FONAMENTS MATEMÀTICS DE L'ENGINYERIA I

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Funcions, límits i continuïtat

Exercicis autoavaluació

Exercici proposat 1: comandes iscont i discont

Hi han dues comandes que ens permeten trobar els punts de discontinuitat d'una funció i saber si la funció és contínua o no **iscont**, **discont**.

- la comanda **iscont** determina si les següents funcions són continues o no en l'interval que s'especifica. Retorna TRUE (veritat) si la funció és contínua en l'interval o FALSE (fals) si l'expressió no és contínua.

- la comanda **discont** retorna un conjunt de valors on la funció donada pot ser discontínua (però no té perquè necessàriament ser-ho)

• Demostrar que l'equació

$$\ln(x) = x^2 - 4x$$

té una solució real a l'interval [1,+∞]. Determinar aquesta solució de manera que l'equació anterior es verifiqui amb un error menor de 0.01.

COMANDES: iscont, biseccio_construct

> $f := x \rightarrow \ln(x) - x^2 + 4 \cdot x$

$$f := x \rightarrow \ln(x) - x^2 + 4x \tag{1.1.1}$$

> $f(1)$

$$3 \tag{1.1.2}$$

> $\lim_{x \rightarrow +\infty} f(x)$

$$-\infty \tag{1.1.3}$$

> $f(5)$

$$\ln(5) - 5 \tag{1.1.4}$$

> $evalf[5](\ (1.1.4)\ )$

$$-3.3906 \tag{1.1.5}$$

> $?iscont$

> $iscont(f(x), x = 0.5..6)$

$$true \tag{1.1.6}$$

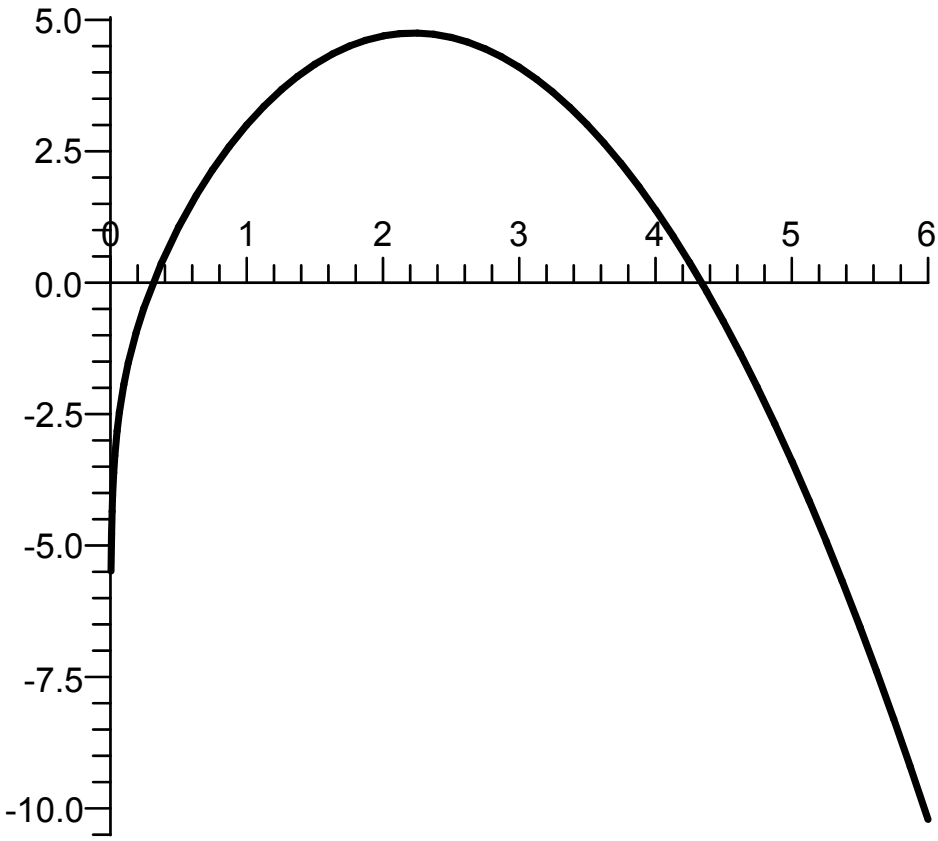
Com que tenim que $f(1) \cdot f(5) < 0$ i a més f és contínua en [1,5] per ser-ho en (0.5,6) , pel teorema de Bolzano podem afirmar que $f(x)$ te un zero en l'interval (1,5). Per tant en particular té un zero en l'interval [1,+∞].

