

## MATHEMATICAL MODELING OF HYDRODYNAMIC PROCESSES IN GEOTHERMAL PLANT

MARIJONAS BOGDEVIČIUS<sup>\*</sup>, JOLANTA JANUTĖNIENĖ<sup>†</sup>, SAULIUS RAZMAS<sup>†</sup>  
AND MINDAUGAS DRAKŠAS<sup>†</sup>

<sup>\*</sup> Mechatronics Science Institute  
Klaipeda University  
Bijunu st. 17 - 103, LT-91255 Klaipeda, Lithuania  
e-mail: mmi@ku.lt, web page: <http://www.ku.lt/mmi/>

<sup>†</sup> Department of Mechanical Engineering  
Klaipeda University  
Bijunu st. 17 - 103, LT-91255 Klaipeda, Lithuania  
Email: mik.jtf@ku.lt- Web page: <http://www.ku.lt/jtf/mechanika/>

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**Summary.** *Researches the physical properties of the mixture of water underground and gases is investigated in this paper. Also paper analyzes an existing geothermal energy extraction system, which consists of a long pipe system and a large number of hydraulic and mechanical elements. The mathematical model of the geothermal system “depth centrifugal pump-pipe (pipe system)” was made to assess the characteristics of an extraction depth centrifugal pump and their correlation with impeller frequency, discharge and other system parameters. Based on this mathematical model the instability zones of the system were determined. A universal mathematical model of depth centrifugal pump was made. This model could be used to describe hydrodynamic processes of injection pumps in the pipe systems, where large amounts of released gases influences the productivity of the pump and geothermal energy extraction system overall.*

### 1 INTRODUCTION

One of renewable energy sources is geothermal energy. Scientific research on geothermal energy is aiming for refinement of extraction technology and expansion of the field of use such as centralized heating or generation of electricity. An investigation of the unsteady flow behaviour centrifugal pump is presented in the work by Barrio [1]. The numerical domain was composed of several modules: suction duct, impeller, volute, diffuser and outlet duct. The authors made use of and solved the Navier–Stokes equations for three-dimensional unsteady flow. A numerical simulation of the three-dimensional fluid flow inside a centrifugal pump is performed in the work of Stickland [2]. The simulation was made with an unsteady calculation, using the sliding mesh technique in order to take into account the impeller-volute interaction. The effects of the blade outlet angle and passage width on the performance of a centrifugal pump have been investigated in the work of Shojaeefard et al.[3]. The numerical

and experimental investigation of the 3-D flow in the centrifugal pump along with the volute was numerically simulated. The finite volume method was used for the discretization of the governing equations. The proposal of a physical model to study the heat transfer and oil flow of oil pipeline under normal operation is presented in the work of Yu et al.[4]. Numerical simulations in a wide range of operating conditions were conducted.

The investigated system is relatively complex, when investigating the hydrodynamic processes of the entire system. Therefore, in the first stage of investigation, assesment of the main energy sources (centrifugal pumps) and the influence of the physical properties of the transported fluid on hydrodynamic processes is needed.

## 2 THE PHYSICAL PROPERTIES OF THE GEOTHERMAL WATER

During experimental studies was determined the dependence between the pressure and the amount of gasses in liquid. The results of experimental investigation is presented in Fig.1.

