideal gas for the pressure p and an exponential law for the viscosity μ versus the temperature T are used:

$$p \equiv R_g \rho T = R_g e^R T \quad , \qquad \mu = \mu_\infty \left[\frac{T}{T_\infty} \right]^{n_1} \quad . \tag{3a,b}$$

Here R_w and T_w are the given values of R and T at the wall, R_g and T_∞ the universal gas constant and the absolute temperature of the undisturbed flow and n_1 the viscosity exponent, u_e, v_e, w_e, R_e and T_e are the values of u, v, w, R and T at the NSL's edge, obtained from the outer inviscid hyperbolical potential flow and u_i, v_i, w_i, r_i and t_i are their free spectral coefficients, which are used to fulfill the NSL's PDEs.

The non-slip condition on the FC's surface (η=0) is automatically satisfied by the equations (2a-c) and the boundary conditions for the velocity's components at the NSL's edge, written in explicit forms, are the following:

$$u_{N-2} = \alpha_{0,N-2} + \sum_{i=1}^{N-3} \alpha_{i,N-2}\, u_i \quad , \qquad v_{N-2} = \alpha_{0,N-2} + \sum_{i=1}^{N-3} \alpha_{i,N-2}\, v_i \quad ,$$

$$u_{N-1} = \alpha_{0,N-1} + \sum_{i=1}^{N-3} \alpha_{i,N-1}\, u_i \quad , \qquad v_{N-1} = \alpha_{0,N-1} + \sum_{i=1}^{N-3} \alpha_{i,N-1}\, v_i \quad ,$$

$$u_{N} = \alpha_{0,N} + \sum_{i=1}^{N-3} \alpha_{i,N}\, u_i \quad , \qquad v_{N} = \alpha_{0,N} + \sum_{i=1}^{N-3} \alpha_{i,N}\, v_i \quad ,$$

$$w_N = \gamma_{0,N} + \sum_{i=1}^{N-1} \gamma_{i,N}\, w_i \quad . \qquad \text{(4a-g)}$$

Hereby the coefficients $\alpha_{i,j}$ and $\gamma_{i,j}$ are of the forms:

$$\alpha_{0,N-2} = \frac{N^2 - N}{2} \quad , \qquad \alpha_{i,N-2} = -\frac{1}{2} \left\lfloor\, N^2 - N\,(1+2i) + i^2 + i \,\right\rfloor \quad ,$$

$$\alpha_{0,N-1} = -N^2 + 2N \quad , \qquad \alpha_{i,N-1} = N^2 - 2N\,(i+1) + i^2 + 2i \quad ,$$

$$\alpha_{0,N} = \frac{N^2 - 3N + 2}{2} \quad , \qquad \alpha_{i,N} = -\frac{1}{2} \left\lfloor\, N^2 - N\,(3+2i) + i^2 + 3i + 2 \,\right\rfloor \;,$$

$$\gamma_{0,N} = 1 \quad , \qquad \gamma_{i,N} = -1 \quad . \qquad \text{(5a-h)}$$

Additionally, the boundary conditions for the absolute temperature and for the density functions, at the NSL's edge, written in implicit forms, are:

$$\sum_{i=1}^{N} r_i = 1 \quad , \qquad \sum_{i=1}^{N} t_i = 1 \quad . \qquad \text{(6a,b)}$$

The equations (4a-g) are used for the elimination of the seven corresponding spectral coefficients of the velocity's components from the NSL's PDEs and for the update their values in the different steps of iteration.

The PDE of continuity and the boundary condition (6a) are used for the computation of spectral coefficients r_i of the here introduced density function $R = \ln r$

For this purpose the collocation method is used. These coefficients can be expressed only as functions of the spectral coefficients of the velocity's components by solving of a linear algebraic system, namely:

$$\sum_{i=1}^{N} g_{ip}\, r_i = \gamma_p \qquad (p=1,\ 2,\ ...,\ N) \quad , \tag{7a}$$

$$r_p = \frac{\Delta_p}{\Delta} \quad . \qquad (p=1,\ 2,\ ...,\ N) \tag{7b}$$

Further, the following notations are introduced:

$$L = u_\delta \frac{\partial R_w}{\partial x_1} + v_\delta \frac{\partial R_w}{\partial x_2} + w_\delta \frac{\partial R_w}{\partial x_3} \quad , \qquad Q = -\left[\frac{\partial u_\delta}{\partial x_1} + \frac{\partial v_\delta}{\partial x_2} + \frac{\partial w_\delta}{\partial x_3}\right] \quad ,$$

