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EFFECTS OF SLIP CONDITION ON FLOW NEAR THE SURFACE OF HYDRO-GEL

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Abstract. The influence of surface properties of a wall of the body on the flow behavior is investigated numerically by the Moving Particle Semi-implicit (MPS) method. Considering the difference between the no-slip condition and the with-slip condition of a wall, a new method to calculate the flow near the hydro-gel wall and that inside the hydro-gel is proposed in this paper.

1 INTRODUCTION

Most of fluid simulation has been calculated under the boundary condition which is no-slip condition between fluid and structure. The wall is usually assumed in numerical simulation as that water does not permeate and it satisfies non-slip condition. But, though the non-slip condition in numerical simulations expresses the existence of surface of a body, it cannot describe the surface properties. Namely, it means that we cannot distinguish the difference between the qualities of the iron and the skin of an animal in water.

The problem of interface between solid and liquid is a significant topic from the viewpoint of water-repellent and drag reduction for material design of ship, car, micro channel and so on. Manservisi et al.[1] dealt with the interaction between a droplet and the surface of a wall and numerically simulated the spreading of a single droplet impacting over horizontal dry surfaces. Many studies focused on the contact angle of a droplet on a solid surface [2-4], and the surface tension is also well studied with numerical system as the problem of multiphase flow[5,6]. Rebouillat et al. [7] and Idelsohn et al. [8] calculated the sloshing of a liquid filled partially in a container as problems of solid and fluid interaction. They used the Finite Element Method (FEM) to calculate the free surface with large deformation and many droplets. Alam et al. [9] indicated that the surface tension of a splash becomes prominent in small-scale phenomena.

From the viewpoint of the field of bionics, the interaction between flow and object such as a cell or living things was studied [10,11]. Kikuchi et al. [11] investigated the flow on a hydrogel surface as strong hydrophilic material to clarify the effect of the surface flow. They found the slip flow on the hydrogel surface, and determined the relation between the strength of the hydrophilicity and the slip velocity. And, creatures living in water such as frog or fish have a slimy mucus skin, and the principal ingredient of mucus is a hydrogel known as mucin[12].

In order to calculate free surface of water, the Moving Particle Semi-implicit (MPS) method is useful. Some studies were reported for splash and droplet dealing with free surface flow [13-15]. However, they ignored surface properties related to different materials, so the effects of a surface on splash are not clear yet. Therefore, since there is no distinction of a surface condition, the form of the splash by the hydrogel object and the splash by the acrylic resin object becomes the same. The purpose of the present paper is to discuss how to express the difference of the surface conditions when using the MPS method. Introducing the interaction models between flow and wall in order to describe the wall conditions, we consider the effect of the slip ratio as hydrophilicity in our numerical simulation.

2 SWELLING DEGREE AND SLIP RATIO

The governing equation of a flow is the incompressible Navier-Stokes equations as the following,

$$\frac{Du}{Dt} = F - \frac{1}{\rho}\nabla P + \nu\nabla^2 u \tag{1}$$

Here, u is the velocity vector of fluid, ρ is the density of fluid, P is the pressure, ν is the kinematic viscosity of fluid. The F is added to the Navier-Stokes equation as the external force such as gravity.

Employed in the present paper is the MPS method, which is one of the particle methods. Assuming two particles i and j, which possess scalar quantities of pressure p_i and p_j, the gradient model between these two particles is written as

$$\nabla P_i = \frac{d}{n^0}\sum_{j\neq i}\frac{p_j - p_i}{|\vec{r}_j - \vec{r}_i|^2}(\vec{r}_j - \vec{r}_i)(|\vec{r}_j - \vec{r}_i|) \tag{2}$$

Here, d is the number of the space dimension. Parameter n^0 is called particle number density in MPS method. Since each water particle has the same mass, every particle number density should be constant and equal to n^0.

The Laplacian model in MPS method is written as

$$\nabla^2 u_i = \frac{2d}{\lambda n^0}\sum_{j\neq i}(u_j - u_i)\kappa(|\vec{r}_j - \vec{r}_i|) \tag{3}$$

where λ is a parameter which is introduced in order to coincide the statistical distribution with an analytic solution [13]. In the MPS method, the interaction between particles is evaluated by weight function κ as follows,

$$\kappa(r) = \begin{cases} \frac{r_e}{r} - 1 & (0 \le r \le r_e) \\ 0 & (r_e \le r) \end{cases} \tag{4}$$

Here, r is the distance between two particles and r_e is the cut-off radius.