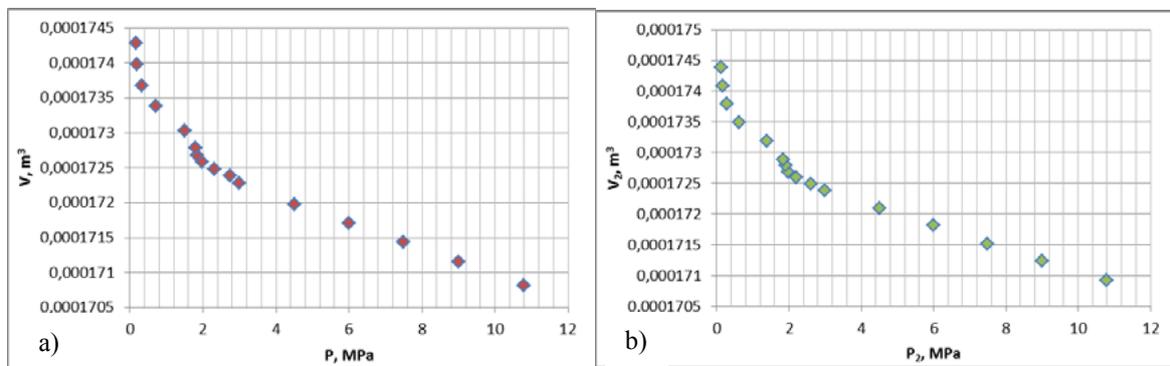


Figure.1. The results of amount of gasses depend on the pressure

The regression relation of the amount of gas and the pressure is deduced. The dependence is described using five degree polynomial function:

$$V(p) = 0,17 \cdot 10^{-3} - 0,14 \cdot 10^{-5} p + 0,41 \cdot 10^{-6} p^2 - 0,66 \cdot 10^{-7} p^3 + 0,51 \cdot 10^{-8} p^4 - 0,15 \cdot 10^{-9} p^5 \quad (1)$$

here  $p$  – the pressure, Pa.

The bulk modulus of fluid is deduced from the experimental results.

## 3 INVESTIGATION OF STABILITY OF THE DEPTH CENTRIFUGAL PUMP

The pressure and flow rate dependence on the angular velocity of pump impeller is shown in Fig.2. The main characteristic of the centrifugal extraction pump dependence of developed pressure on flow rate  $p = \pi(Q_s)$ .

The computational scheme of the analyzed system is shown in Figure 3. This system consists of an extractive well, in which at depth  $H_0$  the deep well centrifugal pump and vertical pipe are immersed.

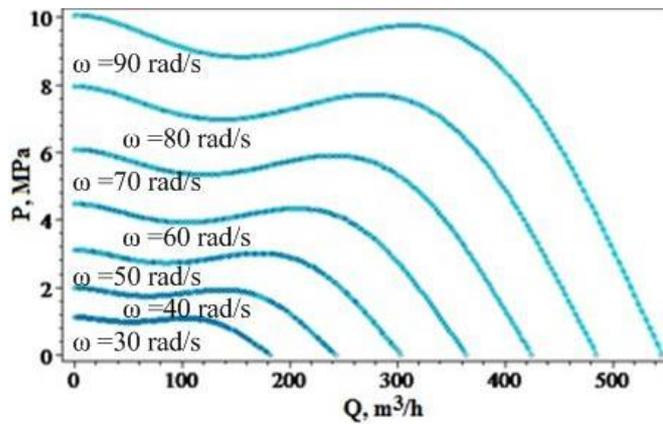


Figure 2. The pressure and flow rate dependence on the impeller angular velocity

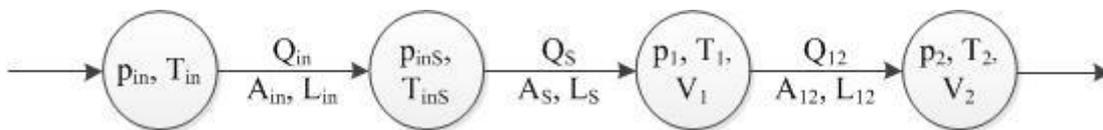


Figure 3. System “deep well centrifugal pump-pipe” stability determination scheme

Mathematical model of the geothermal system “depth centrifugal pump – pipe (pipe system)” was made. Flow rate  $Q_{in}$  change in the suction pipe can be described by the equation:

$$\dot{Q}_{in} = \frac{1}{m_{in}} \left( p_{in} - p_{inS} - p_{inH} - \frac{1}{2} \rho_{in} \xi_{in} \left( \frac{Q_{in}}{A_{in}} \right)^2 \text{sign}(Q_{in}) \right), \quad (2)$$

here  $m_{in} = \rho_{in} \frac{L_{in}}{A_{in}}$ ;  $p_{in}, p_{inS}, p_{inH}$  – the fluid pressures in the suction pipe, pump suction cavity and hydrostatic pressure.

$$p_{inH} = g \rho_{in} (p_{inH}, T) L_{in}, \quad (3)$$

$\rho_{in}$  – pressure;  $L_{in}, A_{in}$  – the length and cross-sectional area of suction pipe;  $\xi_{in}$  – pressure loss coefficient.