$$M = u_\delta \left[\frac{\partial R_e}{\partial x_1} - \frac{\partial R_w}{\partial x_1}\right] + v_\delta \left[\frac{\partial R_e}{\partial x_2} - \frac{\partial R_w}{\partial x_2}\right] + w_\delta \left[\frac{\partial R_e}{\partial x_3} - \frac{\partial R_w}{\partial x_3}\right] \quad ,$$

$$S = \left(R_e - R_w\right)\left[u_\delta \frac{\partial \eta}{\partial x_1} + v_\delta \frac{\partial \eta}{\partial x_2} + w_\delta \frac{\partial \eta}{\partial x_3}\right] \quad .$$

It results in:

$$g_{ip} = \left(\eta^{i-1}(M\ \eta + i\ S)\right)_p \quad , \qquad \gamma_p = (Q-L)_p \qquad \text{if} \qquad 1 \le p \le N-1 ,$$

$$g_{ip} = 1 \quad , \qquad \gamma_p = 1 \qquad \text{if} \qquad p = N \ .$$

In this moment the NSL's PDEs are split.

The PDE of the absolute temperature and the boundary condition (6b) are used for the computation of spectral coefficients t_i of the absolute temperature T. For this purpose the collocation method is also used. These coefficients can be expressed only as functions of the spectral coefficients of the velocity's components by numerical solving of a transcendental algebraic system, it is:

$$\sum_{i=1}^{N} h_i\, t_i + h_0\, T^{n_1} = \theta \quad , \qquad \sum_{i=1}^{N} h_{ip}\, t_i + h_{0p} \left(T^{n_1}\right)_p = \theta_p \quad . \qquad (p=1,\ 2,\ ...,\ N) \tag{8a,b}$$

$$h_{ip} = \left(h_i\right)_p \quad , \qquad h_{0p} = \left(h_0\right)_p \qquad \theta_p = (\theta)_p \qquad \text{if} \qquad 1 \le p \le N-1 .$$

$$h_{ip} = 1 \quad , \qquad h_{0p} = 0 \qquad \theta_p = 1 \qquad \text{if} \qquad p = N \quad .$$

The impulse PDEs are used for the determination of the spectral coefficients of the velocity's components. If the spectral forms (2a-c) of velocity's components, (2d,e) of the density function and of absolute temperature are taken into consideration, the seven spectral coefficients, obtained by writing the boundary conditions in explicit form (4a-g), are eliminated and the collocation method is used, the spectral coefficients of the velocity's components are obtained by the iterative solving of a linear algebraic system with variable coefficients:

$$\sum_{i=1}^{N-3} (\widetilde{A}_{ik}^{(1)} u_i + \widetilde{B}_{ik}^{(1)} v_i) + \sum_{i=1}^{N-1} \widetilde{C}_{ik}^{(1)} w_i = \sum_{i=1}^{N-3} u_i \left[\sum_{j=1}^{N-3} (\widetilde{A}_{ijk}^{(1)} u_j + \widetilde{B}_{ijk}^{(1)} v_j) + \sum_{j=1}^{N-1} \widetilde{C}_{ijk}^{(1)} w_j \right] - \widetilde{D}_k^{(1)} ,$$

$$\sum_{i=1}^{N-3} (\widetilde{A}_{ik}^{(2)} u_i + \widetilde{B}_{ik}^{(2)} v_i) + \sum_{i=1}^{N-1} \widetilde{C}_{ik}^{(2)} w_i = \sum_{i=1}^{N-3} v_i \left[\sum_{j=1}^{N-3} (\widetilde{A}_{ijk}^{(2)} u_j + \widetilde{B}_{ijk}^{(2)} v_j) + \sum_{j=1}^{N-1} \widetilde{C}_{ijk}^{(2)} w_j \right] - \widetilde{D}_k^{(2)},$$

$$\sum_{i=1}^{N-3} (\widetilde{A}_{ik_1}^{(3)} u_i + \widetilde{B}_{ik_1}^{(3)} v_i) + \sum_{i=1}^{N-1} \widetilde{C}_{ik_1}^{(3)} w_i = \sum_{i=1}^{N-3} w_i \left[\sum_{j=1}^{N-3} (\widetilde{A}_{ijk_1}^{(3)} u_j + \widetilde{B}_{ijk_1}^{(3)} v_j) + \sum_{j=1}^{N-1} \widetilde{C}_{ijk_1}^{(3)} w_j \right] - \widetilde{D}_{k_1}^{(3)}. \tag{9}$$

3 COMPUTATION OF AERODYNAMIC CHARACTERISTICS OF A WEDGED DELTA WING MODEL AND THE COMPARISON WITH EXPERIMENTAL RESULTS

Let us consider the wedged delta wing model presented in the (Fig. 1).