Per determinar la solució usarem la comanda biseccio_construct i per això cal carregar el fitxer: F_MAPLE_Funcions.mpl

> $read$ "F_MAPLE_Funcions .mpl"

```
Warning, the previous binding of the name arrow has been
removed and it now has an assigned value
Warning, the previous binding of the name arrow has been
removed and it now has an assigned value
```

> $biseccio\_construct(f, 1, 5, .01);$



$$\begin{bmatrix} x0 & x1 & f(x0) & f(x1) \\ 1 & 5 & 3. & -3.390562088 \\ 3 & 5 & 4.098612289 & -3.390562088 \\ 4 & 5 & 1.386294361 & -3.390562088 \\ 4 & \frac{9}{2} & 1.386294361 & -.745922603 \\ \frac{17}{4} & \frac{9}{2} & 0.384418983 & -.745922603 \\ \frac{17}{4} & \frac{35}{8} & 0.384418983 & -.164718480 \\ \frac{69}{16} & \frac{35}{8} & 0.113861532 & -.164718480 \\ \frac{69}{16} & \frac{139}{32} & 0.113861532 & -0.024426032 \\ \frac{277}{64} & \frac{139}{32} & 0.044968407 & -0.024426032 \\ \frac{555}{128} & \frac{139}{32} & 0.010333846 & -0.024426032 \\ \frac{555}{128} & \frac{1111}{256} & 0.010333846 & -0.007030429 \end{bmatrix}$$

$$\left[x=, \frac{1111}{256}, f(x) = -0.007030429\right] \tag{1.1.7}$$

> $sol := \frac{1111}{256}$

$$sol := \frac{1111}{256} \tag{1.1.8}$$

> $\ln(sol) = sol^2 - 4 \cdot sol$

$$\ln\left(\frac{1111}{256}\right) = \frac{96657}{65536} \tag{1.1.9}$$

> $evalf((1.1.9))$

$$1.467838345 = 1.474868774 \tag{1.1.10}$$

> $1.467838345 - 1.474868774$

$$-0.007030429 \tag{1.1.11}$$

>

Exercici en Math 1D

```
> f:=x->ln(x)-x^2+4*x:
> f(1);
```

$$3 \tag{1.1.12}$$

```
> limit(f(x),x=+infinity);
```

$$-\infty \tag{1.1.13}$$

```
> evalf(f(5));
```

$$-3.390562088 \tag{1.1.14}$$

```
> iscont(f(x),x=0.5..6);
```

$$true \tag{1.1.15}$$

```
> read(`F_MAPLE_Funcions.mpl`);
> biseccio_construct(f,1,5,0.01);
> sol:=1111/256;
```

$$sol := \frac{1111}{256} \tag{1.1.16}$$

```
> evalf(ln(sol)=sol^2-4*sol);
```

$$1.467838345 = 1.474868774 \tag{1.1.17}$$

```
>
```

Exercici proposat 2:

• Classificar les discontinuïtats de la següent funció:

$$\begin{cases} -\frac{1}{25}\,(x+9)^2\,(x+3) & x \le -4 \\ -\frac{2}{x+4}+2 & x < -2 \\ 0 & x = -2 \\ \frac{2}{x}+2 & x < 0 \\ x & x < 1 \\ -x+3 & 1 \le x \\ 0 & otherwise \end{cases}$$

