Dispersion measuring techniques

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A study of chromatic dispersion (CD) in optical fibers, its effects on signal propagation, the devices to compensate it and the methods for its characterization. We have presented the experimental setups for the most popular frequency domain methods for CD measurement, Peucheret and Modulation Phase Shift Method (MPSM), as well as for an advanced method with improved performance, the Asymmetric Mode and Bias Control method (AMBC), whose theoretical basis have also been described. Experimental measures of the D parameter of CD for both 75km of standard single-mode fiber and a Dispersion Compensating Fiber Bragg Grating (DC-FBG) with all 3 methods have shown good agreement.

I. INTRODUCTION

Chromatic dispersion causes propagation delay differences between the spectral components of the signal transmitted throughout optical fiber, with a net result of a loss in the data rate that can be achieved.[1] To overcome this, an accurate characterization of CD in fiber as well as in the devices to compensate it (namely Dispersion Compensating Fiber Bragg Grating DC-FBG and Dispersion Compensating Fiber DCF) is essential. In this work, we start with the theoretical background needed (section II) and we try experimentally the standard methods of measurement MPSM and Peucheret (section III). Then, we develop an advanced method to overcome the limitations of standard methods (section IV), and show its experimental results.

II. THEORETICAL BACKGROUND

Propagation through a single-mode fiber may be modeled as a frequency dependent phase $\phi(\omega)$ [1]. Neglecting loses, we can model this a the following transfer function such as:

$$H(\omega) = \exp(j\phi(\omega)) = \exp(-j\beta(\omega)L) \tag{1}$$

Where $\beta(\omega) = \frac{-\phi(\omega)}{L}$ is the propagation constant and L is the length of the fiber. Considering a signal with bandwidth $\Delta\omega$ centered at frequency ω_0, such that $\Delta\omega \ll \omega_0$, we may write:

$$H(\omega) \approx \exp\left(-jL\left(\beta_0 + \beta_1(\omega-\omega_0) + \frac{\beta_2}{2}(\omega-\omega_0)^2\right)\right) \tag{2}$$

Where $\beta_i = \frac{\partial^i \beta}{\partial \omega^i}\big|_{\omega=\omega_0}$.

Given a Gaussian input pulse of width $\sigma = \frac{\text{FWHM}}{2\sqrt{2\ln 2}}$ and normalized amplitude: $E = e^{\frac{-t^2}{2\sigma^2}}$

The output pulse is obtained as:

$$E_{out} = E * h(t) = \frac{1}{\sqrt[4]{1+\gamma^2}} \exp\left(\frac{-\widetilde{t}^2}{2\sigma^2(1+\gamma^2)}\right) e^{j\kappa\widetilde{t}^2} e^{j\Phi} \tag{3}$$

Where we have defined:

$$\gamma(L) = \frac{\beta_2 L}{\sigma^2} \tag{4}$$

$$\Phi(L) = -\beta_0 L - \frac{\arctan(\gamma)}{2} \tag{5}$$

$$\kappa(L) = \frac{\beta_2 L}{2(\sigma^4 + \beta_2^2 L^2)} \tag{6}$$

$$\widetilde{t}(L) = t - \beta_1 L \tag{7}$$

We can see that the pulse broadens a factor $\sqrt{1+\gamma^2}$ and its amplitude decreases (while maintaining the total energy constant), and a group time delay $\tau_g = \beta_1 L$ appears. We can also see that the signal acquires a phase chirp.

