

Ressolució Exercicis Bàsics Matrius

```
> restart;
> with(linalg):
Warning, the protected names norm and trace have been redefined and unprotected
```

Exercici:

Sigui la matriu $A := \begin{bmatrix} 2 & 1 \\ -2 & 3 \end{bmatrix}$. Determineu els valors de x , y que satisfan $A^2 + xA + Iy = N$, on I i N són la matriu identitat i la matriu nul·la d'ordre 2.

```
> A:=matrix(2,2,[2,1,-2,3]);
Id:=Matrix(2,2,shape=identity);
N:=matrix(2,2,0);
```

$$A := \begin{bmatrix} 2 & 1 \\ -2 & 3 \end{bmatrix}$$

$$Id := \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$N := \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

```
> evalm(A&*A+x*A+y*Matrix(2,2,shape=identity));
```

$$\begin{bmatrix} 2+2x+y & 5+x \\ -10-2x & 7+3x+y \end{bmatrix}$$

```
> solve({2+2*x+y=0, 5+x=0,-10-2*x=0, 7+3*x+y=0},{x,y});
{x=-5,y=8}
>
```

Exercici:

Si A és una matriu quadrada d'ordre 2 no nul·la i x és un nombre real, determineu aquests elements per tal que e compleixi:

$$\begin{bmatrix} 1 & 2 \\ 3 & x \end{bmatrix} A = \begin{bmatrix} 2 & 4 \\ 6 & 8 \end{bmatrix} A$$

```
> A:=matrix(2,2,[a11,a12],[a21,a22]);
```

$$A := \begin{bmatrix} a11 & a12 \\ a21 & a22 \end{bmatrix}$$

```
> M1:=matrix(2,2,[1,2],[3,x]);
M2:=matrix(2,2,[2,4],[6,8]);
```

$$M1 := \begin{bmatrix} 1 & 2 \\ 3 & x \end{bmatrix}$$

$$M2 := \begin{bmatrix} 2 & 4 \\ 6 & 8 \end{bmatrix}$$

```
> evalm(M1&*A-M2&*A);
```

$$\begin{bmatrix} -a11-2a21 & -a12-2a22 \\ -3a11+x a21-8a21 & -3a12+x a22-8a22 \end{bmatrix}$$

```
> solve({-a11-2*a21=0,-a12-2*a22=0,-3*a11+x*a21-8*a21=0,-3*a12+x*a22-8*a22=0},{a11,a12,a21,a22,x});
{a12=0,a11=0,a22=0,a21=0,x=x},{x=2,a12=-2a22,a11=-2a21,a21=a21,a22=a22}
```

Comprovació

```
> A:=matrix(2,2,[-2*a21,-2*a22,a21,a22]);
```

$$A := \begin{bmatrix} -2 a21 & -2 a22 \\ a21 & a22 \end{bmatrix}$$

> x:=2;

> evalm(M1&*A);

$$\begin{bmatrix} 0 & 0 \\ -4 a21 & -4 a22 \end{bmatrix}$$

> evalm(M2&*A);

$$\begin{bmatrix} 0 & 0 \\ -4 a21 & -4 a22 \end{bmatrix}$$

>

Exercici:

Donades les matrius

$$A := \begin{bmatrix} 2 & 1 & 2 \\ 3 & 1 & 2 \end{bmatrix} \quad B := \begin{bmatrix} 0 & 2 & -1 \\ 2 & 2 & 0 \end{bmatrix}$$

efectueu les operacions següents, si són possibles:

A+B, A-B, 2A-3B, A B^T, A^T B, A B, B A

```
> A:=matrix(2,3,[[2,1,2],[3,1,2]]);
> B:=matrix(2,3,[[0,2,-1],[2,2,0]]);
```

$$A := \begin{bmatrix} 2 & 1 & 2 \\ 3 & 1 & 2 \end{bmatrix}$$

$$B := \begin{bmatrix} 0 & 2 & -1 \\ 2 & 2 & 0 \end{bmatrix}$$

```
> evalm(A+B);
```

$$\begin{bmatrix} 2 & 3 & 1 \\ 5 & 3 & 2 \end{bmatrix}$$

```
> evalm(A-B);
```

$$\begin{bmatrix} 2 & -1 & 3 \\ 1 & -1 & 2 \end{bmatrix}$$

```
> evalm(2*A-3*B);
```

$$\begin{bmatrix} 4 & -4 & 7 \\ 0 & -4 & 4 \end{bmatrix}$$

```
> evalm(A&*transpose(B));
```

$$\begin{bmatrix} 0 & 6 \\ 0 & 8 \end{bmatrix}$$

```
> evalm(transpose(A) &*B);
```

$$\begin{bmatrix} 6 & 10 & -2 \\ 2 & 4 & -1 \\ 4 & 8 & -2 \end{bmatrix}$$

```
> evalm(A&*B);
```

Error, (in linalg:-multiply) non matching dimensions for vector/matrix product

```
> evalm(B&*A);
```

Error, (in linalg:-multiply) non matching dimensions for vector/matrix product

```
>
```

Exercici:

Calculeu la matriu $A^3 - 3A^2 - 5A + 2I$ en els casos:

$$A := \begin{bmatrix} 1 & 0 \\ -2 & -1 \end{bmatrix} \quad A := \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ -2 & -1 & 0 \end{bmatrix}$$

```
> I2:=Matrix(2,2,shape=identity);
> I3:=Matrix(3,3,shape=identity);
```

$$I2 := \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$I3 := \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

```
> A:=matrix(2,2,[[1,0],[-2,-1]]);
```

$$A := \begin{bmatrix} 1 & 0 \\ -2 & -1 \end{bmatrix}$$

```
> evalm(A^3-3*A^2-5*A+2*I2);
```

$$\begin{bmatrix} -5 & 0 \\ 8 & 3 \end{bmatrix}$$

```
> A:=matrix(3,3,[[0,0,0],[1,0,0],[-2,-1,0]]);
```

$$A := \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ -2 & -1 & 0 \end{bmatrix}$$

```
> evalm(A^3-3*A^2-5*A+2*I3);
```

$$\begin{bmatrix} 2 & 0 & 0 \\ -5 & 2 & 0 \\ 13 & 5 & 2 \end{bmatrix}$$

Exercici:

Comproveu que les matrius del tipus $\begin{bmatrix} a & b \\ b & a \end{bmatrix}$ commuten per al producte.

```
> M:=matrix(2,2,[[a1,b1],[b1,a1]]);
```

$$M := \begin{bmatrix} a1 & b1 \\ b1 & a1 \end{bmatrix}$$

```
> A:=matrix(2,2,[a2,b2,b2,a2]);
```

$$A := \begin{bmatrix} a2 & b2 \\ b2 & a2 \end{bmatrix}$$

```
> evalm(M&*A-A&*M);
```

$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

```
>
```

Exercici:

Determineu totes les matrius de d'ordre 2 tals que el seu quadrat és la matriu identitat d'ordre 2.

```
> A:=matrix(2,2,[a11,a12,a21,a22]);
I2:=Matrix(2,2,shape=identity);
```

$$A := \begin{bmatrix} a11 & a12 \\ a21 & a22 \end{bmatrix}$$

$$I2 := \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

```
> evalm(A^2-I2);
```

$$\begin{bmatrix} a11^2 + a12 a21 - 1 & a11 a12 + a12 a22 \\ a21 a11 + a22 a21 & a12 a21 + a22^2 - 1 \end{bmatrix}$$

```
> solve({a11^2+a12*a21-1=0,a11*a12+a12*a22=0,a21*a11+a22*a21=0,a12*a21+a2^2-1=0},{a11,a12,a21,a22});
```

$$\left\{ \begin{array}{l} \{a22 = 1, a12 = 0, a11 = 1, a21 = 0\}, \{a22 = -1, a12 = 0, a21 = 0, a11 = -1\}, \\ \{a22 = -1, a11 = 1, a21 = 0, a12 = a12\}, \{a22 = 1, a21 = 0, a12 = a12, a11 = -1\}, \\ \left\{ a12 = -\frac{-1 + a22^2}{a21}, a22 = a22, a11 = -a22, a21 = a21 \right\} \end{array} \right.$$

```
> A1:=matrix(2,2,[1,0,0,1]);
A2:=matrix(2,2,[-1,0,0,-1]);
A3:=matrix(2,2,[1,0,a,-1]);
A4:=matrix(2,2,[-1,0,a,1]);
A5:=matrix(2,2,[-a,b,(1-a^2)/b,a]);
```

$$A1 := \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$A2 := \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

$$A3 := \begin{bmatrix} 1 & 0 \\ a & -1 \end{bmatrix}$$

$$A4 := \begin{bmatrix} -1 & 0 \\ a & 1 \end{bmatrix}$$

$$A5 := \begin{bmatrix} -a & b \\ \frac{1-a^2}{b} & a \end{bmatrix}$$

[comprovació

```
> [evalm(A1^2), evalm(A1^3), evalm(A1^4), evalm(A1^5)];
```

$$\left[\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \right]$$

```
>
```

Exercici:

Dues matrius es diuen que commuten amb el producte si es compleix $AB=BA$. Trobeu totes les matrius de

d'ordre 2 que commuten amb el producte amb la matriu $A := \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

```
> A:=matrix(2,2,[1,2,3,4]);
```

$$A := \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

```
> B:=matrix(2,2,[a,b,c,d]);
```

$$B := \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

```
> evalm(A*B-B*A);
```

$$\begin{bmatrix} 2c-3b & -3b+2d-2a \\ 3a+3c-3d & 3b-2c \end{bmatrix}$$

```
> solve({2*c-3*b=0,-3*b+2*d-2*a=0,3*a+3*c-3*d=0,3*b-2*c=0},{a,b,c,d});
```

$$\left\{ a = -\frac{3}{2}b + d, c = \frac{3}{2}b, b = b, d = d \right\}$$

```
> M:=matrix(2,2,[-c+d,2/3*c,d]);
```

$$M := \begin{bmatrix} -c+d & \frac{2}{3}c \\ c & d \end{bmatrix}$$

[comprovació

```
> evalm(A*M);
```

$$\begin{bmatrix} c+d & \frac{2}{3}c+2d \\ c+3d & 2c+4d \end{bmatrix}$$

```
> evalm(M*A);
```

$$\begin{bmatrix} c+d & \frac{2}{3}c+2d \\ c+3d & 2c+4d \end{bmatrix}$$

```
>
```

Exercici:

Calcular mitjançant el mètode de Gauss el rang de les següents matrius:

$$\begin{bmatrix} -1 & 0 & 1 \\ 3 & 2 & 1 \\ 1 & -1 & 2 \end{bmatrix} \begin{bmatrix} 2 & 4 & 0 & 7 \\ 1 & 3 & 1 & 8 \\ 1 & 0 & -2 & -9 \end{bmatrix} \begin{bmatrix} 1 & 2 & 5 \\ 1 & -3 & -6 \\ -6 & -2 & -8 \\ 0 & 5 & 11 \\ 2 & -1 & -1 \end{bmatrix}$$

```
> M:=matrix(3,3,[-1,0,1,3,2,1,1,-1,2]);
```

$$M := \begin{bmatrix} -1 & 0 & 1 \\ 3 & 2 & 1 \\ 1 & -1 & 2 \end{bmatrix}$$

```
> gausselim(M);
```

$$\begin{bmatrix} -1 & 0 & 1 \\ 0 & 2 & 4 \\ 0 & 0 & 5 \end{bmatrix}$$

```
> rank(M);
```

3

```
> M:=matrix(3,3,[1,1,1,0,1,1,3,0,-6]);
```

$$M := \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 3 & 0 & -6 \end{bmatrix}$$

```
> gausselim(M);
```

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & -6 \end{bmatrix}$$

```
> rank(M);
```

3

```
> M:=matrix(3,4,[2,4,0,7,1,3,1,8,1,0,-2,-9]);
```

$$M := \begin{bmatrix} 2 & 4 & 0 & 7 \\ 1 & 3 & 1 & 8 \\ 1 & 0 & -2 & -9 \end{bmatrix}$$

```
> gausselim(M);
```

$$\begin{bmatrix} 2 & 4 & 0 & 7 \\ 0 & 1 & 1 & \frac{9}{2} \\ 0 & 0 & 0 & \frac{-7}{2} \end{bmatrix}$$

```
> rank(M);
```

3

```
> M:=matrix(5,3,[1,2,5,1,-3,-6,-6,-2,-8,0,5,11,2,-1,-1]);
```

$$M := \begin{bmatrix} 1 & 2 & 5 \\ 1 & -3 & -6 \\ -6 & -2 & -8 \\ 0 & 5 & 11 \\ 2 & -1 & -1 \end{bmatrix}$$

```
> gausselim(M);
```

$$\begin{bmatrix} 1 & 2 & 5 \\ 0 & -5 & -11 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

```
> rank(M);
```

2

```
>
```

Exercici:

Determinar els valors d'a pels quals rang(A) és diferent de 3, on:

$$A = \begin{bmatrix} 1 & a & 1 \\ a+1 & 1 & -a \\ 2 & 1 & -1 \end{bmatrix}$$

```
> A:=matrix(3,3,[1,a,1,a+1,1,-a,2,1,-1]);
```

$$A := \begin{bmatrix} 1 & a & 1 \\ a+1 & 1 & -a \\ 2 & 1 & -1 \end{bmatrix}$$

```
> det(A);
```

$$-2+3a-a^2$$

```
> solve(det(A),a);
```

1,2

```
> addcol(A,3,1,1);
```

$$\begin{bmatrix} 2 & a & 1 \\ 1 & 1 & -a \\ 1 & 1 & -1 \end{bmatrix}$$

```
> addrow(% ,1,2,-1/2);
```

$$\begin{bmatrix} 2 & a & 1 \\ 0 & -\frac{1}{2}a+1 & -\frac{1}{2}-a \\ 1 & 1 & -1 \end{bmatrix}$$

```
> addrow(% ,1,3,-1/2);
```

$$\begin{bmatrix} 2 & a & 1 \\ 0 & -\frac{1}{2}a+1 & -\frac{1}{2}-a \\ 0 & -\frac{1}{2}a+1 & \frac{-3}{2} \end{bmatrix}$$

```
> addrow(% ,2,3,-1);
```

$$\begin{bmatrix} 2 & a & 1 \\ 0 & -\frac{1}{2}a+1 & -\frac{1}{2}-a \\ 0 & 0 & -1+a \end{bmatrix}$$

```
> a:=1:
```

```
matrix([[2, a, 1], [0, -1/2*a+1, -1/2-a], [0, 0, -1+a]]);
```

$$\begin{bmatrix} 2 & 1 & 1 \\ 0 & \frac{1}{2} & \frac{-3}{2} \\ 0 & 0 & 0 \end{bmatrix}$$

```
> a:=2:
```

```
matrix([[2, a, 1], [0, -1/2*a+1, -1/2-a], [0, 0, -1+a]]);
```

$$\begin{bmatrix} 2 & 2 & 1 \\ 0 & 0 & \frac{-5}{2} \\ 0 & 0 & 1 \end{bmatrix}$$

```
> addrow(% ,2,3,2/5);
```

$$\begin{bmatrix} 2 & 2 & 1 \\ 0 & 0 & \frac{-5}{2} \\ 0 & 0 & 0 \end{bmatrix}$$

```
>
```

Exercici:

Calcular les inverses de les matrius:

$$\begin{bmatrix} -2 & 3 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} 1 & 3 & 1 \\ 2 & -1 & 2 \\ 3 & 2 & -3 \end{bmatrix} \begin{bmatrix} 1 & 2 & -3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

```
> A:=matrix(2,2,[-2,3,1,-1]);
`A`^-1`=inverse(A);
```

$$A := \begin{bmatrix} -2 & 3 \\ 1 & -1 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 1 & 3 \\ 1 & 2 \end{bmatrix}$$

```
> A:=matrix(3,3,[1,3,1,2,-1,2,3,2,-3]);
`A`^-1`=inverse(A);
```

$$A := \begin{bmatrix} 1 & 3 & 1 \\ 2 & -1 & 2 \\ 3 & 2 & -3 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} -\frac{1}{42} & \frac{11}{42} & \frac{1}{6} \\ \frac{2}{7} & -\frac{1}{7} & 0 \\ \frac{1}{6} & \frac{1}{6} & -\frac{1}{6} \end{bmatrix}$$

```
> A:=matrix(3,3,[1,2,-3,0,1,2,0,0,1]);
`A`^-1`=inverse(A);
```

$$A := \begin{bmatrix} 1 & 2 & -3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} 1 & -2 & 7 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}$$

```
>
```

Exercici:

Troba, pels valors de k que sigui possible, la matriu inversa de:

$$\begin{bmatrix} 1 & k & k \\ 1 & 2 & 4 \\ 1 & 3 & 9 \end{bmatrix}$$

```
> A:=matrix(3,3,[1,k,k,1,2,4,1,3,9]);
```

$$A := \begin{bmatrix} 1 & k & k \\ 1 & 2 & 4 \\ 1 & 3 & 9 \end{bmatrix}$$

```
> det(A);
```

$$6 - 4k$$

```
> solve({6-4*k=0},k);
```

$$\left\{ k = \frac{3}{2} \right\}$$

```
> swaprow(A,1,3);
```

$$\begin{bmatrix} 1 & 3 & 9 \\ 1 & 2 & 4 \\ 1 & k & k \end{bmatrix}$$

```
> addrow(% ,1,2,-1);
```

$$\begin{bmatrix} 1 & 3 & 9 \\ 0 & -1 & -5 \\ 1 & k & k \end{bmatrix}$$

```
> addrow(% ,1,3,-1);
```

$$\begin{bmatrix} 1 & 3 & 9 \\ 0 & -1 & -5 \\ 0 & -3+k & -9+k \end{bmatrix}$$

```
> addrow(% ,2,3,-3);
```

$$\begin{bmatrix} 1 & 3 & 9 \\ 0 & -1 & -5 \\ 0 & k & 6+k \end{bmatrix}$$

```
> addrow(% ,2,3,k);
```

$$\begin{bmatrix} 1 & 3 & 9 \\ 0 & -1 & -5 \\ 0 & 0 & 6-4k \end{bmatrix}$$

> solve(6-4*k=0,k);

$$\frac{3}{2}$$

> k:=3/2:

> A:=matrix(3,3,[1,k,k,1,2,4,1,3,9]);

$$A := \begin{bmatrix} 1 & \frac{3}{2} & \frac{3}{2} \\ 1 & 2 & 4 \\ 1 & 3 & 9 \end{bmatrix}$$

> rank(A);

$$2$$

>

Exercici:

Calculeu els determinants de les següents matrius:

$$\begin{bmatrix} \cos(\alpha) & -\sin(\alpha) \\ \sin(\alpha) & \cos(\alpha) \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ x & y & z \\ x^2 & y^2 & z^2 \end{bmatrix} \begin{bmatrix} 2 & 1 & 3 & -1 \\ 3 & 1 & 2 & 1 \\ 1 & 2 & 3 & 1 \\ 1 & -1 & 1 & -1 \end{bmatrix}$$

> M:=matrix(2,2,[cos(alpha),-sin(alpha),sin(alpha),cos(alpha)]);
simplify(det(M));

$$M := \begin{bmatrix} \cos(\alpha) & -\sin(\alpha) \\ \sin(\alpha) & \cos(\alpha) \end{bmatrix}$$

$$1$$

> M:=matrix(3,3,[1,1,1,x,y,z,x^2,y^2,z^2]);

simplify(det(M));

$$M := \begin{bmatrix} 1 & 1 & 1 \\ 2 & y & z \\ 4 & y^2 & z^2 \end{bmatrix}$$

$$yz^2 - zy^2 - 2z^2 + 2y^2 + 4z - 4y$$

> M:=matrix(4,4,[2,1,3,-1,3,1,2,1,1,2,3,1,1,-1,1,-1]);
simplify(det(M));

$$M := \begin{bmatrix} 2 & 1 & 3 & -1 \\ 3 & 1 & 2 & 1 \\ 1 & 2 & 3 & 1 \\ 1 & -1 & 1 & -1 \end{bmatrix}$$

$$15$$

>

Exercici:

Calculeu els determinants següents, desenvolupant per la columna o fila més adequada:

$$\begin{bmatrix} 3 & -2 & 0 \\ -3 & 9 & 5 \\ 6 & 5 & 0 \end{bmatrix} \begin{bmatrix} 2 & 1 & -6 \\ 3 & -1 & 7 \\ 0 & 0 & 4 \end{bmatrix} \begin{bmatrix} 3 & -1 & 5 \\ 0 & 2 & 0 \\ 1 & 1 & -1 \end{bmatrix}$$

> M:=matrix(3,3,[3,-2,0,-3,9,5,6,5,0]);
simplify(det(M));

$$M := \begin{bmatrix} 3 & -2 & 0 \\ -3 & 9 & 5 \\ 6 & 5 & 0 \end{bmatrix}$$

$$-135$$

> M:=matrix(3,3,[2,1,-6,3,-1,7,0,0,4]);
simplify(det(M));

$$M := \begin{bmatrix} 2 & 1 & -6 \\ 3 & -1 & 7 \\ 0 & 0 & 4 \end{bmatrix}$$

```
> M:=matrix(3,3,[3,-1,5,0,2,0,1,1,-1]);
simplify(det(M));
```

$$M := \begin{bmatrix} 3 & -1 & 5 \\ 0 & 2 & 0 \\ 1 & 1 & -1 \end{bmatrix}$$

-20

-16

Exercici:

Calculeu la inversa de la matriu $\begin{bmatrix} 2 & 3 \\ 1 & -1 \end{bmatrix}$, i comproveu que $\det(A) = 1/\det(A)$.

```
> A:=matrix(2,2,[2,3,1,-1]);
```

$$A := \begin{bmatrix} 2 & 3 \\ 1 & -1 \end{bmatrix}$$

```
> inverse(A);
```

$$\begin{bmatrix} \frac{1}{5} & \frac{3}{5} \\ \frac{1}{5} & -\frac{2}{5} \end{bmatrix}$$

```
> det(A);
det(inverse(A));
```

-5

$-\frac{1}{5}$

Exercici:

Calculeu el rang de les matrius llistades a sota.

```
> A:=matrix(3,3,[1,2,1,-1,0,1,3,2,2]);
```

$$A := \begin{bmatrix} 1 & 2 & 1 \\ -1 & 0 & 1 \\ 3 & 2 & 2 \end{bmatrix}$$

```
> addrow(A,1,2,1);
```

$$\begin{bmatrix} 1 & 2 & 1 \\ 0 & 2 & 2 \\ 3 & 2 & 2 \end{bmatrix}$$

```
> addrow(% ,1,3,-3);
```

$$\begin{bmatrix} 1 & 2 & 1 \\ 0 & 2 & 2 \\ 0 & -4 & -1 \end{bmatrix}$$

```
> addrow(% ,2,3,2);
```

$$\begin{bmatrix} 1 & 2 & 1 \\ 0 & 2 & 2 \\ 0 & 0 & 3 \end{bmatrix}$$

```
> rank(A);
```

3

```
> A:=matrix(4,4,[2,1,3,-1,3,-1,2,0,1,3,4,-2,4,-3,1,1]);
```

$$A := \begin{bmatrix} 2 & 1 & 3 & -1 \\ 3 & -1 & 2 & 0 \\ 1 & 3 & 4 & -2 \\ 4 & -3 & 1 & 1 \end{bmatrix}$$

```
> mulrow(A,2,2);
```

$$\begin{bmatrix} 2 & 1 & 3 & -1 \\ 6 & -2 & 4 & 0 \\ 1 & 3 & 4 & -2 \\ 4 & -3 & 1 & 1 \end{bmatrix}$$

```
> addrow(% ,1,2,-3);
```

$$\begin{bmatrix} 2 & 1 & 3 & -1 \\ 0 & -5 & -5 & 3 \\ 1 & 3 & 4 & -2 \\ 4 & -3 & 1 & 1 \end{bmatrix}$$

```
> mulrow(% ,3,2);
```

$$\begin{bmatrix} 2 & 1 & 3 & -1 \\ 0 & -5 & -5 & 3 \\ 2 & 6 & 8 & -4 \\ 4 & -3 & 1 & 1 \end{bmatrix}$$

```
> addrow(% ,1,3,-1);
```

$$\begin{bmatrix} 2 & 1 & 3 & -1 \\ 0 & -5 & -5 & 3 \\ 0 & 5 & 5 & -3 \\ 4 & -3 & 1 & 1 \end{bmatrix}$$

```
> addrow(% ,1,4,-2);
```

$$\begin{bmatrix} 2 & 1 & 3 & -1 \\ 0 & -5 & -5 & 3 \\ 0 & 5 & 5 & -3 \\ 0 & -5 & -5 & 3 \end{bmatrix}$$

```
> addrow(% ,2,3,1);
```

$$\begin{bmatrix} 2 & 1 & 3 & -1 \\ 0 & -5 & -5 & 3 \\ 0 & 0 & 0 & 0 \\ 0 & -5 & -5 & 3 \end{bmatrix}$$

```
> addrow(% ,2,4,-1);
```

$$\begin{bmatrix} 2 & 1 & 3 & -1 \\ 0 & -5 & -5 & 3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

```
> rank(A);
```

2

```
> gausselim(A);
```

$$\begin{bmatrix} 2 & 1 & 3 & -1 \\ 0 & -5 & -5 & 3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

```
>
```

```
> A:=matrix(4,6,[2,1,1,1,1,1,3,1,1,2,1,1,1,4,1,3,1,1,1,1,5,4,1]);
```

$$A := \begin{bmatrix} 2 & 1 & 1 & 1 & 1 & 1 \\ 1 & 3 & 1 & 1 & 2 & 1 \\ 1 & 1 & 4 & 1 & 3 & 1 \\ 1 & 1 & 1 & 5 & 4 & 1 \end{bmatrix}$$

```
> swaprow(A,4,1);
```

```

> addrow(% ,1,2,-1);

$$\begin{bmatrix} 1 & 1 & 1 & 5 & 4 & 1 \\ 1 & 3 & 1 & 1 & 2 & 1 \\ 1 & 1 & 4 & 1 & 3 & 1 \\ 2 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

> addrow(% ,1,3,-1);

$$\begin{bmatrix} 1 & 1 & 1 & 5 & 4 & 1 \\ 0 & 2 & 0 & -4 & -2 & 0 \\ 1 & 1 & 4 & 1 & 3 & 1 \\ 2 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

> addrow(% ,1,4,-2);

$$\begin{bmatrix} 1 & 1 & 1 & 5 & 4 & 1 \\ 0 & 2 & 0 & -4 & -2 & 0 \\ 0 & 0 & 3 & -4 & -1 & 0 \\ 2 & 1 & 1 & 1 & 1 & 1 \end{bmatrix}$$

> mulrow(% ,4,2);

$$\begin{bmatrix} 1 & 1 & 1 & 5 & 4 & 1 \\ 0 & 2 & 0 & -4 & -2 & 0 \\ 0 & 0 & 3 & -4 & -1 & 0 \\ 0 & -2 & -2 & -18 & -14 & -2 \end{bmatrix}$$

> addrow(% ,2,4,1);