The density of the fluid  $\rho_{in}(p_{inH}, T)$  depends on the pressure and the temperature, therefore the hydrostatic pressure  $p_{inH}$  can be determined from the (3) equation:

$$p_{inH} - g \rho_{in} (p_{inH}, T) L_{in} = 0 \quad (4)$$

The pressure change in the pump suction cavity can be described by the equation:

$$\dot{p}_{inS} = \frac{1}{C_{inS}} (Q_{in} - Q_S(p_{inS})), \quad (5)$$

here  $C_{inS} = \frac{V_{inS}}{K_{inS}(p_{inS})}$ ,  $V_{inS}$  – the volume of the pump suction cavity;  $K_{inS}(p_{inS})$  – the bulk modulus of fluid.

The flow generated by the centrifugal pump is equal to:

$$\dot{Q}_s = \frac{1}{m_s} \left( p_s(Q_s) - p_1 - p_{sH} - \frac{1}{2} \rho_s \xi_s \left( \frac{Q_s}{A_s} \right)^2 \text{sign}(Q_s) \right), \quad (6)$$

here  $m_s = \rho_s \frac{L_s}{A_s}$  – acoustic mass of the pipeline;  $L_s, A_s$  – high pressure pipe length and cross-sectional area;  $\xi_s$  – pressure loss coefficient.

Pressure change in cavity No. 1 can be described by the equation:

$$\dot{p}_1 = \frac{1}{C_1} (Q_s - Q_{12}(p_2)), \quad (7)$$

here

$$C_1 = \frac{V_1}{K_1(p_1)},$$

$$Q_{12}(p_2) = A_{12} \mu_{12} \sqrt{\frac{2}{\rho} |p_1 - p_2|} \text{sign}(p_1 - p_2), \quad (8)$$

$p_2$  – System load pressure;  $A_{12}$  – cross-sectional area valve;  $\mu_{12}$  – permeability coefficient.

Pressure generated by deep well centrifugal pump is equal to:

$$p_s = \pi(Q_s) + p_{in}. \quad (9)$$

From the equation (2) we find the pressure  $p_{inS}$ . We assume that  $p_s = p_{inS}$  and using the equation (6) we get:

$$\dot{Q}_s = \frac{1}{m_{ins}} (F_1(Q_s) - p_1)$$

here

$$F_1(Q_s) = p_{in} + \pi(Q_s) - p_{inH} - p_{sH} - \frac{1}{2} \rho_{in} \xi_{in} \left( \frac{Q_s}{A_{in}} \right)^2 \cdot \text{sign}(Q_s) - \frac{1}{2} \rho_s \xi_s \left( \frac{Q_s}{A_s} \right)^2 \text{sign}(Q_s) \quad (10)$$

It was determined that the hydrodynamic processes of the analyzed system “deep well centrifugal pump-pipe” can be described with the two-equation ((9) and (6)) system:

$$\begin{cases} \dot{Q}_s = \frac{1}{m_{ins}}(F_1(Q_s) - p_1) \\ \dot{p}_1 = \frac{1}{C_1}(Q_s - Q_{12}(p_2)) \end{cases} \quad (11)$$

The parameters of static process that characterize the hydrodynamic process of the system “The deep well centrifugal pump-pipe”, can be determined from the equation system (11):

$$\begin{cases} F_1(Q_s) - p_1 = 0 \\ Q_s - Q_{12}(p_2) = 0 \end{cases} \quad (12)$$