We see that the parameter that quantifies dispersion is β_2. In practice, D is a more useful parameter:

$$D = \frac{\partial \tau_g}{\partial \lambda} = \frac{\partial \beta_1}{\partial \omega}\frac{-L\omega_0^2}{2\pi c} = -\beta_2 \frac{L\omega_0^2}{2\pi c} \tag{8}$$

A. Time-based measuring methods

The result of propagation of a Gaussian pulse shows that the pulse broadens. This suggests that a method for determining D would be to measure the FWHM of a Gaussian pulse before and after the dispersive device. In fact:

$$\text{FWHM}_d = 2\sqrt{2\ln(2)}\sigma\sqrt{1+\gamma^2} \tag{9}$$

$$\gamma = \sqrt{\left(\frac{\text{FWHM}_d}{\text{FWHM}_0}\right)^2 - 1} \tag{10}$$

And the expression for D will be:

$$D = \frac{-\omega_0^2 \text{FWHM}_0^2}{16\pi c \ln 2}\sqrt{\left(\frac{\text{FWHM}_d}{\text{FWHM}_0}\right)^2 - 1} \tag{11}$$

B. Frequency-based measuring methods

Frequency-based methods use a modulated signal with carrier whose envelop is:

$$E = 1 + m\cos(\omega_{RF}t + \varphi) \tag{12}$$

Where $m \ll 1$ is the index of modulation (small-signal approximation), ω_{RF} is a modulation frequency in the radiofrequency range and φ is the starting phase which can be arbitrarily set to zero.

If we develop the effects of propagation in this signal (with $\phi^+ = \phi(\omega_0 + \omega_{RF})$ and $\phi^- = \phi(\omega_0 - \omega_{RF})$) we get for the output envelop:

$$E_{out} = E * h(t) = 1 + \frac{m}{2}\left(e^{j\omega_{RF}t}e^{j\phi^+} + e^{-j\omega_{RF}t}e^{j\phi^-}\right) \tag{13}$$

The detected photocurrent is proportional to the square modulus of the signal:

$$|E_{out}|^2 \approx 1 + \frac{m}{2}e^{j\omega_{RF}t}e^{j\phi^+} + \frac{m}{2}e^{-j\omega_{RF}t}e^{j\phi^-} \tag{14}$$

$$= 1 + m\left(\cos\left(\frac{\phi^+ + \phi^-}{2}\right)\cos\left(\omega_{RF}t + \frac{\phi^+ - \phi^-}{2}\right)\right) \tag{15}$$

As seen, the RF term contains depends on the dispersion both on its amplitude and its phase, giving rise to two different approaches to determine D, which we will proceed to explain next.

1. Peucheret's method

If we recall our expression for $\phi(\omega)$ equation (2), we have that:

$$\frac{\phi^+ + \phi^-}{2} = \beta_0 L + \frac{1}{2}\beta_2 L\omega_{RF}^2 \tag{16}$$

$$= \beta_0 L + \frac{\pi\lambda_0^2 D f_{RM}^2}{c} \tag{17}$$

So, expression (15) will be at a minimum when:

$$\beta_0 L + \frac{\pi\lambda_0^2 D {f_{RM}^{(n)}}^2}{c} = \pi/2 + n\pi \tag{18}$$

So, by subtracting two consecutive zeros, we will get:

$$D = \frac{c}{\lambda_0^2\left({f_{RF}^{(n+1)}}^2 - {f_{RF}^{(n)}}^2\right)} \tag{19}$$

One shortcoming of this method is that we cannot get the sign of D.

The main limitation of Peucheret's method is that, for small values of D, the radio-frequencies at which a zero appears may be very far apart, which implies a loss in spectral resolution. It also implies the need of an instrument capable of measuring large ranges of frequency, which may be expensive.

2. MPSM

From the expression (15), we have that, if we measure the RF phase shift with a network analyzer, we will get $\psi = \frac{\phi^+ - \phi^-}{2}$. We also have $\tau_g = -\frac{\partial\phi}{\partial\omega} \approx -\frac{\phi^+ - \phi^-}{2\omega_m} = -\frac{\psi}{\omega_m}$ [4] (central difference formula for approximation of derivatives).

The Modulation Phase Shift Method consists on finding D by varying λ_0, (by using a tunable laser, for example).