```

```

> mulrow(% ,4,3);

$$\begin{bmatrix} 1 & 1 & 1 & 5 & 4 & 1 \\ 0 & 2 & 0 & -4 & -2 & 0 \\ 0 & 0 & 3 & -4 & -1 & 0 \\ 0 & 0 & -2 & -22 & -16 & -2 \end{bmatrix}$$

> addrow(% ,3,4,2);

$$\begin{bmatrix} 1 & 1 & 1 & 5 & 4 & 1 \\ 0 & 2 & 0 & -4 & -2 & 0 \\ 0 & 0 & 3 & -4 & -1 & 0 \\ 0 & 0 & 0 & -74 & -50 & -6 \end{bmatrix}$$

> rank(A);
4
>
> A:=matrix(4,3,[1,1,1,1,-1,1,1,1,-1,3,1,1]);

$$A := \begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \\ 3 & 1 & 1 \end{bmatrix}$$

> gausselim(A);

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & -2 & 0 \\ 0 & 0 & -2 \\ 0 & 0 & 0 \end{bmatrix}$$

> rank(A);

```

```

>
> A:=matrix(3,5,[1,1,2,-1,0,0,-1,1,1,1,0,0,2,-1]);
A:=
[ 1  1  2 -1  0
  0 -1  1  1  1
  1  0  0  2 -1]
> gausselim(A);
[ 1  1  2 -1  0
  0 -1  1  1  1
  0  0 -3  2 -2]
> rank(A);
3
>
> A:=matrix(4,4,[1,-1,-2,1,0,1,0,0,-2,0,4,-1,-1,2,2,0]);
A:=
[ 1 -1 -2  1
  0  1  0  0
 -2  0  4 -1
 -1  2  2  0]
> gausselim(A);
[ 1 -1 -2  1
  0  1  0  0
  0  0  0  1
  0  0  0  0]
> rank(A);
3
>
> A:=matrix(3,3,[1,1,0,2,-1,0,1,1,1]);

```

```

A:=
[ 1  1  0
  2 -1  0
  1  1  1]
> gausselim(A);
[ 1  1  0
  0 -3  0
  0  0  1]
> rank(A);
3
>
> A:=matrix(3,4,[2,1,3,4,-1,0,1,2,-6,-2,-4,-4]);
A:=
[ 2  1  3  4
 -1  0  1  2
 -6 -2 -4 -4]
> gausselim(A);
[ 2  1  3  4
  0  1  5  8
  0  0  0  0]
> rank(A);
2
>
> A:=matrix(4,4,[1,0,3,5,-1,0,-3,-5,3,1,5,9,2,2,1,1]);
A:=
[ 1  0  3  5
 -1  0 -3 -5
  3  1  5  9
  2  2  1  1]
> gausselim(A);

```

$$\begin{bmatrix} 1 & 0 & 3 & 5 \\ 0 & 1 & -4 & -6 \\ 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

```
> rank(A);
```

3

```
>
```

```
> unassign('a');
```

```
> A:=matrix(3,3,[a,1,1,1,a,1,1,1,a]);
```

$$A := \begin{bmatrix} a & 1 & 1 \\ 1 & a & 1 \\ 1 & 1 & a \end{bmatrix}$$

```
> det(A);
```

$a^3 - 3a + 2$

```
> factor(det(A));
```

$(a+2)(a-1)^2$

```
> swaprow(A,1,3);
```

$$\begin{bmatrix} 1 & 1 & a \\ 1 & a & 1 \\ a & 1 & 1 \end{bmatrix}$$

```
> addrow(% ,1,2,-1);
```

$$\begin{bmatrix} 1 & 1 & a \\ 0 & a-1 & -a+1 \\ a & 1 & 1 \end{bmatrix}$$

```
> addrow(% ,1,3,-a);
```

$$\begin{bmatrix} 1 & 1 & a \\ 0 & a-1 & -a+1 \\ 0 & -a+1 & 1-a^2 \end{bmatrix}$$

```
> addrow(% ,2,3,1);
```

$$\begin{bmatrix} 1 & 1 & a \\ 0 & a-1 & -a+1 \\ 0 & 0 & -a+2-a^2 \end{bmatrix}$$

```
> solve(-a+2-a^2=0,a);
```

-2, 1

```
> a:=1:
```

```
matrix([[1, 1, a], [0, -1+a, -a+1], [0, 0, -a+2-a^2]]);
```

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

```
> a:=-2:
```

```
matrix([[1, 1, a], [0, -1+a, -a+1], [0, 0, -a+2-a^2]]);
```

$$\begin{bmatrix} 1 & 1 & -2 \\ 0 & -3 & 3 \\ 0 & 0 & 0 \end{bmatrix}$$

```
>
```

```
> unassign('a');
```

```
> A:=matrix(3,3,[a,1,-1,1,-a,1,1,1,a]);
```

$$A := \begin{bmatrix} a & 1 & -1 \\ 1 & -a & 1 \\ 1 & 1 & a \end{bmatrix}$$

```
> swaprow(A,1,3);
```

$$\begin{bmatrix} 1 & 1 & a \\ 1 & -a & 1 \\ a & 1 & -1 \end{bmatrix}$$

```
> addrow(%1,2,-1);
```

$$\begin{bmatrix} 1 & 1 & a \\ 0 & -a-1 & -a+1 \\ a & 1 & -1 \end{bmatrix}$$

```
> addrow(%1,3,-a);
```

$$\begin{bmatrix} 1 & 1 & a \\ 0 & -a-1 & -a+1 \\ 0 & -a+1 & -a^2-1 \end{bmatrix}$$

```
> addrow(%2,3,-1);
```

$$\begin{bmatrix} 1 & 1 & a \\ 0 & -a-1 & -a+1 \\ 0 & 2 & a-2-a^2 \end{bmatrix}$$

```
> swaprow(%2,3);
```

$$\begin{bmatrix} 1 & 1 & a \\ 0 & 2 & a-2-a^2 \\ 0 & -a-1 & -a+1 \end{bmatrix}$$

```
> addrow(%2,3,(1+a)/2);
```

$$\begin{bmatrix} 1 & 1 & a \\ 0 & 2 & a-2-a^2 \\ 0 & 0 & \left(\frac{1}{2}a+\frac{1}{2}\right)(a-2-a^2)-a+1 \end{bmatrix}$$

```
> simplify(%);
```

$$\begin{bmatrix} 1 & 1 & a \\ 0 & 2 & a-2-a^2 \\ 0 & 0 & -\frac{3}{2}a-\frac{1}{2}a^3 \end{bmatrix}$$

```
> mulrow(%3,2);
```

$$\begin{bmatrix} 1 & 1 & a \\ 0 & 2 & a-2-a^2 \\ 0 & 0 & -3a-a^3 \end{bmatrix}$$

```
> solve(-3*a-a^3,a);
```

$$0, I\sqrt{3}, -I\sqrt{3}$$

```
> a:=0:
```

```
matrix([[1, 1, a], [0, 2, -2+a-a^2], [0, 0, -3*a-a^3]]);
```

$$\begin{bmatrix} 1 & 1 & 0 \\ 0 & 2 & -2 \\ 0 & 0 & 0 \end{bmatrix}$$

```
>
```

Exercici:

Troba, si és possible, les matrius inverses de les matrius llistades a sota.

```
> A:=matrix(3,3,[1,0,0,-1,1,0,1,1,1]);
```

$$A := \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

```
> I3:=Matrix(3,3,shape=identity);
```

$$I3 := \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

```
> AI3:=concat(A,I3);
```

$$A I3 := \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ -1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

```
> addrow(% , 1, 2, 1);
```

$$\begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

```
> addrow(% , 1, 3, -1);
```

$$\begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 1 & 1 & -1 & 0 & 1 \end{bmatrix}$$

```
> addrow(% , 2, 3, -1);
```

$$\begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & -2 & -1 & 1 \end{bmatrix}$$

```
> inverse(A);
```

$$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ -2 & -1 & 1 \end{bmatrix}$$

```
>
```

```
> A:=matrix(3,3,[1,2,3,-1,0,1,0,1,2]);
```

$$A := \begin{bmatrix} 1 & 2 & 3 \\ -1 & 0 & 1 \\ 0 & 1 & 2 \end{bmatrix}$$

```
> I3:=Matrix(3,3,shape=identity);
```

$$I3 := \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

```
> AI3:=concat(A,I3);
```

$$A I3 := \begin{bmatrix} 1 & 2 & 3 & 1 & 0 & 0 \\ -1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 2 & 0 & 0 & 1 \end{bmatrix}$$

```
> addrow(% , 1, 2, 1);
```

$$\begin{bmatrix} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 2 & 4 & 1 & 1 & 0 \\ 0 & 1 & 2 & 0 & 0 & 1 \end{bmatrix}$$

```
> mulrow(% , 3, -2);
```

$$\begin{bmatrix} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 2 & 4 & 1 & 1 & 0 \\ 0 & -2 & -4 & 0 & 0 & -2 \end{bmatrix}$$

```
> addrow(% , 2, 3, 1);
```

$$\begin{bmatrix} 1 & 2 & 3 & 1 & 0 & 0 \\ 0 & 2 & 4 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & -2 \end{bmatrix}$$

```
> det(A);
```

0

```
>
```

```
> A:=matrix(4,4,[1,2,-1,1,-1,1,2,1,1,0,3,1,-1,2,1,4]);
```

$$A := \begin{bmatrix} 1 & 2 & -1 & 1 \\ -1 & 1 & 2 & 1 \\ 1 & 0 & 3 & 1 \\ -1 & 2 & 1 & 4 \end{bmatrix}$$

```
> Id:=Matrix(4,4,shape=identity);
```

$$Id := \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

```
> AId:=concat(A,Id);
```

$$AId := \begin{bmatrix} 1 & 2 & -1 & 1 & 1 & 0 & 0 & 0 \\ -1 & 1 & 2 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 3 & 1 & 0 & 0 & 1 & 0 \\ -1 & 2 & 1 & 4 & 0 & 0 & 0 & 1 \end{bmatrix}$$

```
> addrow(% ,1,2,1);
```

$$\begin{bmatrix} 1 & 2 & -1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 3 & 1 & 2 & 1 & 1 & 0 & 0 \\ 1 & 0 & 3 & 1 & 0 & 0 & 1 & 0 \\ -1 & 2 & 1 & 4 & 0 & 0 & 0 & 1 \end{bmatrix}$$

```
> addrow(% ,1,3,-1);
```

$$\begin{bmatrix} 1 & 2 & -1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 3 & 1 & 2 & 1 & 1 & 0 & 0 \\ 0 & -2 & 4 & 0 & -1 & 0 & 1 & 0 \\ -1 & 2 & 1 & 4 & 0 & 0 & 0 & 1 \end{bmatrix}$$

```
> addrow(% ,1,4,1);
```

$$\begin{bmatrix} 1 & 2 & -1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 3 & 1 & 2 & 1 & 1 & 0 & 0 \\ 0 & -2 & 4 & 0 & -1 & 0 & 1 & 0 \\ 0 & 4 & 0 & 5 & 1 & 0 & 0 & 1 \end{bmatrix}$$

```
> mulrow(% ,3,3);
```

$$\begin{bmatrix} 1 & 2 & -1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 3 & 1 & 2 & 1 & 1 & 0 & 0 \\ 0 & -6 & 12 & 0 & -3 & 0 & 3 & 0 \\ 0 & 4 & 0 & 5 & 1 & 0 & 0 & 1 \end{bmatrix}$$

```
> mulrow(% ,4,-3);
```

$$\begin{bmatrix} 1 & 2 & -1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 3 & 1 & 2 & 1 & 1 & 0 & 0 \\ 0 & -6 & 12 & 0 & -3 & 0 & 3 & 0 \\ 0 & 36 & 0 & 45 & 9 & 0 & 0 & 9 \end{bmatrix}$$

```
> addrow(% ,2,3,2);
```

$$\begin{bmatrix} 1 & 2 & -1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 3 & 1 & 2 & 1 & 1 & 0 & 0 \\ 0 & 0 & 14 & 4 & -1 & 2 & 3 & 0 \\ 0 & 36 & 0 & 45 & 9 & 0 & 0 & 9 \end{bmatrix}$$

```
> addrow(% ,2,4,4);
```

$$\begin{bmatrix} 1 & 2 & -1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 3 & 1 & 2 & 1 & 1 & 0 & 0 \\ 0 & 0 & 14 & 4 & -1 & 2 & 3 & 0 \\ 0 & 48 & 4 & 53 & 13 & 4 & 0 & 9 \end{bmatrix}$$

```
> addrow(% ,3,4,-4/14);
```

$$\begin{bmatrix} 1 & 2 & -1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 3 & 1 & 2 & 1 & 1 & 0 & 0 \\ 0 & 0 & 14 & 4 & -1 & 2 & 3 & 0 \\ 0 & 48 & 0 & \frac{363}{7} & \frac{93}{7} & \frac{24}{7} & \frac{-6}{7} & 9 \end{bmatrix}$$

```
> mulrow(% ,2,1/3);
```

$$\begin{bmatrix} 1 & 2 & -1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & \frac{1}{3} & \frac{2}{3} & \frac{1}{3} & \frac{1}{3} & 0 & 0 \\ 0 & 0 & 14 & 4 & -1 & 2 & 3 & 0 \\ 0 & 0 & 0 & \frac{-57}{7} & \frac{9}{7} & \frac{24}{7} & \frac{-6}{7} & -3 \end{bmatrix}$$

> addrow(% ,2,1,-2);

$$\begin{bmatrix} 1 & 0 & \frac{-5}{3} & \frac{-1}{3} & \frac{1}{3} & \frac{-2}{3} & 0 & 0 \\ 0 & 1 & \frac{1}{3} & \frac{2}{3} & \frac{1}{3} & \frac{1}{3} & 0 & 0 \\ 0 & 0 & 14 & 4 & -1 & 2 & 3 & 0 \\ 0 & 0 & 0 & \frac{-57}{7} & \frac{9}{7} & \frac{24}{7} & \frac{-6}{7} & -3 \end{bmatrix}$$