The swelling degree is employed here as the slip ratio for treating the hydrophilic surface of an object, which is related to the weight of water included in a hydrogel [16], defined as the ratio of the weight of water against that of agar as follows,

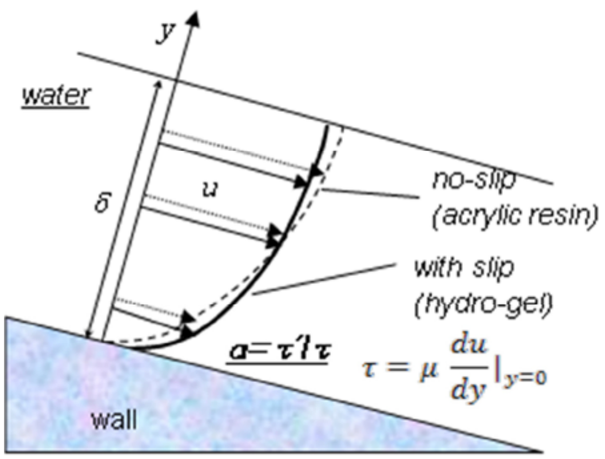

Fig.1 Schema of experiment of a film flow with slip at the wall. Slip ratio α was obtained from the ratio of wall shear stress. The τ is wall shear stress on the slope with no-slip condition. The τ' is that on the slope with hydrogel (agar).

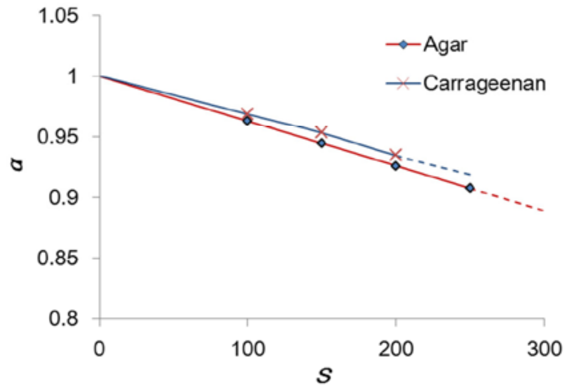

Fig.2 Relationship of swelling degree S of agar and slip ratio α, which was obtained experimentally.

$$S = (m_{\text{water}} + m_{\text{gel}}) / m_{\text{gel}} \tag{5}$$

where m_{water} is the weight of water and the m_{gel} is the weight of hydrogel.

The increase of S means the water contained in the agar increases. Agar is a hydrogel, and is convenient for making a suitable shape and it's easy to control its degree of swelling. Agar has the effect to increase the slip on the surface of an object. The difference of the velocity of water flow versus S was investigated by Kikuchi and Mochizuki [11], who measured the flow velocity u at the distance y from the surface of the acrylic slope and the agar slope as shown in Fig. 2. The τ is the wall shear stress on the no-slip condition, and τ' is that on the agar slope. The coefficient α is defined as the parameter on the slip condition wall as follows,

$$\alpha = \tau' / \tau \qquad (6)$$

Figure 3 shows the experimental relation between the S and α. The solid line is the observed data, and the dotted line is an estimated data. The τ' was smaller than τ, since α decreased with the increasing of S. This relationship can be expressed as

$$\alpha = 1 - \beta S \qquad (7)$$

where β is estimated to be 3.7×10^{-4}. It is noted that larger S gives more slip on the surface. When the boundary condition is complete-slip, α is 0. This means that shearing stress decreases by about 10%, when S is 250.

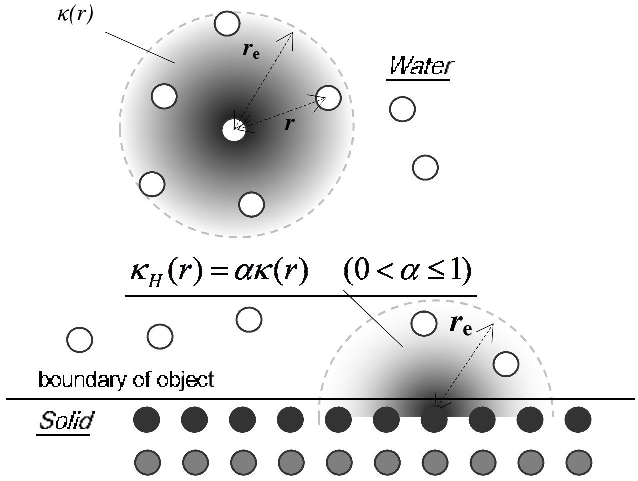

Fig.3 Difference of weight functions κ_H and κ. The distance between particles denoted by r. The κ_H is used between the particle on the boundary of the hydrogel object and the virtual water particle near the boundary. Here, κ is used between the other particles, and α is the coordinate as a ratio of κ_H and κ.