We have:

$$D = \frac{\partial\tau_g}{\partial\lambda} \approx \frac{\tau_g(\lambda_2) - \tau_g(\lambda_1)}{\lambda_2 - \lambda_1} = -\frac{\psi(\lambda_2) - \psi(\lambda_1)}{\omega_m(\lambda_2 - \lambda_1)} \tag{20}$$

One clear shortcoming of this method is the fact that we need to measure the phase of a signal. The phase is only distinguishable up to a 2π additive factor. This means that there is a maximum measurable value of D, when $\psi(\lambda_2) - \psi(\lambda_1) = \pm\pi$:

$$|D_{max}| = \frac{1}{2f_{RF}(\lambda_2 - \lambda_1)}$$

Another limitation of the MPSM is the trade off between the values of f_{RF} and $\Delta\lambda = \lambda_2 - \lambda_1$. Since we are approximating derivatives with increments, we need a small frequency to minimize the error of $\tau_g \approx -\frac{\psi}{\omega_m}$, but we need a high frequency, since the magnitude measured experimentally (and thus object to experimental errors) is the phase ψ.

III. EXPERIMENTAL RESULTS

For this work, we focused on the frequency-based methods. We will test a DC-FBG (Pirelli CDCM-04074, with $D = -1252.35$ps/nm) as well as 75km of standard single-mode fiber ($D = 1275$ps/nm).

A. Mach-Zehnder modulator

For all of our methods of measuring dispersion, we will use a Mach-Zehnder modulator. The input waveguide is split up into two waveguides interferometer arms. If a voltage is applied across one of the arms, a phase shift is induced for the wave passing through that arm. When the two arms are recombined, the phase difference between the two waves is converted to an amplitude modulation. The transfer function of the Mach-Zehnder modulator is:

$$E_{out} = E_{in}\cos\left(\frac{V_B + V_{RF}}{V_\pi}\pi\right) \tag{21}$$

With $\theta_i = \frac{\pi}{V_\pi}V_i$, and by using trigonometry identities and small signal approximation, we have:

$$E_{out} \approx E_{in}\left[\cos(\theta_B)\left(1 - \frac{1}{2}\theta_{RF}^2\right) - \sin(\theta_B)\theta_{RF}\right] \tag{22}$$

Which at quadrature point ($\theta_B = \frac{\pi}{4}$) is:

$$\approx \frac{E_{in}}{\sqrt{2}}\left[1 - m\cos(\omega_{RF}t) - \frac{m^2}{2}\cos(2\omega_{RF}t)\right] \quad (23)$$

Where $m = \frac{V_{RF}}{V_\pi} \ll 1$. If we neglect terms of order m^2, and synchronize the phase we have:

$$E_{out} \approx \frac{E_{in}}{\sqrt{2}}\left[1 + m\cos(\omega_{RF}t)\right] \quad (24)$$

Which is exactly the modulated signal with carrier that we need for our frequency-based methods.

B. Peucheret's Method

For this method, we used a tunable laser (NewFocus 6427) to generate our signal Mach-Zender Modulator (Fujitsu FTM7921ER) to modulate our signal, and a Network Analyzer (HP8510B), to measure the amplitude of the S21 as a function of f_{RF}, with a photo-detector (HP83440D). For the measure of the 75km of fiber, we used an EDFA to amplify our signal due to loses. We used a wavelength of $\lambda = 1559$nm.

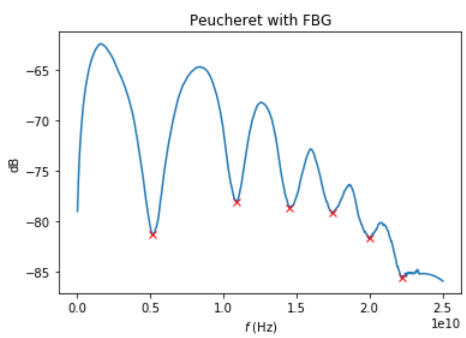

Figure 1: S21 amplitude for a DC-FBG.