> mulrow(% ,3,1/14);

$$\begin{bmatrix} 1 & 0 & \frac{-5}{3} & \frac{-1}{3} & \frac{1}{3} & \frac{-2}{3} & 0 & 0 \\ 0 & 1 & \frac{1}{3} & \frac{2}{3} & \frac{1}{3} & \frac{1}{3} & 0 & 0 \\ 0 & 0 & 1 & \frac{2}{7} & \frac{-1}{14} & \frac{1}{7} & \frac{3}{14} & 0 \\ 0 & 0 & 0 & \frac{-57}{7} & \frac{9}{7} & \frac{24}{7} & \frac{-6}{7} & -3 \end{bmatrix}$$

> addrow(% ,3,1,5/3);

$$\begin{bmatrix} 1 & 0 & 0 & \frac{1}{7} & \frac{3}{14} & \frac{-3}{7} & \frac{5}{14} & 0 \\ 0 & 1 & \frac{1}{3} & \frac{2}{3} & \frac{1}{3} & \frac{1}{3} & 0 & 0 \\ 0 & 0 & 1 & \frac{2}{7} & \frac{-1}{14} & \frac{1}{7} & \frac{3}{14} & 0 \\ 0 & 0 & 0 & \frac{-57}{7} & \frac{9}{7} & \frac{24}{7} & \frac{-6}{7} & -3 \end{bmatrix}$$

> addrow(% ,3,2,-1/3);

$$\begin{bmatrix} 1 & 0 & 0 & \frac{1}{7} & \frac{3}{14} & \frac{-3}{7} & \frac{5}{14} & 0 \\ 0 & 1 & 0 & \frac{4}{7} & \frac{5}{14} & \frac{2}{7} & \frac{-1}{14} & 0 \\ 0 & 0 & 1 & \frac{2}{7} & \frac{-1}{14} & \frac{1}{7} & \frac{3}{14} & 0 \\ 0 & 0 & 0 & \frac{-57}{7} & \frac{9}{7} & \frac{24}{7} & \frac{-6}{7} & -3 \end{bmatrix}$$

> mulrow(% ,4,-7/57);

$$\begin{bmatrix} 1 & 0 & 0 & \frac{1}{7} & \frac{3}{14} & \frac{-3}{7} & \frac{5}{14} & 0 \\ 0 & 1 & 0 & \frac{4}{7} & \frac{5}{14} & \frac{2}{7} & \frac{-1}{14} & 0 \\ 0 & 0 & 1 & \frac{2}{7} & \frac{-1}{14} & \frac{1}{7} & \frac{3}{14} & 0 \\ 0 & 0 & 0 & 1 & \frac{-3}{19} & \frac{-8}{19} & \frac{2}{19} & \frac{7}{19} \end{bmatrix}$$

> addrow(% ,4,1,-1/7);

$$\begin{bmatrix} 1 & 0 & 0 & 0 & \frac{9}{38} & \frac{-7}{19} & \frac{13}{38} & \frac{-1}{19} \\ 0 & 1 & 0 & \frac{4}{7} & \frac{5}{14} & \frac{2}{7} & \frac{-1}{14} & 0 \\ 0 & 0 & 1 & \frac{2}{7} & \frac{-1}{14} & \frac{1}{7} & \frac{3}{14} & 0 \\ 0 & 0 & 0 & 1 & \frac{-3}{19} & \frac{-8}{19} & \frac{2}{19} & \frac{7}{19} \end{bmatrix}$$

> addrow(% ,4,2,-4/7);

$$\begin{bmatrix} 1 & 0 & 0 & 0 & \frac{9}{38} & -\frac{7}{19} & \frac{13}{38} & -\frac{1}{19} \\ 0 & 1 & 0 & 0 & \frac{17}{38} & \frac{10}{19} & -\frac{5}{38} & -\frac{4}{19} \\ 0 & 0 & 1 & \frac{2}{7} & -\frac{1}{14} & \frac{1}{7} & \frac{3}{14} & 0 \\ 0 & 0 & 0 & 1 & -\frac{3}{19} & -\frac{8}{19} & \frac{2}{19} & \frac{7}{19} \end{bmatrix}$$

> addrow(% ,4,3,-2/7);

$$\begin{bmatrix} 1 & 0 & 0 & 0 & \frac{9}{38} & -\frac{7}{19} & \frac{13}{38} & -\frac{1}{19} \\ 0 & 1 & 0 & 0 & \frac{17}{38} & \frac{10}{19} & -\frac{5}{38} & -\frac{4}{19} \\ 0 & 0 & 1 & 0 & -\frac{1}{38} & \frac{5}{19} & \frac{7}{38} & -\frac{2}{19} \\ 0 & 0 & 0 & 1 & -\frac{3}{19} & -\frac{8}{19} & \frac{2}{19} & \frac{7}{19} \end{bmatrix}$$

> inverse(A);

$$\begin{bmatrix} \frac{9}{38} & -\frac{7}{19} & \frac{13}{38} & -\frac{1}{19} \\ \frac{17}{38} & \frac{10}{19} & -\frac{5}{38} & -\frac{4}{19} \\ -\frac{1}{38} & \frac{5}{19} & \frac{7}{38} & -\frac{2}{19} \\ -\frac{3}{19} & -\frac{8}{19} & \frac{2}{19} & \frac{7}{19} \end{bmatrix}$$

>

> A:=matrix(4,4,[1,1,0,0,0,1,1,0,0,0,1,1,1,0,0,1]);

$$A := \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \end{bmatrix}$$

> Id:=Matrix(4,4,shape=identity);

$$Id := \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

> AId:=concat(A,Id);

$$AId := \begin{bmatrix} 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

> addrow(% ,1,4,-1);

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 1 & -1 & 0 & 0 & 1 \end{bmatrix}$$

> addrow(% ,2,4,1);

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & -1 & 1 & 0 & 1 \end{bmatrix}$$

> addrow(% ,3,4,-1);

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 & -1 & 1 \end{bmatrix}$$

> det(A);

0

>

> A:=matrix(3,3,[2,4,3,0,1,1,2,2,-1]);

$$A := \begin{bmatrix} 2 & 4 & 3 \\ 0 & 1 & 1 \\ 2 & 2 & -1 \end{bmatrix}$$

> I3:=Matrix(3,3,shape=identity);

$$I3 := \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

> AI3:=concat(A,I3);

$$AI3 := \begin{bmatrix} 2 & 4 & 3 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 2 & 2 & -1 & 0 & 0 & 1 \end{bmatrix}$$

> addrow(% ,1,3,-1);

$$\begin{bmatrix} 2 & 4 & 3 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & -2 & -4 & -1 & 0 & 1 \end{bmatrix}$$

> mulrow(% ,2,2);

$$\begin{bmatrix} 2 & 4 & 3 & 1 & 0 & 0 \\ 0 & 2 & 2 & 0 & 2 & 0 \\ 0 & -2 & -4 & -1 & 0 & 1 \end{bmatrix}$$

> addrow(% ,2,3,1);

$$\begin{bmatrix} 2 & 4 & 3 & 1 & 0 & 0 \\ 0 & 2 & 2 & 0 & 2 & 0 \\ 0 & 0 & -2 & -1 & 2 & 1 \end{bmatrix}$$

> addrow(% ,2,1,-2);

$$\begin{bmatrix} 2 & 0 & -1 & 1 & -4 & 0 \\ 0 & 2 & 2 & 0 & 2 & 0 \\ 0 & 0 & -2 & -1 & 2 & 1 \end{bmatrix}$$

> addrow(% ,3,2,1);

$$\begin{bmatrix} 2 & 0 & -1 & 1 & -4 & 0 \\ 0 & 2 & 0 & -1 & 4 & 1 \\ 0 & 0 & -2 & -1 & 2 & 1 \end{bmatrix}$$

> mulrow(% ,3,-1/2);

$$\begin{bmatrix} 2 & 0 & -1 & 1 & -4 & 0 \\ 0 & 2 & 0 & -1 & 4 & 1 \\ 0 & 0 & 1 & \frac{1}{2} & -1 & \frac{-1}{2} \end{bmatrix}$$

> addrow(% ,3,1,1);

$$\begin{bmatrix} 2 & 0 & 0 & \frac{3}{2} & -5 & \frac{-1}{2} \\ 0 & 2 & 0 & -1 & 4 & 1 \\ 0 & 0 & 1 & \frac{1}{2} & -1 & \frac{-1}{2} \end{bmatrix}$$

> mulrow(% ,1,1/2);

$$\begin{bmatrix} 1 & 0 & 0 & \frac{3}{4} & \frac{-5}{2} & \frac{-1}{4} \\ 0 & 2 & 0 & -1 & 4 & 1 \\ 0 & 0 & 1 & \frac{1}{2} & -1 & \frac{-1}{2} \end{bmatrix}$$

> mulrow(% ,2,1/2);

$$\begin{bmatrix} 1 & 0 & 0 & \frac{3}{4} & -\frac{5}{2} & -\frac{1}{4} \\ 0 & 1 & 0 & -\frac{1}{2} & 2 & \frac{1}{2} \\ 0 & 0 & 1 & \frac{1}{2} & -1 & -\frac{1}{2} \end{bmatrix}$$

> inverse(A);

$$\begin{bmatrix} \frac{3}{4} & -\frac{5}{2} & -\frac{1}{4} \\ -\frac{1}{2} & 2 & \frac{1}{2} \\ \frac{1}{2} & -1 & -\frac{1}{2} \end{bmatrix}$$

>

> A:=matrix(4,4,[0,-1,0,1,0,2,-1,1,1,-2,1,1,-1,0,1,3]);

$$A := \begin{bmatrix} 0 & -1 & 0 & 1 \\ 0 & 2 & -1 & 1 \\ 1 & -2 & 1 & 1 \\ -1 & 0 & 1 & 3 \end{bmatrix}$$

> Id:=Matrix(4,4,shape=identity);

$$Id := \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

> AId:=concat(A,Id);

$$AId := \begin{bmatrix} 0 & -1 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 2 & -1 & 1 & 0 & 1 & 0 & 0 \\ 1 & -2 & 1 & 1 & 0 & 0 & 1 & 0 \\ -1 & 0 & 1 & 3 & 0 & 0 & 0 & 1 \end{bmatrix}$$

> addrow(% ,2,3,1);

$$\begin{bmatrix} 0 & -1 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 2 & -1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 2 & 0 & 1 & 1 & 0 \\ -1 & 0 & 1 & 3 & 0 & 0 & 0 & 1 \end{bmatrix}$$

> addrow(% ,1,2,2);

$$\begin{bmatrix} 0 & -1 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 3 & 2 & 1 & 0 & 0 \\ 1 & 0 & 0 & 2 & 0 & 1 & 1 & 0 \\ -1 & 0 & 1 & 3 & 0 & 0 & 0 & 1 \end{bmatrix}$$

> addrow(% ,3,4,1);

$$\begin{bmatrix} 0 & -1 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 3 & 2 & 1 & 0 & 0 \\ 1 & 0 & 0 & 2 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 5 & 0 & 1 & 1 & 1 \end{bmatrix}$$

> swaprow(% ,1,3);

$$\begin{bmatrix} 1 & 0 & 0 & 2 & 0 & 1 & 1 & 0 \\ 0 & 0 & -1 & 3 & 2 & 1 & 0 & 0 \\ 0 & -1 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 5 & 0 & 1 & 1 & 1 \end{bmatrix}$$

> swaprow(% ,2,3);

$$\begin{bmatrix} 1 & 0 & 0 & 2 & 0 & 1 & 1 & 0 \\ 0 & -1 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 3 & 2 & 1 & 0 & 0 \\ 0 & 0 & 1 & 5 & 0 & 1 & 1 & 1 \end{bmatrix}$$

> addrow(% ,3,4,1);

```

> mulrow(%2,-1);

$$\begin{bmatrix} 1 & 0 & 0 & 2 & 0 & 1 & 1 & 0 \\ 0 & -1 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 3 & 2 & 1 & 0 & 0 \\ 0 & 0 & 0 & 8 & 2 & 2 & 1 & 1 \end{bmatrix}$$

> mulrow(%3,-1);

$$\begin{bmatrix} 1 & 0 & 0 & 2 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & -1 & -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 3 & 2 & 1 & 0 & 0 \\ 0 & 0 & 0 & 8 & 2 & 2 & 1 & 1 \end{bmatrix}$$

> mulrow(%4,1/8);

$$\begin{bmatrix} 1 & 0 & 0 & 2 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & -1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -3 & -2 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & \frac{1}{4} & \frac{1}{4} & \frac{1}{8} & \frac{1}{8} \end{bmatrix}$$

> addrow(%4,1,-2);

$$\begin{bmatrix} 1 & 0 & 0 & 0 & \frac{-1}{2} & \frac{1}{2} & \frac{3}{4} & \frac{-1}{4} \\ 0 & 1 & 0 & -1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & -3 & -2 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & \frac{1}{4} & \frac{1}{4} & \frac{1}{8} & \frac{1}{8} \end{bmatrix}$$

> addrow(%4,2,1);

```

```

> addrow(%4,3,3);

$$\begin{bmatrix} 1 & 0 & 0 & 0 & \frac{-1}{2} & \frac{1}{2} & \frac{3}{4} & \frac{-1}{4} \\ 0 & 1 & 0 & 0 & \frac{-3}{4} & \frac{1}{4} & \frac{1}{8} & \frac{1}{8} \\ 0 & 0 & 1 & -3 & -2 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 & \frac{1}{4} & \frac{1}{4} & \frac{1}{8} & \frac{1}{8} \end{bmatrix}$$

> inverse(A);

$$\begin{bmatrix} \frac{-1}{2} & \frac{1}{2} & \frac{3}{4} & \frac{-1}{4} \\ \frac{-3}{4} & \frac{1}{4} & \frac{1}{8} & \frac{1}{8} \\ \frac{-5}{4} & \frac{-1}{4} & \frac{3}{8} & \frac{3}{8} \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{8} & \frac{1}{8} \end{bmatrix}$$

>