COMANDES: iscont, discont, limit

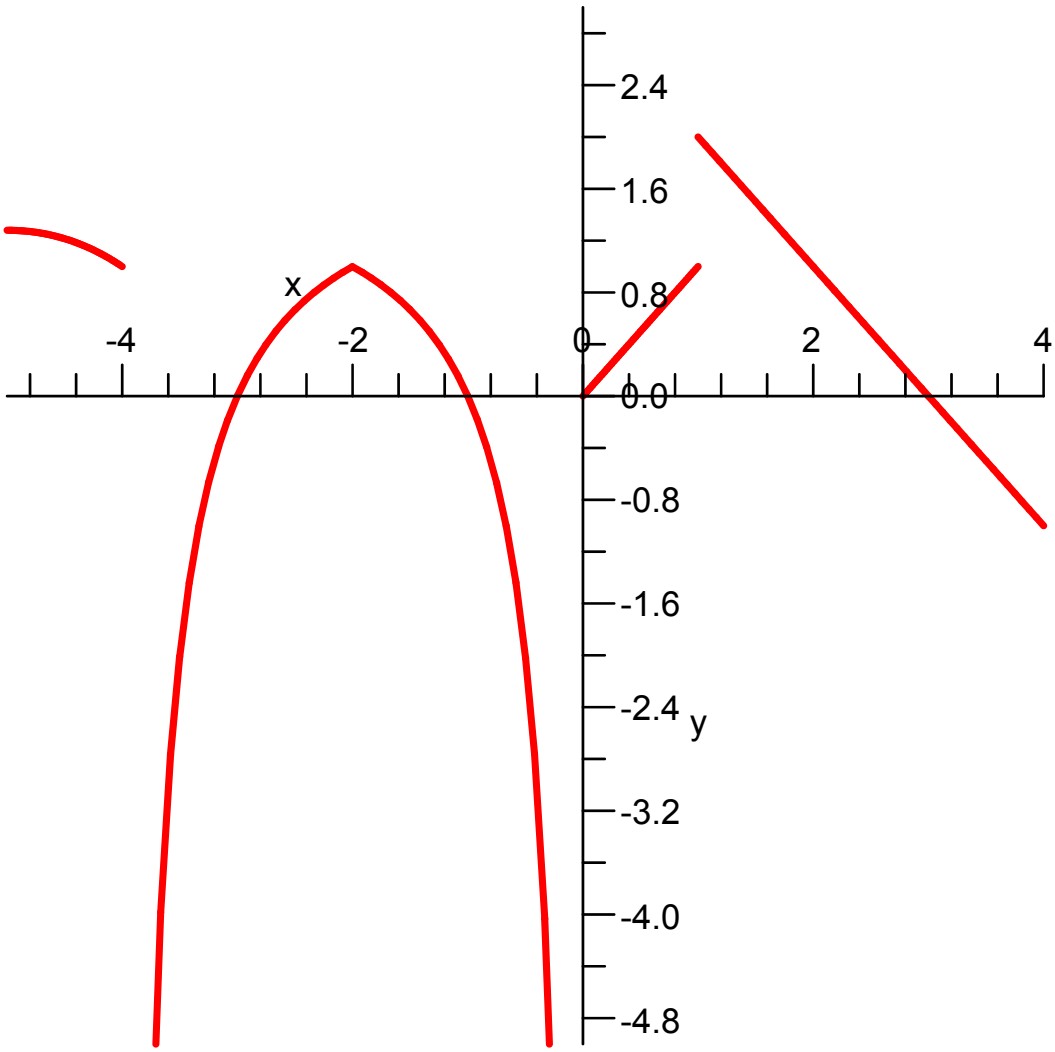
```
> f:= x → piecewise(x≤ -4, -1 (x+9)^2 (x+3)/25, x < -2, -2/(4+x) +2, x < 0, 2/x
      +2, x < 1, x, 1 ≤ x, -x+3, 0):
  f(x)
```

$$\begin{cases} -\frac{1}{25}\,(x+9)^2\,(x+3) & x \le -4 \\ -\frac{2}{4+x}+2 & x < -2 \\ \frac{2}{x}+2 & x < 0 \\ x & x < 1 \\ -x+3 & 1 \le x \\ 0 & otherwise \end{cases} \tag{1.2.1}$$