For the measurement of the DC-FBG, with equation (19), we obtained an average of $|D| = 1316.195$ps/nm

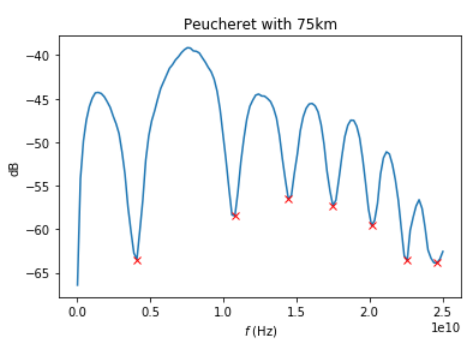

Figure 2: S21 amplitude for 75km of fiber.

For the measurement of the DC-FBG, with equation (19), we obtained an average $|D| = 1339.226$ps/nm

C. MPSM

For this method, we used the same setup as with Peucheret Method (III B), but we fixed a radiofrequency $f_{RF} = 500$GHz, and varied λ. To do this, we used GPIB connection to control the tunable laser with a computer.

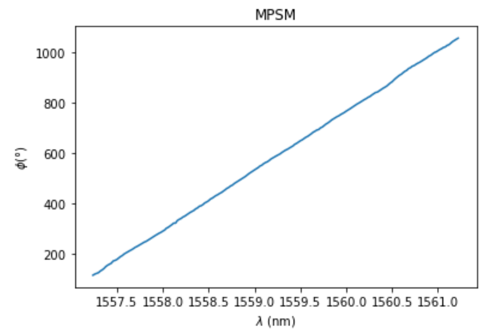

Figure 3: S21 phase-shift delay as a function of λ.

Then, by using equation (20), we obtained on average a value of $D = -1315.609$ps/nm

IV. ADVANCED METHODS

Until now, we have been using the Mach-Zehnder modulator in quadrature point. Now, we will use the modulator in asymmetric mode (by controlling each branch independently), and use the bias as a control parameter. The resulting expression from the modulator will now be:

$$E_{out} = E_{in}\left(e^{j\theta_m} + e^{j\theta_B}\right) = E'_{in}\left(e^{j\theta_m - \theta_B} + 1\right) \quad (25)$$

$$\approx E''_{in}\left[2\cos(\theta_B/2) + e^{-j\theta_B/2}\frac{jm}{\pi}\cos(\omega_m t)\right] \quad (26)$$

Where we have made the approximation $e^{j\theta_{RF}} \approx 1 + j\theta_{RF}$, and we have taken constant phases into E''_{in}

The effect of dispersion over this signal is:

$$\begin{aligned} E_d = E''_{out} * h(t) \cong E_{in}\Big[&2cos(\theta_B/2) + \\ &+ e^{-j\theta_B/2}\frac{jm}{2\pi}\left(e^{j\omega_m t}e^{j\phi^+} + e^{-j\omega_m t}e^{j\phi^-}\right)\Big] \\ &\cong E_{in}\Big[2cos(\theta_B/2) - \\ -\frac{m}{2\pi}\sin(\omega_m t + \phi^+ - \theta_B/2) &+ \frac{m}{2\pi}\sin(\omega_m t - \phi^- + \theta_B/2)\Big] \end{aligned} \quad (27)$$

Where, as before, we have synchronized the phase and the group delay, and $\phi^\pm = \phi(\omega_0 \pm \omega_m)$

And the detected electric signal will be:

$$I_d \propto E_d^2 \propto \Big[2cos(\theta_B/2)+ \\ \frac{m}{2\pi}\left(-\sin(\omega_m t+\phi^+-\theta_B/2)+\sin(\omega_m t-\phi^-+\theta_B/2)\right)\Big]^2 \tag{28}$$

And approximating to first order in m we get for I_d:

$$\cos(\theta_B/2)\sin\left(\frac{\phi^+ + \phi^-}{2}-\theta_B/2\right)\cos\left(\omega_m t+\frac{\phi^+ - \phi^-}{2}\right) \tag{29}$$

Compared with the equation for the quadrature point:

$$I_d \propto \cos\left(\frac{\phi^+ + \phi^-}{2}\right)\cos\left(\omega_{RF} t+\frac{\phi^+ - \phi^-}{2}\right) \tag{30}$$

Now, we have terms with θ_B that depend only on the bias voltage, and allow us to measure $\frac{\phi^+ + \phi^-}{2}$ (and thus D) without changing the frequency. This allow us to overcome the main limitation of Peucheret's method, which was the need of reaching high frequencies to measure small values of D. It also allows us to measure the sign of D.