```

Exercici:

Donades les matrius

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

calcula: la inversa de la matriu I-A, la inversa de la matriu I+A, (I+A)(I-A)^(-1).

```
> Id:=Matrix(4,4,shape=identity);
```

$$Id := \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

```
> A:=matrix(4,4,[0,1,0,0,0,0,1,0,0,0,0,1,0,0,0]);
```

$$A := \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

```
> inverse(Id-A);
```

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

```
> inverse(Id+A);
```

$$\begin{bmatrix} 1 & -1 & 1 & -1 \\ 0 & 1 & -1 & 1 \\ 0 & 0 & 1 & -1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

```
> evalm((Id+A)*inverse(Id-A));
```

$$\begin{bmatrix} 1 & 2 & 2 & 2 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

```
>
```

Exercici:

Aplicant la definició, calculeu el determinant de les matrius següents:

$$\begin{bmatrix} 1 & 2 \\ 5 & 3 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 1 \\ 0 & 2 & 4 \end{bmatrix} \begin{bmatrix} 2 & 1 & 1 \\ 1 & 0 & -3 \\ 3 & -2 & 1 \end{bmatrix}$$

```
> M:=matrix(2,2,[1,2,5,3]);
det(M);
```

$$M := \begin{bmatrix} 1 & 2 \\ 5 & 3 \end{bmatrix}$$

-7

```
> M:=matrix(3,3,[1,2,3,1,1,1,0,2,4]);
det(M);
```

$$M := \begin{bmatrix} 1 & 2 & 3 \\ 1 & 1 & 1 \\ 0 & 2 & 4 \end{bmatrix}$$

0

```
> M:=matrix(3,3,[2,1,1,1,0,-3,3,-2,1]);
det(M);
```

$$M := \begin{bmatrix} 2 & 1 & 1 \\ 1 & 0 & -3 \\ 3 & -2 & 1 \end{bmatrix}$$

-24

[>

Exercici:

Calculeu els següents determinants, transformant-los prèviament en d'altres m'les simples

$$\begin{bmatrix} 2 & 3 & -2 & 4 \\ 3 & -2 & 1 & 2 \\ 3 & 2 & 3 & 4 \\ -2 & 4 & 0 & 5 \end{bmatrix} \begin{bmatrix} 1 & 2 & 1 & 2 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 2 \\ 1 & 2 & 2 & 1 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 & 1 & 1 \\ 1 & 2 & 1 & 1 \\ 1 & 1 & 2 & 1 \\ 1 & 1 & 1 & 2 \end{bmatrix}$$

```
> M:=matrix(4,4,[2,3,-2,4,3,-2,1,2,3,2,3,4,-2,4,0,5]);
det(M);
```

$$M := \begin{bmatrix} 2 & 3 & -2 & 4 \\ 3 & -2 & 1 & 2 \\ 3 & 2 & 3 & 4 \\ -2 & 4 & 0 & 5 \end{bmatrix}$$

-286

```
> M:=matrix(5,5,[1,2,1,2,1,0,0,1,1,1,1,1,0,0,0,0,0,1,1,2,1,2,2,1,1]);
det(M);
```

$$M := \begin{bmatrix} 1 & 2 & 1 & 2 & 1 \\ 0 & 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 2 \\ 1 & 2 & 2 & 1 & 1 \end{bmatrix}$$

2

```
> M:=matrix(4,4,[a,1,1,1,1,a,1,1,1,1,a,1,1,1,1,a]);
det(M);
```

$$M := \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}$$

-3

[>

Exercici:

Comproveu que: $\text{Det} \begin{pmatrix} 1 & 2 & b+c \\ 1 & b & 2+c \\ 1 & c & 2+b \end{pmatrix} = 0$

```
> M:=matrix(3,3,[1,a,b+c,1,b,a+c,1,c,a+b]);
```

$$M := \begin{bmatrix} 1 & 0 & b+c \\ 1 & b & c \\ 1 & c & b \end{bmatrix}$$

```
> det(M);
```

0

[>

Exercici bàsic sistemes 56:

Discuti i resoleu els següents sistemes d'equacions lineals:

Apartat (a)

$$x + y + z = 6$$

$$\begin{aligned} 2x - y &= 0 \\ 3y - 2z &= 0 \end{aligned}$$

```
> A:=matrix(3,3,[1,1,1,2,-1,0,0,3,-2]);
B:=matrix(3,1,[6,0,0]);
AB:=concat(A,B);
```

$$AB := \begin{bmatrix} 1 & 1 & 1 & 6 \\ 2 & -1 & 0 & 0 \\ 0 & 3 & -2 & 0 \end{bmatrix}$$

```
> addrow(% ,1,2,-2);
```

$$\begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & -3 & -2 & -12 \\ 0 & 3 & -2 & 0 \end{bmatrix}$$

```
> addrow(% ,2,3,1);
```

$$\begin{bmatrix} 1 & 1 & 1 & 6 \\ 0 & -3 & -2 & -12 \\ 0 & 0 & -4 & -12 \end{bmatrix}$$

```
> z:=-12/(-4);
y:=-1/3*(-12+2*z);
x:=6-y-z;
```

$$\begin{aligned} z &:= 3 \\ y &:= 2 \\ x &:= 1 \end{aligned}$$

```
> [det(A),rank(A),rank(AB)];
```

[12, 3, 3]

```
> linsolve(A,B);
```

$$\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

Apartat (b)

$$\begin{aligned} 2x + 3y + z &= 4 \\ x - 2y + z &= -2 \\ 8x + 5y + 5z &= 1 \end{aligned}$$

```
> A:=matrix(3,3,[2,3,1,1,-2,1,8,5,5]);
B:=matrix(3,1,[4,-2,1]);
AB:=concat(A,B);
```

$$AB := \begin{bmatrix} 2 & 3 & 1 & 4 \\ 1 & -2 & 1 & -2 \\ 8 & 5 & 5 & 1 \end{bmatrix}$$

```
> swaprow(AB,1,2);
```

$$\begin{bmatrix} 1 & -2 & 1 & -2 \\ 2 & 3 & 1 & 4 \\ 8 & 5 & 5 & 1 \end{bmatrix}$$

```
> addrow(% ,1,2,-2);
```

$$\begin{bmatrix} 1 & -2 & 1 & -2 \\ 0 & 7 & -1 & 8 \\ 8 & 5 & 5 & 1 \end{bmatrix}$$

```
> addrow(% ,1,3,-8);
```

$$\begin{bmatrix} 1 & -2 & 1 & -2 \\ 0 & 7 & -1 & 8 \\ 0 & 21 & -3 & 17 \end{bmatrix}$$

```
> addrow(% ,2,3,-3);
```

$$\begin{bmatrix} 1 & -2 & 1 & -2 \\ 0 & 7 & -1 & 8 \\ 0 & 0 & 0 & -7 \end{bmatrix}$$

```
> [det(A),rank(A),rank(AB)];
```

[0, 2, 3]

```
> linsolve(A,B);
```

```
>
```

Apartat (c)

$$\begin{aligned} x-2y+z-t &= 1 \\ 2x+y+3z &= 2 \\ -x+3y-z+4t &= -1 \end{aligned}$$

```
> A:=matrix(3,4,[1,-2,1,-1,2,1,3,0,-1,3,-1,4]);
B:=matrix(3,1,[1,2,-1]);
AB:=concat(A,B);
```

$$AB := \begin{bmatrix} 1 & -2 & 1 & -1 & 1 \\ 2 & 1 & 3 & 0 & 2 \\ -1 & 3 & -1 & 4 & -1 \end{bmatrix}$$

```
> addrow(AB,1,2,-2);
```

$$\begin{bmatrix} 1 & -2 & 1 & -1 & 1 \\ 0 & 5 & 1 & 2 & 0 \\ -1 & 3 & -1 & 4 & -1 \end{bmatrix}$$

```
> addrow(% ,1,3,1);
```

$$\begin{bmatrix} 1 & -2 & 1 & -1 & 1 \\ 0 & 5 & 1 & 2 & 0 \\ 0 & 1 & 0 & 3 & 0 \end{bmatrix}$$

```
> swaprow(% ,2,3);
```

$$\begin{bmatrix} 1 & -2 & 1 & -1 & 1 \\ 0 & 1 & 0 & 3 & 0 \\ 0 & 5 & 1 & 2 & 0 \end{bmatrix}$$

```
> addrow(% ,2,3,-5);
```

$$\begin{bmatrix} 1 & -2 & 1 & -1 & 1 \\ 0 & 1 & 0 & 3 & 0 \\ 0 & 0 & 1 & -13 & 0 \end{bmatrix}$$

```
> [rank(A),rank(AB)];
```

[3,3]

```
> linsolve(A,B);
```

$$\begin{bmatrix} 1 - \frac{18}{13}t_1 \\ -\frac{3}{13}t_1 \\ -t_1 \\ \frac{1}{13}t_1 \end{bmatrix}$$

Apartat (d)

$$\begin{aligned} x+y+z &= 1 \\ 2x-2y-z &= 0 \\ x+3y+5z &= 2 \\ 5x+3y+6z &= 4 \end{aligned}$$

```
> A:=matrix(4,3,[1,1,1,2,-2,-1,1,3,5,5,3,6]);
B:=matrix(4,1,[1,0,2,4]);
AB:=concat(A,B);
```

$$AB := \begin{bmatrix} 1 & 1 & 1 & 1 \\ 2 & -2 & -1 & 0 \\ 1 & 3 & 5 & 2 \\ 5 & 3 & 6 & 4 \end{bmatrix}$$

```
> addrow(AB,1,2,-2);
```

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & -4 & -3 & -2 \\ 1 & 3 & 5 & 2 \\ 5 & 3 & 6 & 4 \end{bmatrix}$$

```
> addrow(% ,1,3,-1);
```

```

> addrow(% , 1, 4, -5);

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & -4 & -3 & -2 \\ 0 & 2 & 4 & 1 \\ 5 & 3 & 6 & 4 \end{bmatrix}$$

> swaprow(% , 2, 4);

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & -4 & -3 & -2 \\ 0 & 2 & 4 & 1 \\ 0 & -2 & 1 & -1 \end{bmatrix}$$

> addrow(% , 2, 3, 1);

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & -2 & 1 & -1 \\ 0 & 0 & 5 & 0 \\ 0 & -4 & -3 & -2 \end{bmatrix}$$

> addrow(% , 2, 4, -2);

$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & -2 & 1 & -1 \\ 0 & 0 & 5 & 0 \\ 0 & 0 & -5 & 0 \end{bmatrix}$$

> addrow(% , 3, 4, 1);

```

```


$$\begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & -2 & 1 & -1 \\ 0 & 0 & 5 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

> [rank(A), rank(AB)];
[3, 3]
> linsolve(A,B);

$$\begin{bmatrix} \frac{1}{2} \\ \frac{1}{2} \\ 0 \end{bmatrix}$$

>
Apartat (e)

$$\begin{aligned} 3x + 2y + 2z &= 4 \\ 2x + y - 3z &= 1 \\ x - 3y + 4z &= 2 \end{aligned}$$

> A:=matrix(3,3,[3,-2,2,2,1,-3,1,-3,4]);
B:=matrix(3,1,[4,1,2]);
AB:=concat(A,B);

$$AB := \begin{bmatrix} 3 & -2 & 2 & 4 \\ 2 & 1 & -3 & 1 \\ 1 & -3 & 4 & 2 \end{bmatrix}$$

> swaprow(AB,1,3);

$$\begin{bmatrix} 1 & -3 & 4 & 2 \\ 2 & 1 & -3 & 1 \\ 3 & -2 & 2 & 4 \end{bmatrix}$$

> addrow(% , 1, 2, -2);

```

```

> addrow(% , 1, 3, -3);

$$\begin{bmatrix} 1 & -3 & 4 & 2 \\ 0 & 7 & -11 & -3 \\ 3 & -2 & 2 & 4 \end{bmatrix}$$

> addrow(% , 2, 3, -1);

$$\begin{bmatrix} 1 & -3 & 4 & 2 \\ 0 & 7 & -11 & -3 \\ 0 & 7 & -10 & -2 \end{bmatrix}$$

> [det(A) , rank(A) , rank(AB) ];

$$[-7, 3, 3]$$

> linsolve(A,B);

$$\begin{bmatrix} \frac{10}{7} \\ \frac{8}{7} \\ 1 \end{bmatrix}$$

>

```

Apartat (f)

$$\begin{aligned} x - 2y + z + 2t &= 2 \\ -2x + y + 3z - t &= 0 \\ 3x - y - 2z - t &= 2 \\ -1x + 5y - 1z - 3t &= -1 \end{aligned}$$

```

> A:=matrix(4,4,[1,-2,1,2,-2,1,3,-1,3,-1,-2,-1,-1,5,-1,-3]);
B:=matrix(4,1,[2,0,2,-1]);
AB:=concat(A,B);