> $iscont(f(x), x = -5..2)$

$$false \tag{1.2.2}$$

> $plot(f(x), x = -5..4, y = -5..3, discont = true, thickness = 2)$



> $?discont$

> $discont(f(x), x)$

$$\{-4, -2, 0, 1\} \tag{1.2.3}$$

$x = -4$ discontinuitat de salt infinit o essencial

> $\lim_{x \to -4^-} f(x)$

$$1 \tag{1.2.4}$$

> $\lim_{x \to -4^+} f(x)$

$$-\infty \tag{1.2.5}$$

$x = -2$ contínua

> $\lim_{x \to -2^-} f(x)$

$$1 \tag{1.2.6}$$

> $\lim_{x \to -2^+} f(x)$

$$1 \tag{1.2.7}$$

> $f(-2)$

$$1 \tag{1.2.8}$$

$x = 0$ discontinuitat de salt infinit o essencial

> $\lim_{x \to 0^-} f(x)$

$$-\infty \tag{1.2.9}$$

> $\lim_{x \to 0^+} f(x)$

$$0 \tag{1.2.10}$$

$x = 1$ discontinuitat de salt finit

> $\lim_{x \to 1^-} f(x)$

$$1 \tag{1.2.11}$$

> $\lim_{x \to 1^+} f(x)$

$$2 \tag{1.2.12}$$

>

Exercici en Math 1D

```
> f:=x->piecewise(x <= -4, -1/25*(x+9)^2*(x+3), x < -2, -2/
  (4+x)+2, x < 0, 2/x+2, x < 1, x, 1 <= x, -x+3, 0):f(x);
```

$$\begin{cases} -\frac{1}{25}(x+9)^2(x+3) & x \le -4 \\ -\frac{2}{4+x}+2 & x < -2 \\ \frac{2}{x}+2 & x < 0 \\ x & x < 1 \\ -x+3 & 1 \le x \\ 0 & otherwise \end{cases} \tag{1.2.13}$$

```
> iscont(f(x),x=-5..2);
```

$$\mathit{false} \tag{1.2.14}$$

```
> discont(f(x),x);
```

$$\{-4, -2, 0, 1\} \tag{1.2.15}$$

```
> limit(f(x),x=-4,left);
  limit(f(x),x=-4,right);
```

$$1$$
$$-\infty \tag{1.2.16}$$

```
> limit(f(x),x=-2,left);
  limit(f(x),x=-2,right);
```

$$1$$
$$1 \tag{1.2.17}$$

```
> limit(f(x),x=0,left);
  limit(f(x),x=0,right);
```

$$-\infty$$
$$0 \tag{1.2.18}$$

```
> limit(f(x),x=1,left);
  limit(f(x),x=1,right);
```

$$1$$
$$2 \tag{1.2.19}$$

```
>
```

Exercici proposat 3:

- Calcular el següent límit:

$$\lim_{x \to 0} \frac{\sqrt{2+x} - \sqrt{2}}{x}$$

- Representar gràficament la funció mitjançant una línia de punts de color taronja amb una resolució de 2 píxels.