As shown in Fig. 4, the interaction as the shear force between each particle is evaluated by weight function κ. The range, which is the effect of weight function acts, is r_e. The shear force acting between wall and liquid is calculated by the Laplasian term of Eq.(3). Namely, α is multiplied to the κ only for the water particles near the boundary of hydrogel wall, because the effect of slip is caused as the shear force near the hydrogel wall. Thus, Eq. (3) is rewritten using $\kappa_H(r) = \alpha\kappa(r)$ as follows,

$$\nabla^2 u_i = \frac{2d}{\lambda n^0}\sum_{j\neq i}(u_j - u_i)\kappa_H(|\vec{r}_j - \vec{r}_i|) ,$$
$$\kappa_H(r) = \alpha\kappa(r) \quad (0 \le \alpha \le 1) \qquad (8)$$

i: water particle near hydro-gel wall, j: surface particle of hydro-gel wall

Namely, the way for adjusting the slip effect to the calculation in the present study is as follows: i) select the S according to the hydrophilicity of a target object, ii) estimate the α using Eq.(7), iii) apply the α to the weight function of Laplacian term for the calculation of shear force near the hydrogel wall.

3 SIMULATION RESULT

The MPS simulation system was developed in three dimensional domain. The liquid was assumed to be water, namely the density of water ρ_{water} was 1000 kg/m3. The radius R of sphere was 10 mm assumed as small living things. The buoyancy of sphere is considered in the calculation. The range of S was from 1 to 350 in this simulation. The cut-off radius r_e was 2.1.

The initial distance l_0 between particles was 4×10-3m. The bench mark test by the collapse of liquid columns was performed in order to confirm the influence of l_0 on simulation result. The collapse of liquid columns is commonly used for the evaluation of the program of MPS method. With the difference of l_0, there was little difference in the velocity of collapsing speed and in the form of collapse. The formation of collapse was also compared with the experimental result[31]. We confirmed the formation of water collapse was similar to that of experimental result.

3.1 Flow on hydro-gel wall

Figure 4 shows the profile of flow velocity on the hydro-gel wall of a conduit calculated by our simulation. The distributions of velocity of water in the x direction on the no-slip wall and with-slip wall (S = 100 and 250) are shown in Fig.4. Calculation domain is the length = 80 l_0, the height 20 l_0, and the width = 10 l_0. The upper surface of water is a free surface and is given velocity u_0 = 4 cm/s to the x direction as initial velocity. In addition, the distribution of u is normalized by u_0, and height y is normalized by h, respectively. The distribution was obtained on the center of bottom wall.

About the simulation result, as it is known the flow under no-slip conditions becomes the Poiseuille flow as shown in the dashed line by the theoretical value, the velocity distribution of no-slip condition flow by our calculation is almost correct to that theoretical value. From the enlargement figure shown in the right bottom of Fig.4, the flow velocity near the surface of hydro-gel wall increased by the effect of slip on the hydro-gel surface with the increase of a swelling degree. For example, the flow velocity near the wall in the with-slip (S = 250) condition was increasing about 10% from that of no-slip conditions, namely, it was same as the above-mentioned experimental result. The gradient of velocity near the hydro-gel wall is decreasing by the increase of the swelling degree. Since the wall stress was proportional to the velocity gradient near the surface as shown in the formula (6), this result shows that the wall