```

```

AB:=

$$\begin{bmatrix} 1 & -2 & 1 & 2 & 2 \\ -2 & 1 & 3 & -1 & 0 \\ 3 & -1 & -2 & -1 & 2 \\ -1 & 5 & -1 & -3 & -1 \end{bmatrix}$$

> addrow(AB,1,2,2);

$$\begin{bmatrix} 1 & -2 & 1 & 2 & 2 \\ 0 & -3 & 5 & 3 & 4 \\ 3 & -1 & -2 & -1 & 2 \\ -1 & 5 & -1 & -3 & -1 \end{bmatrix}$$

> addrow(% , 1, 3, -3);

$$\begin{bmatrix} 1 & -2 & 1 & 2 & 2 \\ 0 & -3 & 5 & 3 & 4 \\ 0 & 5 & -5 & -7 & -4 \\ -1 & 5 & -1 & -3 & -1 \end{bmatrix}$$

> addrow(% , 1, 4, 1);

$$\begin{bmatrix} 1 & -2 & 1 & 2 & 2 \\ 0 & -3 & 5 & 3 & 4 \\ 0 & 5 & -5 & -7 & -4 \\ 0 & 3 & 0 & -1 & 1 \end{bmatrix}$$

> addrow(% , 2, 4, 1);

$$\begin{bmatrix} 1 & -2 & 1 & 2 & 2 \\ 0 & -3 & 5 & 3 & 4 \\ 0 & 5 & -5 & -7 & -4 \\ 0 & 0 & 5 & 2 & 5 \end{bmatrix}$$

> mulrow(% , 3, 3);

```

$$\begin{bmatrix} 1 & -2 & 1 & 2 & 2 \\ 0 & -3 & 5 & 3 & 4 \\ 0 & 15 & -15 & -21 & -12 \\ 0 & 0 & 5 & 2 & 5 \end{bmatrix}$$

> addrow(% , 2, 3, 5) ;

$$\begin{bmatrix} 1 & -2 & 1 & 2 & 2 \\ 0 & -3 & 5 & 3 & 4 \\ 0 & 0 & 10 & -6 & 8 \\ 0 & 0 & 5 & 2 & 5 \end{bmatrix}$$

> mulrow(% , 3, 1/2) ;

$$\begin{bmatrix} 1 & -2 & 1 & 2 & 2 \\ 0 & -3 & 5 & 3 & 4 \\ 0 & 0 & 5 & -3 & 4 \\ 0 & 0 & 5 & 2 & 5 \end{bmatrix}$$

> addrow(% , 3, 4, -1) ;

$$\begin{bmatrix} 1 & -2 & 1 & 2 & 2 \\ 0 & -3 & 5 & 3 & 4 \\ 0 & 0 & 5 & -3 & 4 \\ 0 & 0 & 0 & 5 & 1 \end{bmatrix}$$

> [det(A) , rank(A) , rank(AB)] ;

[-50, 4, 4]

> linsolve(A,B) ;

$$\begin{bmatrix} \frac{37}{25} \\ \frac{2}{5} \\ \frac{23}{25} \\ \frac{1}{5} \end{bmatrix}$$

[>

Apartat (g)

$$\begin{aligned} -2x - y &= 1 \\ 2x + y + 3z &= 0 \\ z &= 2 \end{aligned}$$

=

> A:=matrix(3,3,[2,1,3,-2,-1,0,0,0,1]) :

B:=matrix(3,1,[0,1,2]) :

AB:=concat(A,B) ;

$$AB := \begin{bmatrix} 2 & 1 & 3 & 0 \\ -2 & -1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

> addrow(AB,1,2,1) ;

$$\begin{bmatrix} 2 & 1 & 3 & 0 \\ 0 & 0 & 3 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

> mulrow(% , 3, 3) ;

$$\begin{bmatrix} 2 & 1 & 3 & 0 \\ 0 & 0 & 3 & 1 \\ 0 & 0 & 3 & 6 \end{bmatrix}$$

> addrow(% , 2, 3, -1) ;

$$\begin{bmatrix} 2 & 1 & 3 & 0 \\ 0 & 0 & 3 & 1 \\ 0 & 0 & 0 & 5 \end{bmatrix}$$

> [det(A) , rank(A) , rank(AB)] ;

[0, 2, 3]

> linsolve(A,B) ;

>

Apartat (h)

$$\begin{aligned} x + 2y &= 1 \\ 2x - z &= 1 \end{aligned}$$

$$\begin{aligned}x+y-z &= 0 \\x-y+2z &= 3\end{aligned}$$

```
> A:=matrix(4,3,[1,2,0,2,0,-1,1,1,-1,1,-1,2]);
B:=matrix(4,1,[1,1,0,3]);
AB:=concat(A,B);
```

$$AB := \begin{bmatrix} 1 & 2 & 0 & 1 \\ 2 & 0 & -1 & 1 \\ 1 & 1 & -1 & 0 \\ 1 & -1 & 2 & 3 \end{bmatrix}$$

```
> addrow(AB,1,2,-2);
addrow(%1,3,-1);
addrow(%1,4,-1);
```

$$\begin{bmatrix} 1 & 2 & 0 & 1 \\ 0 & -4 & -1 & -1 \\ 1 & 1 & -1 & 0 \\ 1 & -1 & 2 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 0 & 1 \\ 0 & -4 & -1 & -1 \\ 0 & -1 & -1 & -1 \\ 1 & -1 & 2 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 0 & 1 \\ 0 & -4 & -1 & -1 \\ 0 & -1 & -1 & -1 \\ 0 & -3 & 2 & 2 \end{bmatrix}$$

```
> mulrow(%3,-1);
```

$$\begin{bmatrix} 1 & 2 & 0 & 1 \\ 0 & -4 & -1 & -1 \\ 0 & 1 & 1 & 1 \\ 0 & -3 & 2 & 2 \end{bmatrix}$$

```
> swaprow(%2,3);
```

$$\begin{bmatrix} 1 & 2 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & -4 & -1 & -1 \\ 0 & -3 & 2 & 2 \end{bmatrix}$$

```
> addrow(%2,3,4);
```

$$\begin{bmatrix} 1 & 2 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 3 & 3 \\ 0 & -3 & 2 & 2 \end{bmatrix}$$

```
> addrow(%2,4,3);
```

$$\begin{bmatrix} 1 & 2 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 3 & 3 \\ 0 & 0 & 5 & 5 \end{bmatrix}$$

```
> mulrow(%3,1/3);
mulrow(%4,1/5);
addrow(%3,4,-1);
```

$$\begin{bmatrix} 1 & 2 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 5 & 5 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

```
> [rank(A),rank(AB)];
```

[3,3]

```
> linsolve(A,B);
```

$$\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

```
>
```

Apartat (i)

$$x+y-z+2t=0$$

$$2x-y-z=1$$

$$3x-z+t=1$$

```
> A:=matrix(3,4,[1,1,-1,2,2,-1,-1,0,3,0,-1,1]);
```

```
B:=matrix(3,1,[0,1,1]);
```

```
AB:=concat(A,B);
```

$$AB := \begin{bmatrix} 1 & 1 & -1 & 2 & 0 \\ 2 & -1 & -1 & 0 & 1 \\ 3 & 0 & -1 & 1 & 1 \end{bmatrix}$$

```
> addrow(AB,1,2,-2);
```

$$\begin{bmatrix} 1 & 1 & -1 & 2 & 0 \\ 0 & -3 & 1 & -4 & 1 \\ 3 & 0 & -1 & 1 & 1 \end{bmatrix}$$

```
> addrow(% ,1,3,-3);
```

$$\begin{bmatrix} 1 & 1 & -1 & 2 & 0 \\ 0 & -3 & 1 & -4 & 1 \\ 0 & -3 & 2 & -5 & 1 \end{bmatrix}$$

```
> addrow(% ,2,3,-1);
```

$$\begin{bmatrix} 1 & 1 & -1 & 2 & 0 \\ 0 & -3 & 1 & -4 & 1 \\ 0 & 0 & 1 & -1 & 0 \end{bmatrix}$$

```
> [rank(A),rank(AB)];
```

[3,3]

```
> linsolve(A,B);
```

$$\begin{bmatrix} \frac{1}{3} \\ -\frac{1}{3}-t_1 \\ -t_1 \\ -t_1 \end{bmatrix}$$

```
>
```

Apartat (j)

$$x+y+z=3$$

$$x+2y-4z=1$$

```
> A:=matrix(2,3,[1,1,1,1,2,-4]);
```

```
B:=matrix(2,1,[3,1]);
```

```
AB:=concat(A,B);
```

$$AB := \begin{bmatrix} 1 & 1 & 1 & 3 \\ 1 & 2 & -4 & 1 \end{bmatrix}$$

```
> addrow(AB,1,2,-1);
```

$$\begin{bmatrix} 1 & 1 & 1 & 3 \\ 0 & 1 & -5 & -2 \end{bmatrix}$$

```
> [rank(A),rank(AB)];
```

[2,2]

```
> linsolve(A,B);
```

$$\begin{bmatrix} 5-6t_1 \\ -2+5t_1 \\ -t_1 \end{bmatrix}$$

>

Apartat (k)

$$\begin{aligned} x+2y-z &= 1 \\ x+y+3z &= 0 \\ x+7z &= -1 \end{aligned}$$

```
> A:=matrix(3,3,[1,2,-1,1,1,3,1,0,7]):
B:=matrix(3,1,[1,0,-1]):
AB:=concat(A,B);
```

$$AB := \begin{bmatrix} 1 & 2 & -1 & 1 \\ 1 & 1 & 3 & 0 \\ 1 & 0 & 7 & -1 \end{bmatrix}$$

```
> addrow(AB,1,2,-1);
```

$$\begin{bmatrix} 1 & 2 & -1 & 1 \\ 0 & -1 & 4 & -1 \\ 1 & 0 & 7 & -1 \end{bmatrix}$$

```
> addrow(% ,1,3,-1);
```

$$\begin{bmatrix} 1 & 2 & -1 & 1 \\ 0 & -1 & 4 & -1 \\ 0 & -2 & 8 & -2 \end{bmatrix}$$

```
> addrow(% ,2,3,-2);
```

$$\begin{bmatrix} 1 & 2 & -1 & 1 \\ 0 & -1 & 4 & -1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

>

Exercici bàsic sistemes 57:

Discuti, segons el valor d'a, els següents sistemes d'equacions i trobeu les solucions en cas d'existir.

Apartat (a)

$$\begin{aligned} x+y+a^2z &= 2+a^2 \\ a^2x+y+z &= 3 \\ x+a^2y+z &= 4-a \end{aligned}$$

```
> unassign('a');
> A:=matrix(3,3,[a^2,1,1,1,a^2,1,1,1,a^2]):
B:=matrix(3,1,[3,4-a,2+a^2]):
AB:=concat(A,B);
```

$$AB := \begin{bmatrix} a^2 & 1 & 1 & 3 \\ 1 & a^2 & 1 & 4-a \\ 1 & 1 & a^2 & 2+a^2 \end{bmatrix}$$

```
> det(A);
```

$$a^6 - 3a^2 + 2$$

```
> solve(det(A),a);
```

$$-1, -1, 1, 1, I\sqrt{2}, -I\sqrt{2}$$

```
> x:=linsolve(A,B):
for i from 1 by 1 to 3 do
x[i,1]:=simplify(factor(x[i,1]));
end:
print(x);
```

$$\begin{bmatrix} \frac{2a+3}{(a+1)(2+a^2)} \\ -\frac{a^2-2a-1}{(a+1)(2+a^2)} \\ \frac{a^3+a^2+4a+5}{(a+1)(2+a^2)} \end{bmatrix}$$

```
> a:=1:
linsolve(matrix(3,3,[a^2,1,1,1,a^2,1,1,1,a^2]),matrix(3,1,[3,4-a,2+a^2]));
```

$$\begin{bmatrix} 3-t_1 & -t_1 \\ 1 & -t_2 \\ -t_1 & 1 \\ -t_1 & 2 \end{bmatrix}$$

```
> a:=-1:
linsolve(matrix(3,3,[a^2,1,1,1,a^2,1,1,1,a^2]),matrix(3,1,[3,4-a,2+a^2]));
```

```
> unassign('a');
```

```
> print(AB);
```

$$\begin{bmatrix} a^2 & 1 & 1 & 3 \\ 1 & a^2 & 1 & 4-a \\ 1 & 1 & a^2 & 2+a^2 \end{bmatrix}$$

```
> swapcol(AB,3,1);
```

$$\begin{bmatrix} 1 & 1 & a^2 & 3 \\ 1 & a^2 & 1 & 4-a \\ a^2 & 1 & 1 & 2+a^2 \end{bmatrix}$$

```
> addrow(% ,1,2,-1);
```

$$\begin{bmatrix} 1 & 1 & a^2 & 3 \\ 0 & a^2-1 & 1-a^2 & 1-a \\ a^2 & 1 & 1 & 2+a^2 \end{bmatrix}$$

```
> addrow(% ,1,3,-a^2);
```

$$\begin{bmatrix} 1 & 1 & a^2 & 3 \\ 0 & a^2-1 & 1-a^2 & 1-a \\ 0 & 1-a^2 & 1-a^4 & -2a^2+2 \end{bmatrix}$$

```
> addrow(% ,2,3,1);
```

$$\begin{bmatrix} 1 & 1 & a^2 & 3 \\ 0 & a^2-1 & 1-a^2 & 1-a \\ 0 & 0 & 2-a^2-a^4 & 3-a-2a^2 \end{bmatrix}$$

```
> solve(2-a^4-a^2,a);
```

$$-1, 1, I\sqrt{2}, -I\sqrt{2}$$

```
> a:=1:
```

```
matrix([[1, 1, a^2, 3], [0, a^2-1, 1-a^2, 1-a], [0, 0, 2-a^4-a^2, 3-a-2*a^2]]);
```

$$\begin{bmatrix} 1 & 1 & 1 & 3 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

```
> a:=-1:
```

```
matrix([[1, 1, a^2, 3], [0, a^2-1, 1-a^2, 1-a], [0, 0, 2-a^4-a^2, 3-a-2*a^2]]);
```

$$\begin{bmatrix} 1 & 1 & 1 & 3 \\ 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 2 \end{bmatrix}$$

```
>
```

Apartat (b)

$$\begin{aligned} x-xa+y &= 0 \\ y-ya+z &= 0 \end{aligned}$$

$$x+z-z a=0$$

```

> unassign('a');
> A:=matrix(3,3,[1-a,1,0,0,1-a,1,1,0,1-a]);
  B:=matrix(3,1,[0,0,0]);
  AB:=concat(A,B);