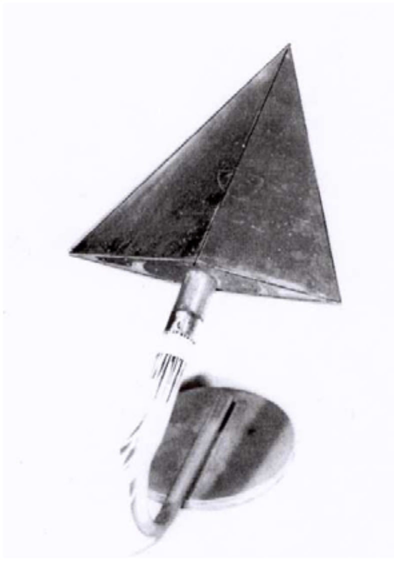


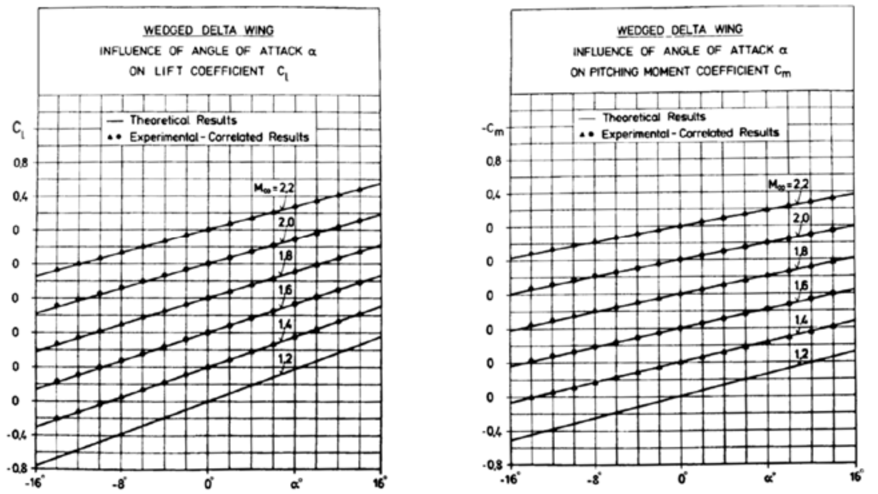
Figure 1: The Wedged Delta Wing Model

This model has the following geometrical characteristics:

- $b = 16.703\ cm$ the maximal span
- $h_1 = 17.362\ cm$ the maximal depth
- $S_0 = 145\ cm^2$ the area of its planform
- $\delta = 25.7^\circ$ the angle of aperture in the planform
- $\gamma = 5.62^\circ$ the angle of aperture in the vertical symmetry plane Ox_1x_3
- $\ell = 0.481$ the dimensionless span
- $\tau = 0.0946$ the relative volume

The lift and pitching moment coefficients, computed by using an own developed inviscid solver, are in very good agreement with the experimental- correlated results obtained in the trisonic wind tunnel of the DLR Cologne, in the frame of one of the research projects of the author, sponsored by the DFG. These agreements can be seen in the (Figs. 2a,b) , for the all ranges of angles of attack ($\alpha = -16^\circ \div 16^\circ$) and Mach numbers ($M_\infty = 1.25 \div 2.2$) taken here into consideration. For these ranges of α and M_∞ the wedged delta wing model has subsonic leading edges. These agreements between theory and experiment lead to the following remarks:

- if the FC is flattened enough and flies at moderate angles of attack, the more economical flight with three-dimensional characteristic surface (instead of the flight with shock waves) occurs and the own developed software, based on it, are confirmed;
- the supersonic flow is laminar, as supposed here and remains attached for larger range of angles of attack than by subsonic flow;
- the influence of friction upon the lift and pitching moment coefficients is neglectable.