A. AMBC

AMBC stands for Asymmetric Mode and Bias Control. For this method, we can fix f_{RF} to obtain a good spectral resolution, and we will vary the bias $\theta_B = \frac{V_B}{V_\pi}\pi$ From (29), we can see that as we change θ_B, we will find zeros when $\theta_B/2 = (2n+1)\pi/2$, and when $\frac{\phi^+ + \phi^-}{2} - \theta_B/2 = n\pi$. This means that, as we increase V_B, we will find many zeros, some of which will depend on the dispersion coefficient and some of which will not (mobile and fixed zeros respectively).

By taking advantage of the mobile ones, we get the following expression for D:

$$\frac{\phi^+ + \phi^-}{2} - \theta_B/2 = \frac{\pi\lambda_0^2 D}{c} f_m^2 - \theta_B/2 = n\pi \tag{31}$$

Now, we can consider the case without DUT ($D \approx 0$) and the case with DUT, and subtract the mobile zeros of each:

$$\frac{\pi\lambda_0^2 D}{c} f_m^2 - \theta_{B2}/2 + \theta_{B1}/2 = 0 \tag{32}$$

And by substituting the value of θ_B with equation 2, we get the expression:

$$D = \frac{c(V_{B1}-V_{B2})}{2V_\pi \lambda_0^2 f_m^2} \tag{33}$$

The main drawback is the slowness of the measure, since many different voltages need to be applied. This is paired with the effect known as Bias Drift [6]. When using a Mach-Zehnder modulator, the values of θ_B "drift" with time. For the other measures, the drift was too slow to affect the final results, but for this one, the bias drift may change our measures. This can be minimized with the use of the fixed zeros. We can impose that the zeros that are not supposed to move with dispersion lay in the same value of V.

B. Experimental results of the AMBC

Here, we have used the same setup as with Peucheret (III B), but we have used the Mach-Zehnder in asymmetric mode with a bias controlled by a programmable power supply (Promax FA-851) controlled by GPIB. We used a fixed radiofrequency of $f_{RF} = 2$GHz and wave-length of $\lambda = 1559$nm. If we look at how the output signal varies as we change the bias, we obtained the following plots:

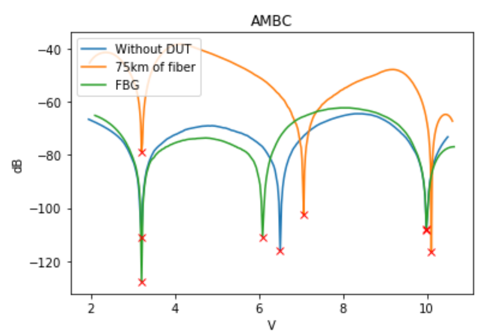

Figure 4: S21 amplitude as a function of V_b.

We can see that the first and third minimums are fixed, and the second one moves as we increase the dispersion. We will use them to calculate $V_{pi} = \frac{V_3 - V_1}{2}$, as well as to minimize the effect of bias drift.

The values of D obtained with equation (33) are $D = 1269.748$ps/nm for the fiber, and $D = -1362.975$ps/nm for the FBG.

V. CONCLUSIONS

We have showed the phenomenon of CD in single-mode optical fiber. We have reviewed the main concepts and definitions related to it and its effects both over pulses and RF envelop propagation. We have built the experimental set ups for the most popular RF modulation methods for measuring CD, and also for an advanced method, called AMBC. The experimental values for the 3 methods implemented, as well as the nominal values of the D parameter of both 75km of fiber and a DC-FBG have good agreement.

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