COMANDES: limit, Plot Builder

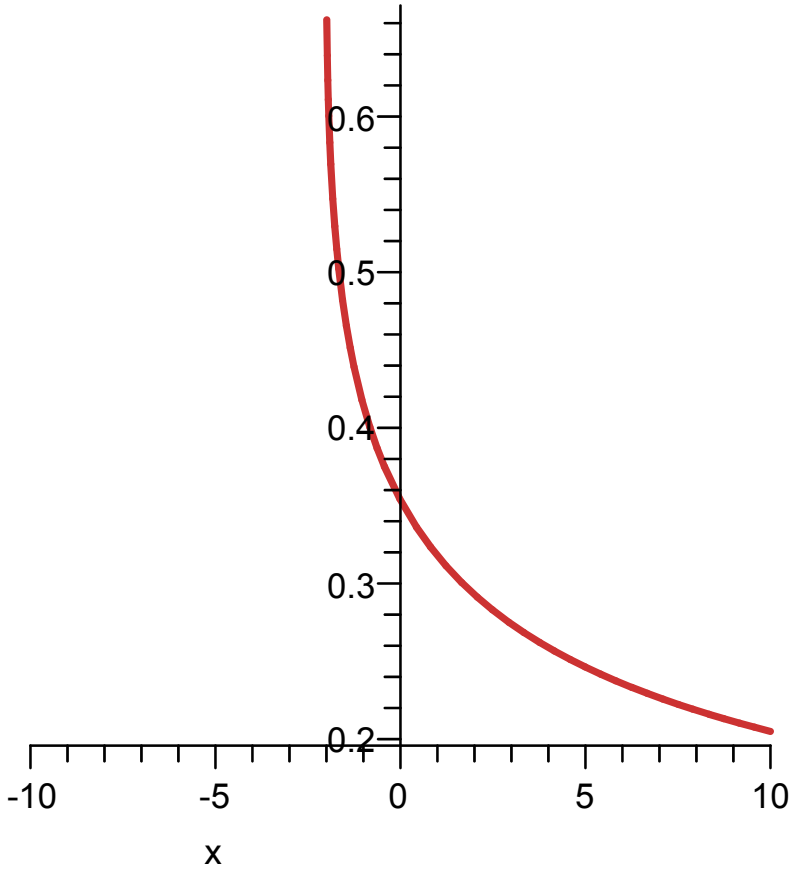
$$> \lim_{x \to 0} \frac{\left(\sqrt{2+x} - \sqrt{2}\right)}{x}$$

$$\frac{1}{4}\sqrt{2} \tag{1.3.1}$$

$$> \frac{\left(\sqrt{2+x} - \sqrt{2}\right)}{x}$$

$$\frac{\sqrt{x+2}-\sqrt{2}}{x} \tag{1.3.2}$$

```
> plot(((x + 2)^(1/2)- 2^(1/2))/x,  x =
      - 10  .. 10,  linestyle = DOT,  thickness = 2,  color = orange)
```




```
>
```

Exercici en Math 1D

```
> limit((sqrt(x+2)-sqrt(2))/x, x = 0);
```

$$\frac{1}{4}\sqrt{2} \tag{1.3.3}$$

```
> plot(((x+2)^(1/2)-2^(1/2))/x, x = -10 .. 10, linestyle =
  DOT,
  thickness = 2, color = orange);
>
```

Exercici proposat 4:

• Calcular el límit de la funció següent en el punt 2 mitjançant la Maplet corresponent

$$\frac{\left(8 \cdot x^4 + 3 \cdot x + 1\right)}{\left(5 \cdot x^4 + 2 \cdot x + 1\right)}$$