The non-linear equation system was solved using Newton’s method. The solution of the equation system (11) can be written:

$$\begin{cases} Q_s = Q_{s0} + Q \\ p_1 = p_{10+p} \end{cases} \quad (13)$$

After linearization of the system of equations (11) in respect of point  $(Q_{s0}, p_{10})$ , the following equation is formed:

$$\begin{cases} \dot{Q} = \frac{1}{m_{ins}} \left( \frac{dF_1(Q_{s0})}{dQ} Q - p \right) \\ \dot{p} = \frac{1}{C_1} \left( Q - \frac{dQ_{12}(p_{10})}{dp} \right) \end{cases} \quad (14)$$

By eliminating the pressure  $p$  from the equation (14), the second-degree differential equation was formed:

$$m_{ins} \ddot{Q} - \left( \frac{dF_1(Q_{s0})}{dQ} - \frac{1}{C_1} \frac{dQ_{12}(p_{10})}{dp} m_{ins} \right) \dot{Q} + \frac{1}{C_1} \left( 1 - \frac{dQ_{12}(p_{10})}{dp} \frac{dF_1(Q_{s0})}{dQ} \right) Q = 0 \quad (15)$$

The following parameters were introduced into the equation (15):

$$K_1 = C_1 \frac{dF_1(Q_{s0})}{dQ} - \frac{dQ_{12}(p_{10})}{dp} m_{ins}; \quad K_2 = 1 - \frac{dQ_{12}(p_{10})}{dp} \frac{dF_1(Q_{s0})}{dQ}; \quad K_3 = \omega^2 = \frac{K_2}{m_{ins} C_1}. \quad (16)$$

The parameter  $K_3$  – natural angular frequency of the system. Instability of hydrodynamic processes occurring in the system “deep well centrifugal pump-pipe” is determined from the conditions:

$K_1 > 0$  – the dynamic instability,  $K_2 \leq 0$  and  $K_3 \leq 0$  – the static instability.

When  $K_1 > 0$  the vibrations occur in the system “deep well centrifugal pump-pipe”. The system becomes dynamically unstable, because of the increased energy level and the increased vibrations amplitude.

When  $K_2 > 0$  system is statically stable. When  $K_1 \leq 0$  the energy level of the system decreases and the vibrations become damped (dynamically stable system). When  $K_1 > 0$  the energy level of the system increases, vibration amplitude increases (dynamically unstable system). System “deep well centrifugal pump-pipe” stability dependencies ( $K_1$ ,  $K_2$  and  $K_3$ ), from fluid flow rate, impeller rpm, when inlet pressure  $p_{in} = 2.0 \text{ MPa}$  and  $p_2 = 1.6 \text{ MPa}$  are shown in Figure 4.

With  $p = \pi(Q_s)$  (Fig. 2) as a main characteristic of the deep well centrifugal pump, with assessment of geometrical parameters of the extraction well and physical properties of transported fluid, the following system instability areas were identified as seen in Fig. 4 in the system “deep well centrifugal pump-pipe” both statically (parameter  $K_1 \geq 0$ ) and dynamic (parameters  $K_2 < 0$  and  $K_3 < 0$ ) instability are possible. The system is unstable when RPM of the impeller of the pump changes in the range of 35-40 Hz.