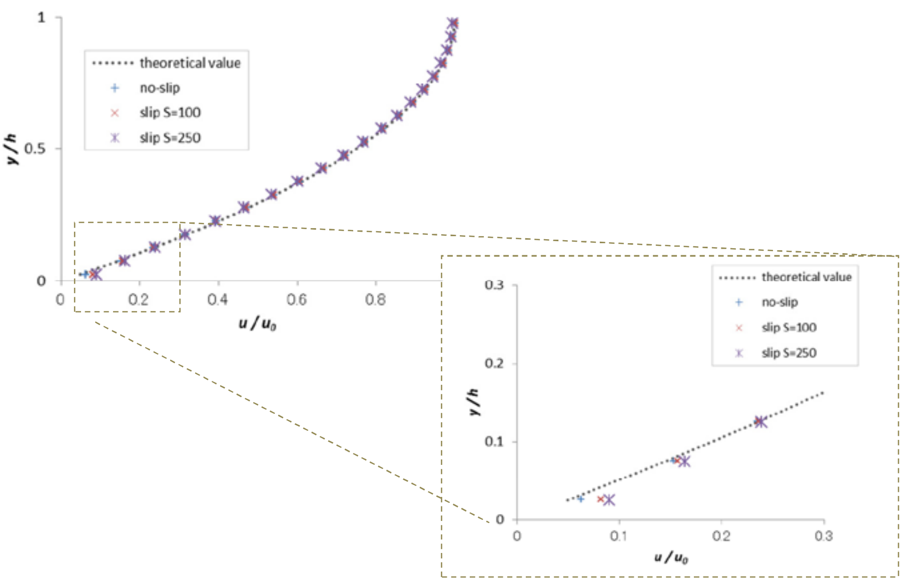

Fig.4 Difference of flow velocity u on the hydro-gel wall (slip wall) and no-slip wall in a conduit by MPS method. The U_0 is the uniform flow and h is the height of conduit.

stress of surface was reduced. By introducing the slip ratio near the surface of a wall, the calculation result which was same as the experimental result was obtained.

3.1 Flow inside of hydro-gel

The simulation schema of the flow inside of hydro-gel is shown in Fig.5. The bottom wall is the fixed wall without slip, and the layer of hydro-gel with the height h is placed on it and the upper plate is placed on the top of hydro-gel. The upper plate was moved at the fixed speed U. The hydro-gel particles were arranged at equal intervals l_H in the distance which was decided in inverse proportion to the degree of swelling by the primitive cubic lattice. Between the lattice the water particle was arranged uniformly, and calculated the motion of the particle of inside hydro-gel. Slip ratio α of the shear force between a hydro-gel particle and a water particle was applied. The rate of the number of particles and that of water particle was same as the swelling degree. The simulation result about the flow velocity distributed in the distance y from a floor in hydro-gel is shown in Fig.6. The flow in hydro-gel was induced by movement of the top plate of a wall, and the profile of velocity was parabola. This result showed same tendency with the experiment result.

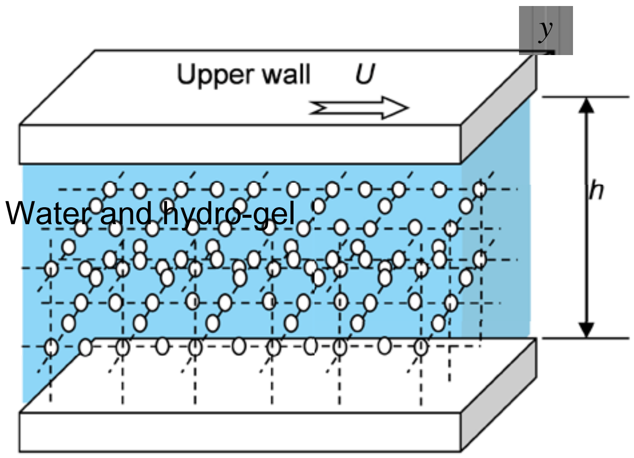

Fig. 5 Simulation condition of hydrogel in water conduit. The white dots are articles as the hydrogel molecule. The other area is filled with water particles.

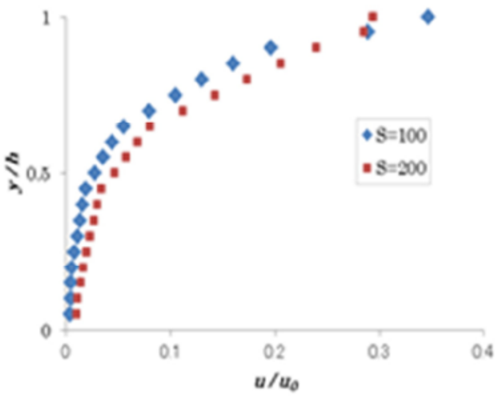

Fig. 6 Simulation result of the velocity of water in the hydrogel.