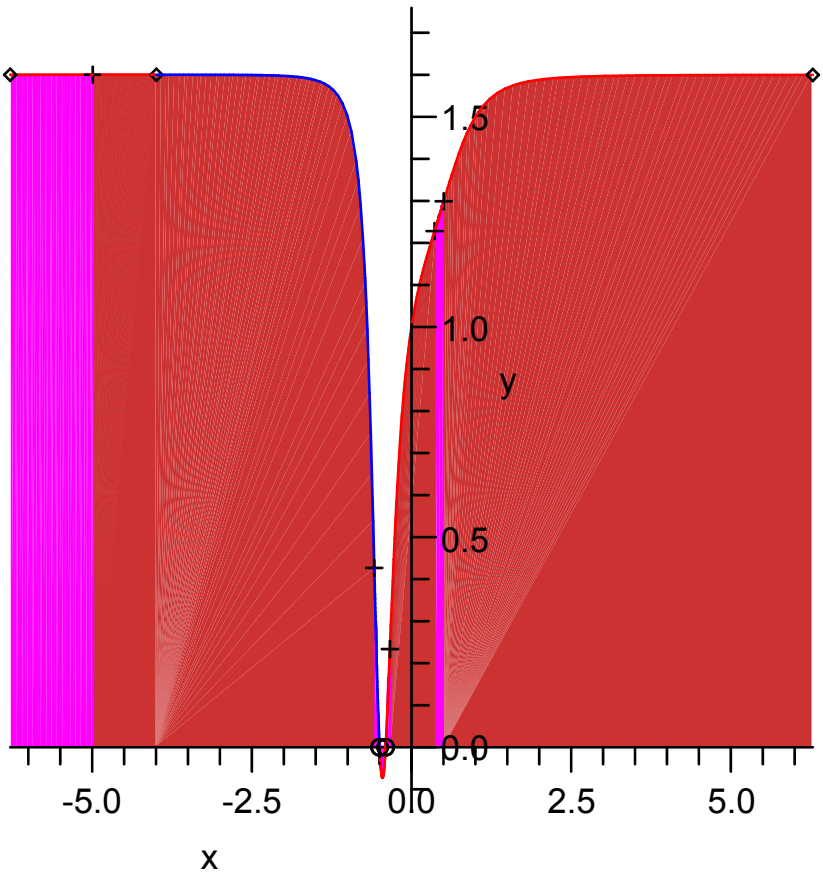
• Representar la funció i determinar-ne els zeros mitjançant la Maplet corresponent

COMANDES: Limit Methods, Curve Analysis

> $Student\left[Calculus1\right]LimitTutor\left(\frac{\left(8 \cdot x^4 + 3 \cdot x + 1\right)}{\left(5 \cdot x^4 + 2 \cdot x + 1\right)}\right);$

$$\lim_{x \to 2} \left(\frac{8\,x^4 + 3\,x + 1}{5\,x^4 + 2\,x + 1}\right) = \frac{27}{17} \tag{1.4.1}$$

> $Student\left[Calculus1\right]CurveAnalysisTutor\left(\frac{\left(8 \cdot x^4 + 3 \cdot x + 1\right)}{\left(5 \cdot x^4 + 2 \cdot x + 1\right)}\right);$



> $\frac{\left(8 \cdot x^4 + 3 \cdot x + 1\right)}{\left(5 \cdot x^4 + 2 \cdot x + 1\right)}$

$$\frac{8\,x^4 + 3\,x + 1}{5\,x^4 + 2\,x + 1} \tag{1.4.2}$$

```
> solve( {(1.4.2)} )
```

$$\left\{x=\frac{-1}{2}\right\}, \left\{x=-\frac{1}{12}\left(244+12\sqrt{417}\right)^{(1/3)}+\frac{2}{3}\frac{1}{\left(244+12\sqrt{417}\right)^{(1/3)}}+\frac{1}{6}\right\}, \left\{x=\frac{1}{24}\left(244+12\sqrt{417}\right)^{(1/3)}-\frac{1}{3}\frac{1}{\left(244+12\sqrt{417}\right)^{(1/3)}}+\frac{1}{6}+\frac{1}{2}\,\mathrm{I}\sqrt{3}\left(-\frac{1}{12}\left(244+12\sqrt{417}\right)^{(1/3)}-\frac{2}{3}\frac{1}{\left(244+12\sqrt{417}\right)^{(1/3)}}\right)\right\}, \left\{x=\frac{1}{24}\left(244+12\sqrt{417}\right)^{(1/3)}-\frac{1}{3}\frac{1}{\left(244+12\sqrt{417}\right)^{(1/3)}}+\frac{1}{6}-\frac{1}{2}\,\mathrm{I}\sqrt{3}\left(-\frac{1}{12}\left(244+12\sqrt{417}\right)^{(1/3)}-\frac{2}{3}\frac{1}{\left(244+12\sqrt{417}\right)^{(1/3)}}\right)\right\} \tag{1.4.3}$$

```
> evalf( [ (1.4.3) ])
```

$$[\{x=-.5000000000\}, \{x=-.4052678569\}, \{x=0.4526339285-0.6418710860\,\mathrm{I}\}, \{x=0.4526339285+0.6418710860\,\mathrm{I}\}] \tag{1.4.4}$$