Figures 2a,b : The Lift- and Pitching Moment Coefficients of the Wedged Delta Wing Model with Subsonic Leading Edges

The developed hybrid NSL's solutions are used for the computation of the total drag, which include the inviscid and the friction drag. The inviscid drag coefficients of the thin, of the thick-symmetrical components and of the delta wings are the following:

$$C_d = 8\ \ell \int_{\tilde{O}\tilde{A}_1\tilde{C}} \tilde{u}\ \tilde{w}\ \tilde{x}_1\, d\tilde{x}_1\, d\tilde{y}\ ,\quad C_d^* = 8\ \ell \int_{\tilde{O}\tilde{A}_1\tilde{C}} \tilde{u}^* \tilde{w}^* \tilde{x}_1\, d\tilde{x}_1\, d\tilde{y}\ ,\ C_d^{(i)} = C_d + C_d^* \qquad (10a\text{-}c)$$

The friction drag and the total drag coefficients are:

$$C_d^{(f)} = 8\ \nu_f u_1 \int_{\tilde{O}\tilde{A}_1\tilde{C}} u_e \tilde{x}_1\, d\tilde{x}_1\, d\tilde{y}\qquad ,\qquad C_d^{(t)} = C_d^{(f)} + C_d^{(i)} \qquad (11a,b)$$

In the (Fig. 3) are represented the dependences of the inviscid and of the total drag coefficients of the wedged delta wing on angle of attack for the Mach number $M_\infty = 2$. It is to remark that the viscous drag coefficient has:

- an important contribution in the total drag coefficient of the wedged delta wing and cannot be neglected ;
- the magnitudes of all three drag coefficients increase when the absolute value of the angle of attack increases, but the increment of friction drag coefficient is much slowly as those of the inviscid drag coefficient;
- all the dependences of drag coefficients of wedged delta wing versus the angle of attack are symmetrical with respect to the value of the angle of attack $\alpha = 0^\circ$.

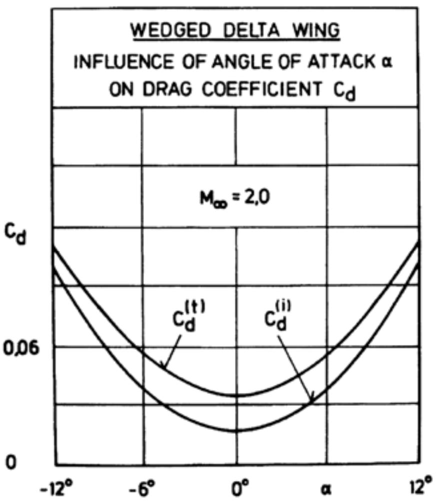


Figure 3: The Dependences of Inviscid and of Total Drag Coefficients $C_d^{(i)}$ and $C_d^{(t)}$ upon the Angle of Attack α at the Mach Number $M_\infty = 2$

Due to analytical hybridization the numerical solutions, presented here, have important analytical properties, namely: they have correct last behaviors, they have correct jumps along the singular lines (like subsonic leading edges, junction lines wing-fuselage and wing-leading edge flaps) according to the minimum singularities (which must fulfill the jumps) and the singularities are balanced. Additionally, for hyperbolic PDE the condition on the characteristic is automatically fulfilled. The viscous-inviscid coupling is realized without interface need, because the hybrid numerical solutions for the NSL's PDEs are prepared for this coupling, due to the hybridization. These hybrid NSL's solutions are also useful for the performing of the viscous global optimal design (GOD) of the shapes of FCs.

4 THE MULTIDISCIPLINARY WEAK INTERACTION AERODYNAMICS-STRUCTURE

A weak interaction is proposed. The influence of structure requests upon the aerodynamics GOD of the FCs shapes with respect to minimum drag at cruise can be transformed in new or modified constraints for the GOD, via iterative optimum-optimorum theory, up its second step of iteration, as proposed in [1,4]. A limitation of the twist can be necessary for FCs, which are flying at higher supersonic Mach number and it can be realized by imposing the constraint of pressure equalization along the subsonic leading edges at a lower supersonic Mach number than the cruising one. A necessary augmentation of thickness distribution can be necessary in the rear part of FC, which can be obtained by artificial prolongation of the depth and, after the optimization of the thickness distribution, the initial depth is restored by cutting and elimination of the prolongation. The influence of aerodynamics upon the structure is obtained by adding the aerodynamic pressure distribution to the load distribution and by computing the new resulting deformation of the structure.

5 CONCLUSIONS

The proposed reinforced hybrid numerical solutions are split, do not need viscous-inviscid interface at the NSL's edge and have important analytical properties, like correct last behaviors, correct jumps along the singular lines (like subsonic leading edges, junction lines wing-fuselage and wing-leading edge flaps), fulfill automatically the non-slip condition on the FC's surface, are meshless, are split, accurate and rapid convergent. Additionally, the derivatives of the velocity's components can be exact computed and the boundary condition on the Mach cone of the apex is automatically fulfilled. A weak interaction aerodynamics-structure is proposed.

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