12 CONCLUSIONS

The influence of slip condition on the flow near the wall was simulated by the Moving Particle Semi-implicit (MPS) method. Considering the difference between the no-slip conition and the with-slip condition of object's wall, a new method to calculate the flow near the hydro-gel wall and inside of hydro-gel is proposed in this paper. From the viewpoint of the field of bionics, the wall condition is an important topic of interaction between the fluid dynamics and the movement dynamics of living things such as frog or fish which have a slimy mucus skin.

Introducing the slip ratio according to the swelling of hydro-gel to the MPS method, the difference of flow near the wall between the no-slip condition and the slip condition such as hydro-gel was investigated numerically. As the result, the water flow in a conduit with the hydro-gel wall obtained by our simulation showed good agreement with the experimental result, which the velocity near the hydrogel wall was higher than that of no-slip wall. Furthermore, we applyed this method to the simulation of the flow inside of the hydro-gel, which the flow became a parabola to the depth and the flow was similar to the experimental result.

REFERENCES

[1] Manservisi S, Scardovelli R. A variational approach to the contact angle dynamics of spreading droplets. *Computers & Fluids* (2009) **38**:406-424.
[2] Tanaka Y, Washio Y, Yoshino M, Hirata T. Numerical simulation of dynamic behavior of droplet on solid surface by the two-phase lattice Boltzmann method. *Computers & Fluids* 2011; **40**:68-78.
[3] Muradoglu M, Tasoglu S. A front-tracking method for computational modeling of impact and spreading of viscous droplets on solid walls. *Computers & Fluids* 2010; **39**:615-625.
[4] Yabe T, Chinda K, Hiraishi T. Computation of surface tension and contact angle and its application to water strider, *Computers & Fluids* 2007; **36**:184-190.
[5] Liu J, Koshizuka S, Oka Y (2005) A hybrid particle-mesh method for viscous, incompressible, multiphase flows. *Journal of Computational Physics* 202:65-93.
[6] Caboussat A. A numerical method for the simulation of free surface flows with surface tension. *Computers & Fluids* 2006; **35**:1205-1216
[7] Rebouillat S, Liksonov D. Fluid–structure interaction in partially filled liquid containers: A comparative review of numerical approaches, *Computers & Fluids* 2010; **39**:739-746.
[8] Idelsohn SR, Onate E, Pin FD. A Lagrangian meshless finite element method applied to fluid structure interaction problems. *Computers & Structures* 2003; **81**:655-671.
[9] Alam A, Kai H, Suzuki, H. Two-dimensional numerical simulation of water splash phenomena with and without surface tension. *J Mar Sci Technol* 2007; **12**:59-71.
[10] Yokoyama M, Mochizuki O, Deformation of a fluid-filled compliant cylinder in a uniform flow. *Journal of Fluids and Structures* 2009; **25**:1049-1064.
[11] Kikuchi K, Mochizuki O. A flow on a hydrogel surface mimicked a living cell. *The 21st International Symposium on Transport Phenomena* 2010; CD-ROM.
[12] Ling SC, Ling TY. Anomalous drag-reducing phenomenon at a water / fish-mucus or polymer interface. *J.Fluid Mech.* 1974; **65**:499-512.
[13] Harada T, Suzuki Y, Koshizuka S, Arakawa T, Shoji S. Simulation of Droplet Generation in Micro Flow Using MPS Method. *JSME International Journal Series B* 2006; **49**:731-736.
[14] Xie H, Koshizuka S, Oka Y. Numerical Simulation of Liquid Drop Deposition in Annular-Mist Flow Regime of Boiling Water Reactor. *Journal of Nuclear Science and Technology* 2004; **41**:569-578.
[15] Nomura K, Koshizuka S, Oka Y, Obata H. Numerical Analysis of Droplet Breakup Behavior using Particle Method. *Journal of Nuclear Science and Technology* 2001; **38**:1057-1064.
[16] Alupeia IC, Marcel Popa M, Hamcerencu M, Abadiec MJM. Super absorbant hydrogels

based on xanthan and poly(vinyl alcohol): 1. The study of the swelling properties. *European Polymer Journal* 2002; **38**:2313-2320.