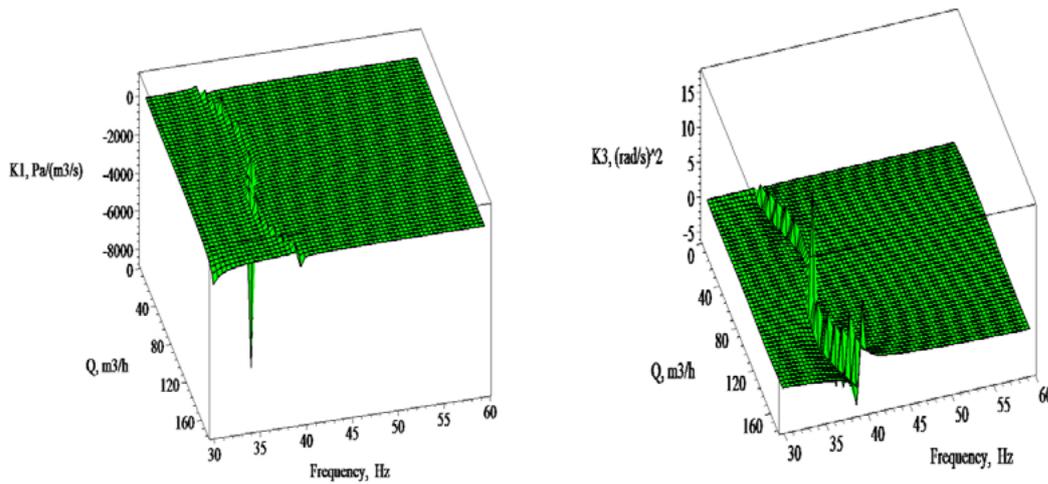


Figure 4. System “deep well centrifugal pump-pipe” stability areas ( $p_{in} = 2.0 \text{ MPa}$  and  $p_2 = 1.6 \text{ MPa}$ )

A numerical solution of the mathematical models of geothermal system was made. Conditions: depth 300 m., nominal speed of the motor 43 Hz, resistance of pressure at the end of pipeline 1.4 MPa. The results of numerical analysis is presented in Fig 5-8.

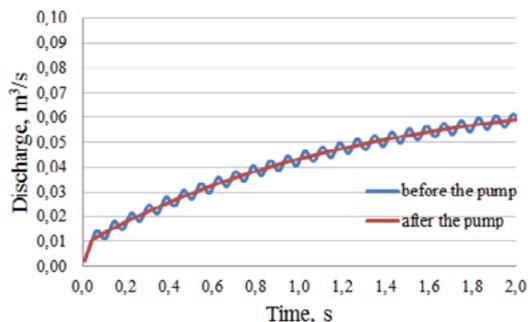


Figure 5. Discharge in the different pipeline points

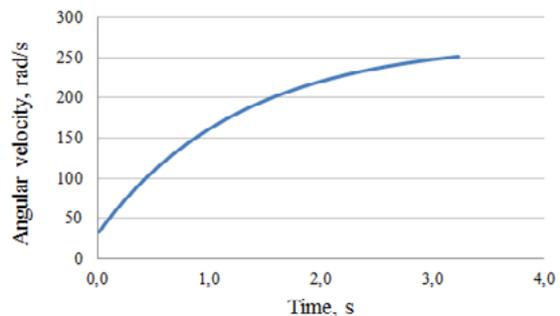


Figure 6. Angular velocity of the pump impeller

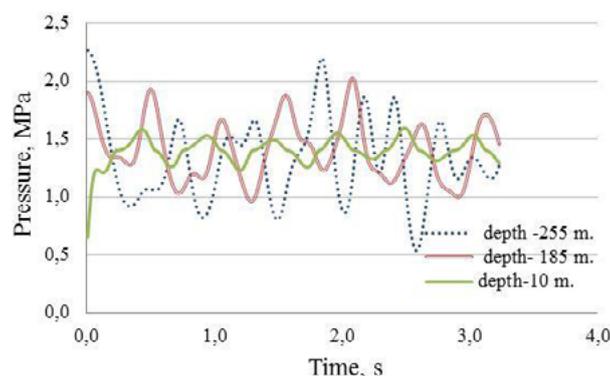


Figure 7. Dependences of the pressure in the different pipeline points

## 5. CONCLUSIONS

- Dependencies amount of gas on the pressure determined with respect to the results of the experiments.
- It was determined that the amount of released gas directly correlates with increased liquid-gas mixture compressibility and slow down of hydrodynamic processes which changes working properties of centrifugal pump – reduces the natural frequency of the system “subsurface centrifugal pump – pipe line/system”.
- Research showed that the natural frequency of the “depth centrifugal pump – pipe” system depends on frequency of the rotation of impeller, discharge and other parameters. It was determined that a low – 1 Hz natural frequency of the system is a result of the combination of high compressibility of the underground water and the large amount of water inside the pipes/tubes.

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