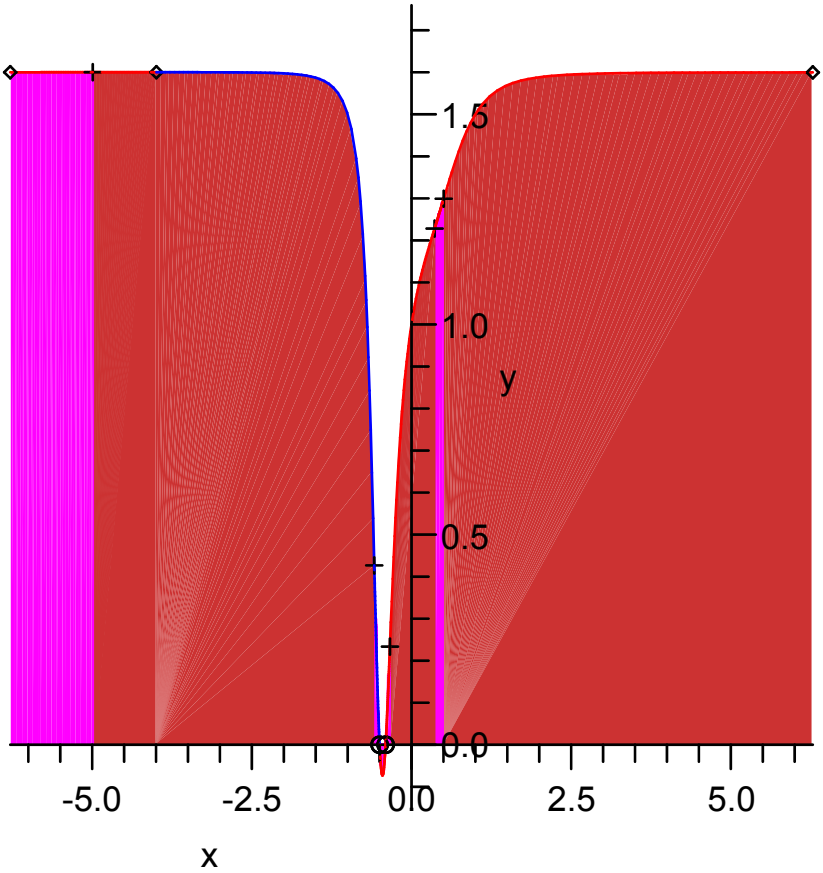
```
>
```

Exercici en Math 1D

```
> ?LimitTutor
> Student[Calculus1]:-LimitTutor((8*x^4+3*x+1)/(5*x^4+2*x+1))
  ;
```

$$\lim_{x\to 0}\left(\frac{8\,x^4+3\,x+1}{5\,x^4+2\,x+1}\right)=1 \tag{1.4.5}$$

```
> ?CurveAnalysisTutor
> Student[Calculus1]:-CurveAnalysisTutor((8*x^4+3*x+1)/(5*
  x^4+2*x+1));
```




```
> ?solve
> solve((8*x^4+3*x+1)/(5*x^4+2*x+1)=0,x);
```

$$\frac{-1}{2}, -\frac{1}{12}\left(244+12\sqrt{417}\right)^{(1/3)}+\frac{2}{3}\frac{1}{\left(244+12\sqrt{417}\right)^{(1/3)}}+\frac{1}{6}, \frac{1}{24}\left(244+12\sqrt{417}\right)^{(1/3)}-\frac{1}{3}\frac{1}{\left(244+12\sqrt{417}\right)^{(1/3)}}+\frac{1}{6}+\frac{1}{2}\,\mathrm{I}\sqrt{3}\left(-\frac{1}{12}\left(244+12\sqrt{417}\right)^{(1/3)}-\frac{2}{3}\frac{1}{\left(244+12\sqrt{417}\right)^{(1/3)}}\right), \frac{1}{24}\left(244+12\sqrt{417}\right)^{(1/3)}-\frac{1}{3}\frac{1}{\left(244+12\sqrt{417}\right)^{(1/3)}}+\frac{1}{6}-\frac{1}{2}\,\mathrm{I}\sqrt{3}\left(-\frac{1}{12}\left(244+12\sqrt{417}\right)^{(1/3)}-\frac{2}{3}\frac{1}{\left(244+12\sqrt{417}\right)^{(1/3)}}\right) \tag{1.4.6}$$

```
> evalf(%);
```

$$-.5000000000, -.4052678569, 0.4526339285-0.6418710860\,\mathrm{I}, 0.4526339285+0.6418710860\,\mathrm{I} \tag{1.4.7}$$

```
>
```

Exercici proposat 5:

- Determinar el valor del paràmetre a per tal que la funció sigui contínua en tot el seu domini

$$f(x) = \frac{(x-a)^2 \sin(x)}{x-1}$$

COMANDES: limit

> $f := x \to \frac{(x-a)^2 \sin(x)}{x-1}$

$$f := x \to \frac{(x-a)^2 \sin(x)}{x-1} \tag{1.5.1}$$

> $?iscont$

> $iscont(f(x), x = -3..3)$

$$false \tag{1.5.2}$$

> $discont(f(x), x)$

$$\{1\} \tag{1.5.3}$$

> $\lim_{x \to 1^+} f(x)$

$$\mathrm{signum}(-1+a)^2\ \infty \tag{1.5.4}$$

> $\lim_{x \to 1^-} f(x)$

$$-\mathrm{signum}(-1+a)^2\ \infty \tag{1.5.5}$$

Si $-1+a \neq 0$ tenim que la funció presenta una discontinuitat essencial en x=1 ja que els límits laterals són ∞

Per tant perquè la funció sigui contínua cal que $a = 1$

> $g := x \to subs(a = 1, f(x))$

$$g := x \to subs(a = 1, f(x)) \tag{1.5.6}$$

> $g(x)$

$$(x-1)\sin(x) \tag{1.5.7}$$

> $iscont(g, x = -1000..1000)$

$$true \tag{1.5.8}$$

>

Exercici en Math 1D

```
> f := x -> (x-a)^2*sin(x)/(x-1);
```

$$f := x \to \frac{(x-a)^2 \sin(x)}{x-1} \tag{1.5.9}$$

```
> iscont(f(x), x = -3 .. 3);
```

$$false \tag{1.5.10}$$

```
> discont(f(x), x);
```

$$\{1\} \tag{1.5.11}$$

```
> limit(f(x), x = 1, right);
```

$$\mathrm{signum}(-1+a)^2\ \infty \tag{1.5.12}$$

```
> limit(f(x), x = 1, left);
```

$$-\mathrm{signum}(-1+a)^2\ \infty \tag{1.5.13}$$

Si $-1+a \neq 0$ tenim que la funció presenta una discontinuitat essencial en x=1 ja que els límits laterals són ∞

Per tant perquè la funció sigui contínua cal que $a = 1$

```
> g := x -> subs(a = 1, f(x));
```

$$g := x \rightarrow subs(a = 1, f(x)) \tag{1.5.14}$$

```
> g(x);
```

$$(x-1)\sin(x) \tag{1.5.15}$$

```
> iscont(g, x = -1000 .. 1000);
```

$$true \tag{1.5.16}$$