```

$$AB := \begin{bmatrix} 1-a & 1 & 0 & 0 \\ 0 & 1-a & 1 & 0 \\ 1 & 0 & 1-a & 0 \end{bmatrix}$$

```

> det(A);

```

$$2-3a+3a^2-a^3$$

```

> solve(det(A),a);

```

$$2, \frac{1}{2} + \frac{1}{2}I\sqrt{3}, \frac{1}{2} - \frac{1}{2}I\sqrt{3}$$

```

> x:=linsolve(A,B);
  for i from 1 by 1 to 3 do
    x[i,1]:=simplify(factor(x[i,1]));
  end:
  print(x);

```

$$\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

```

> a:=2:
  linsolve(matrix(3,3,[1-a,1,0,0,1-a,1,1,0,1-a]),matrix(3,1,[0,0,0]));

```

$$\begin{bmatrix} -1 \\ 1 \\ -1 \\ 1 \end{bmatrix}$$

```

> unassign('a');
  print(AB);

```

$$\begin{bmatrix} 1-a & 1 & 0 & 0 \\ 0 & 1-a & 1 & 0 \\ 1 & 0 & 1-a & 0 \end{bmatrix}$$

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```

> swapcol(AB,2,1);

```

$$\begin{bmatrix} 1 & 1-a & 0 & 0 \\ 1-a & 0 & 1 & 0 \\ 0 & 1 & 1-a & 0 \end{bmatrix}$$

```

> addrow(% ,1,2,-1+a);

```

$$\begin{bmatrix} 1 & 1-a & 0 & 0 \\ 0 & (a-1)(1-a) & 1 & 0 \\ 0 & 1 & 1-a & 0 \end{bmatrix}$$

```

> swapcol(% ,2,3);

```

$$\begin{bmatrix} 1 & 0 & 1-a & 0 \\ 0 & 1 & (a-1)(1-a) & 0 \\ 0 & 1-a & 1 & 0 \end{bmatrix}$$

```

> addrow(% ,2,3,a-1);

```

$$\begin{bmatrix} 1 & 0 & 1-a & 0 \\ 0 & 1 & (a-1)(1-a) & 0 \\ 0 & 0 & (a-1)^2(1-a)+1 & 0 \end{bmatrix}$$

```

> simplify(factor((-1+a)^2*(1-a)+1));

```

$$-(2+a)(a^2-a+1)$$

```

> solve(% ,a);

```

$$2, \frac{1}{2} + \frac{1}{2}I\sqrt{3}, \frac{1}{2} - \frac{1}{2}I\sqrt{3}$$

```

> a:=2:
  matrix([[1, 0, 1-a, 0], [0, 1, (-1+a)*(1-a), 0], [0, 0,
    (-1+a)^2*(1-a)+1, 0]]);

```

$$\begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

```

>

```

Apartat (c)

$$-2x+2y+2za=-4$$

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$$xa - 2y + 4z = a + 2$$

$$x - y + 2z = a$$

```
> unassign('a');
```

```
> A:=matrix(3,3,[a,-2,4,1,-1,2,-2,2,2*a]):
```

```
B:=matrix(3,1,[a+2,a,-4]):
```

```
AB:=concat(A,B);
```

$$AB := \begin{bmatrix} a & -2 & 4 & a+2 \\ 1 & -1 & 2 & a \\ -2 & 2 & 2a & -4 \end{bmatrix}$$

```
> det(A);
```

$$-2a^2 + 8$$

```
> solve(det(A),a);
```

$$-2, 2$$

```
> x:=linsolve(A,B):
```

```
for i from 1 by 1 to 3 do
```

```
  x[i,1]:=simplify(factor(x[i,1])):
```

```
end:
```

```
print(x);
```

$$\begin{bmatrix} -1 \\ -\frac{a^2+a+6}{a+2} \\ \frac{-2+a}{a+2} \end{bmatrix}$$

```
> a:=2:
```

```
linsolve(matrix(3,3,[a,-2,4,1,-1,2,-2,2,2*a]),matrix(3,1,[a+2,a,-4])):
```

```
;
```

$$\begin{bmatrix} 2 + t_1 \\ -t_1 \\ 0 \end{bmatrix}$$

```
> a:=-2:
```

```
linsolve(matrix(3,3,[a,-2,4,1,-1,2,-2,2,2*a]),matrix(3,1,[a+2,a,-4])):
```

```
;
```

```
> unassign('a');
```

```
print(AB);
```

$$\begin{bmatrix} a & -2 & 4 & a+2 \\ 1 & -1 & 2 & a \\ -2 & 2 & 2a & -4 \end{bmatrix}$$

```
> swapcol(AB,1,2);
```

$$\begin{bmatrix} -2 & a & 4 & a+2 \\ -1 & 1 & 2 & a \\ 2 & -2 & 2a & -4 \end{bmatrix}$$

```
> swaprow(% ,1,2);
```

$$\begin{bmatrix} -1 & 1 & 2 & a \\ -2 & a & 4 & a+2 \\ 2 & -2 & 2a & -4 \end{bmatrix}$$

```
> addrow(% ,1,2,-2);
```

$$\begin{bmatrix} -1 & 1 & 2 & a \\ 0 & -2+a & 0 & 2-a \\ 2 & -2 & 2a & -4 \end{bmatrix}$$

```
> addrow(% ,1,3,2);
```

$$\begin{bmatrix} -1 & 1 & 2 & a \\ 0 & -2+a & 0 & 2-a \\ 0 & 0 & 4+2a & 2a-4 \end{bmatrix}$$

```
> a:=-2:
```

```
matrix([[ -1, 1, 2, a], [0, -2+a, 0, -a+2], [0, 0, 4+2*a, 2*a-4]]);
```

$$\begin{bmatrix} -1 & 1 & 2 & -2 \\ 0 & -4 & 0 & 4 \\ 0 & 0 & 0 & -8 \end{bmatrix}$$

```
> a:=2:
```

```
matrix([[ -1, 1, 2, a], [0, -2+a, 0, -a+2], [0, 0, 4+2*a, 2*a-4]]);
```

$$\begin{bmatrix} -1 & 1 & 2 & 2 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 8 & 0 \end{bmatrix}$$

Apartat (d)

$$\begin{aligned} ax+3y+z &= 2 \\ 3x+ay+2z &= 3 \\ -x+y-z &= 1 \end{aligned}$$

```

> unassign('a');
> A:=matrix(3,3,[a,3,1,3,a,2,-1,1,-1]):
B:=matrix(3,1,[2,3,1]):
AB:=concat(A,B);

```

$$AB := \begin{bmatrix} a & 3 & 1 & 2 \\ 3 & a & 2 & 3 \\ -1 & 1 & -1 & 1 \end{bmatrix}$$

```

> det(A);

```

$$-a - a^2 + 6$$

```

> solve(det(A),a);

```

$$-3, 2$$

```

> x:=linsolve(A,B):
for i from 1 by 1 to 3 do
  x[i,1]:=simplify(factor(x[i,1])):
end:
print(x);

```

$$\begin{bmatrix} \frac{3a-14}{(-2+a)(3+a)} \\ \frac{-8+5a}{(-2+a)(3+a)} \\ -\frac{a-4}{-2+a} \end{bmatrix}$$

```

> a:=2:
linsolve(matrix(3,3,[a,3,1,3,a,2,-1,1,-1]),matrix(3,1,[2,3,1]));
> a:=-3:

```

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```
matrix(3,3,[a,3,1,3,a,2,-1,1,-1]),matrix(3,1,[2,3,1]):
```

```
> unassign('a');
print(AB);
```

$$\begin{bmatrix} a & 3 & 1 & 2 \\ 3 & a & 2 & 3 \\ -1 & 1 & -1 & 1 \end{bmatrix}$$

```
> swapcol(AB,1,3);
```

$$\begin{bmatrix} 1 & 3 & a & 2 \\ 2 & a & 3 & 3 \\ -1 & 1 & -1 & 1 \end{bmatrix}$$

```
> addrow(% ,1,2,-2);
```

$$\begin{bmatrix} 1 & 3 & a & 2 \\ 0 & -6+a & -2a+3 & -1 \\ -1 & 1 & -1 & 1 \end{bmatrix}$$

```
> addrow(% ,1,3,1);
```

$$\begin{bmatrix} 1 & 3 & a & 2 \\ 0 & -6+a & -2a+3 & -1 \\ 0 & 4 & a-1 & 3 \end{bmatrix}$$

```
> swaprow(% ,2,3);
```

$$\begin{bmatrix} 1 & 3 & a & 2 \\ 0 & 4 & a-1 & 3 \\ 0 & -6+a & -2a+3 & -1 \end{bmatrix}$$

```
> addrow(% ,2,3,-(-6+a)/4);
```

$$\begin{bmatrix} 1 & 3 & a & 2 \\ 0 & 4 & a-1 & 3 \\ 0 & 0 & \left(\frac{3}{2}-\frac{1}{4}a\right)(a-1)-2a+3 & \frac{7}{2}-\frac{3}{4}a \end{bmatrix}$$

```
> simplify(%);
```

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$$\begin{bmatrix} 1 & 3 & a & 2 \\ 0 & 4 & a-1 & 3 \\ 0 & 0 & -\frac{1}{4}a + \frac{3}{2} - \frac{1}{4}a^2 & \frac{7}{2} - \frac{3}{4}a \end{bmatrix}$$

```
> mulrow(% , 3, 4);
```

$$\begin{bmatrix} 1 & 3 & a & 2 \\ 0 & 4 & a-1 & 3 \\ 0 & 0 & -a - a^2 + 6 & 14 - 3a \end{bmatrix}$$

```
> solve(-a^2-a+6,a);
```

-3,2

```
> a:=2:
```

```
matrix([[1, 3, a, 2], [0, 4, -1+a, 3], [0, 0, -a^2-a+6, 14-3*a]]);
```

$$\begin{bmatrix} 1 & 3 & 2 & 2 \\ 0 & 4 & 1 & 3 \\ 0 & 0 & 0 & 8 \end{bmatrix}$$

```
> a:=-3:
```

```
matrix([[1, 3, a, 2], [0, 4, -1+a, 3], [0, 0, -a^2-a+6, 14-3*a]]);
```

$$\begin{bmatrix} 1 & 3 & -3 & 2 \\ 0 & 4 & -4 & 3 \\ 0 & 0 & 0 & 23 \end{bmatrix}$$

```
>
```

Apartat (e)

$$2x + y - az = 1$$

$$3x + 2y + z = a$$

$$2x + y + z = -a$$

```
> unassign('a');
```

```
> A:=matrix(3,3,[2,1,-a,3,2,1,2,1,1]);
```

```
B:=matrix(3,1,[1,a,-a]);
```

```
AB:=concat(A,B);
```

$$AB := \begin{bmatrix} 2 & 1 & -a & 1 \\ 3 & 2 & 1 & a \\ 2 & 1 & 1 & -a \end{bmatrix}$$

```
> det(A);
```

a+1

```
> solve(det(A),a);
```

-1

```
> x:=linsolve(A,B);
```

```
for i from 1 by 1 to 3 do
```

```
  x[i,1]:=simplify(factor(x[i,1]));
```

```
end;
```

```
print(x);
```

$$\begin{bmatrix} -3a+1 \\ 5a-1 \\ -1 \end{bmatrix}$$

```
> a:=-1:
```

```
linsolve(matrix(3,3,[2,1,-a,3,2,1,2,1,1]),matrix(3,1,[1,a,-a]));
```

$$\begin{bmatrix} 3 - t_1 \\ -5 + t_1 \\ -t_1 \end{bmatrix}$$

```
> unassign('a');
```

```
print(AB);
```

$$\begin{bmatrix} 2 & 1 & -a & 1 \\ 3 & 2 & 1 & a \\ 2 & 1 & 1 & -a \end{bmatrix}$$

```
> swapcol(AB,2,1);
```

$$\begin{bmatrix} 1 & 2 & -a & 1 \\ 2 & 3 & 1 & a \\ 1 & 2 & 1 & -a \end{bmatrix}$$

```
> addrow(% , 1, 2, -2);
```

```

      [ 1  2  -a  1 ]
      [ 0 -1 2a+1 -2+a ]
      [ 1  2  1  -a ]
> addrow(%1,3,-1);
      [ 1  2  -a  1 ]
      [ 0 -1 2a+1 -2+a ]
      [ 0  0  a+1  -a-1 ]
> a:=-1:
matrix([[1, 2, -a, 1], [0, -1, 1+2*a, -2+a], [0, 0, a+1, -a-1]]);
      [ 1  2  1  1 ]
      [ 0 -1 -1 -3 ]
      [ 0  0  0  0 ]
>

```

Apartat (f)

$$\begin{aligned} ax+y+2z &= 1 \\ x+ay+z &= a \\ x+2y+z &= a \end{aligned}$$

```

> unassign('a');
> A:=matrix(3,3,[a,1,2,1,a,1,1,2,1]):
B:=matrix(3,1,[1,a,a]):
AB:=concat(A,B);
      AB:= [ a  1  2  1 ]
           [ 1  a  1  a ]
           [ 1  2  1  a ]
> det(A);
      -4a+a^2+4
> solve(det(A),a);
      2,2
> x:=linsolve(A,B):
for i from 1 by 1 to 3 do
  x[i,1]:=simplify(factor(x[i,1]));
end:

```

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```

print(x);
      [ -2a-1 ]
      [ -2+a ]
      [ 0 ]
      [ a^2-1 ]
      [ -2+a ]
> a:=-2:
linsolve(matrix(3,3,[a,1,2,1,a,1,1,2,1]),matrix(3,1,[1,a,a]));
      [ -5 ]
      [ 4 ]
      [ 0 ]
      [ -3 ]
      [ 4 ]
> unassign('a');
print(AB);
      [ a  1  2  1 ]
      [ 1  a  1  a ]
      [ 1  2  1  a ]
> swapcol(AB,1,3);
      [ 2  1  a  1 ]
      [ 1  a  1  a ]
      [ 1  2  1  a ]
> swaprow(%1,3);
      [ 1  2  1  a ]
      [ 1  a  1  a ]
      [ 2  1  a  1 ]
> swaprow(%2,3);
      [ 1  2  1  a ]
      [ 2  1  a  1 ]
      [ 1  a  1  a ]
> addrow(%1,2,-2);

```

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```

>
<math display="block">\begin{bmatrix} 1 & 1 & a+1 & 2 \\ 0 & -a & -a(a+1)+2 & 1-2a \\ 0 & -3 & -2a+2 & a-4 \end{bmatrix}
> swaprow(% ,2,3);
<math display="block">\begin{bmatrix} 1 & 1 & a+1 & 2 \\ 0 & -3 & -2a+2 & a-4 \\ 0 & -a & -a(a+1)+2 & 1-2a \end{bmatrix}
> mulrow(% ,3,3);
<math display="block">\begin{bmatrix} 1 & 1 & a+1 & 2 \\ 0 & -3 & -2a+2 & a-4 \\ 0 & -3a & -3a(a+1)+6 & 3-6a \end{bmatrix}
> addrow(% ,2,3,-a);
<math display="block">\begin{bmatrix} 1 & 1 & a+1 & 2 \\ 0 & -3 & -2a+2 & a-4 \\ 0 & 0 & -a(-2a+2)-3a(a+1)+6 & -a(a-4)+3-6a \end{bmatrix}
> simplify(%);
<math display="block">\begin{bmatrix} 1 & 1 & a+1 & 2 \\ 0 & -3 & -2a+2 & a-4 \\ 0 & 0 & -a^2-5a+6 & -a^2-2a+3 \end{bmatrix}
> solve(-a^2-5*a+6,a);
-6,1
> a:=-6:
matrix([[1, 1, a+1, 2], [0, -3, -2*a+2, -4+a], [0, 0, -a^2-5*a+6, -2*a-a^2+3]]);
<math display="block">\begin{bmatrix} 1 & 1 & -5 & 2 \\ 0 & -3 & 14 & -10 \\ 0 & 0 & 0 & -21 \end{bmatrix}
> a:=1:
matrix([[1, 1, a+1, 2], [0, -3, -2*a+2, -4+a], [0, 0, -a^2-5*a+6, -2*a-a^2+3]]);

```

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```

>
<math display="block">\begin{bmatrix} 1 & 1 & 2 & 2 \\ 0 & -3 & 0 & -3 \\ 0 & 0 & 0 & 0 \end{bmatrix}
>
Apartat (h)
<math display="block">\begin{aligned} 3x-2y+z &= 1 \\ 4x+y-2z &= 2 \\ 2x-5y-az &= 3 \end{aligned}
> unassign('a');
> A:=matrix(3,3,[3,-2,1,4,1,-2,2,-5,-a]);
B:=matrix(3,1,[1,2,3]);
AB:=concat(A,B);
<math display="block">AB := \begin{bmatrix} 3 & -2 & 1 & 1 \\ 4 & 1 & -2 & 2 \\ 2 & -5 & -a & 3 \end{bmatrix}
> det(A);
-11a-44
> solve(det(A),a);
-4
> x:=linsolve(A,B);
for i from 1 by 1 to 3 do
x[i,1]:=simplify(factor(x[i,1]));
end;
print(x);
<math display="block">\begin{bmatrix} \frac{5a+11}{11(4+a)} \\ \frac{2(a-11)}{11(4+a)} \\ -\frac{3}{4+a} \end{bmatrix}
> a:=-4:
linsolve(matrix(3,3,[3,-2,1,4,1,-2,2,-5,-a]),matrix(3,1,[1,2,3]));
> unassign('a');

```

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```
print(AB);
```

$$\begin{bmatrix} 3 & -2 & 1 & 1 \\ 4 & 1 & -2 & 2 \\ 2 & -5 & -a & 3 \end{bmatrix}$$

```
> mulrow(AB,2,3);
```

$$\begin{bmatrix} 3 & -2 & 1 & 1 \\ 12 & 3 & -6 & 6 \\ 2 & -5 & -a & 3 \end{bmatrix}$$

```
> addrow(%1,2,-4);
```

$$\begin{bmatrix} 3 & -2 & 1 & 1 \\ 0 & 11 & -10 & 2 \\ 2 & -5 & -a & 3 \end{bmatrix}$$

```
> mulrow(%3,3);
```

$$\begin{bmatrix} 3 & -2 & 1 & 1 \\ 0 & 11 & -10 & 2 \\ 6 & -15 & -3a & 9 \end{bmatrix}$$

```
> addrow(%1,3,-2);
```

$$\begin{bmatrix} 3 & -2 & 1 & 1 \\ 0 & 11 & -10 & 2 \\ 0 & -11 & -2-3a & 7 \end{bmatrix}$$

```
> addrow(%2,3,1);
```

$$\begin{bmatrix} 3 & -2 & 1 & 1 \\ 0 & 11 & -10 & 2 \\ 0 & 0 & -12-3a & 9 \end{bmatrix}$$

```
> a:=-4;
```

```
matrix([[3, -2, 1, 1], [0, 11, -10, 2], [0, 0, -12-3*a, 9]]);
```

$$\begin{bmatrix} 3 & -2 & 1 & 1 \\ 0 & 11 & -10 & 2 \\ 0 & 0 & 0 & 9 \end{bmatrix}$$

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```
>
```

Apartat (f)

$$ax+y+2z=1$$

$$x+ay+z=a$$

$$x+2y+z=a$$

```
> unassign('a');
```

```
> A:=matrix(3,3,[a,1,2,1,a,1,1,2,1]);
```

```
B:=matrix(3,1,[1,a,a]);
```

```
AB:=concat(A,B);
```

$$AB := \begin{bmatrix} a & 1 & 2 & 1 \\ 1 & a & 1 & a \\ 1 & 2 & 1 & a \end{bmatrix}$$

```
> det(A);
```

$$-4a+a^2+4$$

```
> solve(det(A),a);
```

$$2,2$$

```
> x:=linsolve(A,B);
```

```
for i from 1 by 1 to 3 do
```

```
  x[i,1]:=simplify(factor(x[i,1]));
```

```
end;
```

```
print(x);
```

$$\begin{bmatrix} -\frac{2a-1}{-2+a} \\ 0 \\ \frac{a^2-1}{-2+a} \end{bmatrix}$$

```
> a:=-2;
```

```
linsolve(matrix(3,3,[a,1,2,1,a,1,1,2,1]),matrix(3,1,[1,a,a]));
```

$$\begin{bmatrix} -\frac{5}{4} \\ 0 \\ -\frac{3}{4} \end{bmatrix}$$

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```
> unassign('a');
print(AB);
```

$$\begin{bmatrix} a & 1 & 2 & 1 \\ 1 & a & 1 & a \\ 1 & 2 & 1 & a \end{bmatrix}$$

```
> swapcol(AB,1,3);
```

$$\begin{bmatrix} 2 & 1 & a & 1 \\ 1 & a & 1 & a \\ 1 & 2 & 1 & a \end{bmatrix}$$

```
> swaprow(% ,1,3);
```

$$\begin{bmatrix} 1 & 2 & 1 & a \\ 1 & a & 1 & a \\ 2 & 1 & a & 1 \end{bmatrix}$$

```
> swaprow(% ,2,3);
```

$$\begin{bmatrix} 1 & 2 & 1 & a \\ 2 & 1 & a & 1 \\ 1 & a & 1 & a \end{bmatrix}$$

```
> addrow(% ,1,2,-2);
```

$$\begin{bmatrix} 1 & 2 & 1 & a \\ 0 & -3 & -2+a & 1-2a \\ 1 & a & 1 & a \end{bmatrix}$$

```
> addrow(% ,1,3,-1);
```

$$\begin{bmatrix} 1 & 2 & 1 & a \\ 0 & -3 & -2+a & 1-2a \\ 0 & -2+a & 0 & 0 \end{bmatrix}$$

```
> a:=2;
```

```
matrix([[1, 2, 1, a], [0, -3, -2+a, 1-2*a], [0, -2+a, 0, 0]]);
```

$$\begin{bmatrix} 1 & 2 & 1 & 2 \\ 0 & -3 & 0 & -3 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

```
[ ]>
```

Exercici bàsic sistemes 58:

Discutiu els sistemes d'equacions lineals següents en funció dels paràmetres reals a,b. Resoleu-lo en els casos en què sigui possible.

Apartat (a)

$$ax+y+z=b$$

$$x+ay+z=b$$

$$x+y+az=b$$

```
> unassign('a');unassign('b');
```

```
> A:=matrix(3,3,[a,1,1,1,a,1,1,1,a]);
```

```
B:=matrix(3,1,[b,b,b]);
```

```
AB:=concat(A,B);
```

$$AB:=\begin{bmatrix} a & 1 & 1 & b \\ 1 & a & 1 & b \\ 1 & 1 & a & b \end{bmatrix}$$

```
> swapcol(AB,1,3);
```

$$\begin{bmatrix} 1 & 1 & a & b \\ 1 & a & 1 & b \\ a & 1 & 1 & b \end{bmatrix}$$

```
> addrow(% ,1,2,-1);
```

$$\begin{bmatrix} 1 & 1 & a & b \\ 0 & a-1 & -a+1 & 0 \\ a & 1 & 1 & b \end{bmatrix}$$

```
> addrow(% ,1,3,-a);
```

```

> addrow(% ,2,3,1);

$$\begin{bmatrix} 1 & 1 & a & b \\ 0 & a-1 & -a+1 & 0 \\ 0 & -a+1 & 1-a^2 & -ab+b \end{bmatrix}$$

> solve(-a+2-a^2,a);
-2,1
a=1
> a:=1:
matrix([[1, 1, a, b], [0, -1+a, -a+1, 0], [0, 0, -a+2-a^2, -a*b+b]])

$$\begin{bmatrix} 1 & 1 & 1 & b \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

a=-2
> a:=-2:
matrix([[1, 1, a, b], [0, -1+a, -a+1, 0], [0, 0, -a+2-a^2, -a*b+b]])
>

$$\begin{bmatrix} 1 & 1 & -2 & b \\ 0 & -3 & 3 & 0 \\ 0 & 0 & 0 & 3b \end{bmatrix}$$

> a:=1:
linsolve(matrix(3,3,[a,1,1,1,a,1,1,1,a]),matrix(3,1,[b,b,b]));

$$\begin{bmatrix} b-t_1 & -t_1 \\ -t_1 & 1 \\ -t_1 & 2 \end{bmatrix}$$

> a:=-2:b:=0:
linsolve(matrix(3,3,[a,1,1,1,a,1,1,1,a]),matrix(3,1,[b,b,b]));

```

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```

> unassign('a');unassign('b');
linsolve(matrix(3,3,[a,1,1,1,a,1,1,1,a]),matrix(3,1,[b,b,b]));

$$\begin{bmatrix} -t_1 \\ -t_1 \\ -t_1 \end{bmatrix}$$


$$\begin{bmatrix} \frac{b}{a+2} \\ \frac{b}{a+2} \\ \frac{b}{a+2} \end{bmatrix}$$

>
Apartat (b)

$$\begin{aligned} x+by+az &= 1 \\ ax+by+z &= 1 \\ x+aby+z &= b \end{aligned}$$

> unassign('a');unassign('b');
> A:=matrix(3,3,[a,b,1,1,a*b,1,1,b,a]):
B:=matrix(3,1,[1,b,1]):
AB:=concat(A,B);

$$AB := \begin{bmatrix} a & b & 1 & 1 \\ 1 & ab & 1 & b \\ 1 & b & a & 1 \end{bmatrix}$$

> swapcol(AB,1,3);

$$\begin{bmatrix} 1 & b & a & 1 \\ 1 & ab & 1 & b \\ a & b & 1 & 1 \end{bmatrix}$$

> addrow(% ,1,2,-1);

```

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$$\begin{aligned} ax+y-z &= 1 \\ x-ay+z &= 4 \end{aligned}$$

```
> unassign('a'); unassign('b');
> A:=matrix(3,3,[a,1,-1,1,-a,1,1,1,a]);
B:=matrix(3,1,[1,4,b]);
AB:=concat(A,B);
```

$$AB := \begin{bmatrix} a & 1 & -1 & 1 \\ 1 & -a & 1 & 4 \\ 1 & 1 & a & b \end{bmatrix}$$

```
> swaprow(AB,1,3);
```

$$\begin{bmatrix} 1 & 1 & a & b \\ 1 & -a & 1 & 4 \\ a & 1 & -1 & 1 \end{bmatrix}$$

```
> addrow(% ,1,2,-1);
```

$$\begin{bmatrix} 1 & 1 & a & b \\ 0 & -a-1 & -a+1 & -b+4 \\ a & 1 & -1 & 1 \end{bmatrix}$$

```
> addrow(% ,1,3,-a);
```

$$\begin{bmatrix} 1 & 1 & a & b \\ 0 & -a-1 & -a+1 & -b+4 \\ 0 & -a+1 & -a^2-1 & -ab+1 \end{bmatrix}$$

```
> addrow(% ,2,3,-1);
```

$$\begin{bmatrix} 1 & 1 & a & b \\ 0 & -a-1 & -a+1 & -b+4 \\ 0 & 2 & a-2-a^2 & b-3-ab \end{bmatrix}$$

```
> swaprow(% ,2,3);
```

$$\begin{bmatrix} 1 & 1 & a & b \\ 0 & 2 & a-2-a^2 & b-3-ab \\ 0 & -a-1 & -a+1 & -b+4 \end{bmatrix}$$

```
> simplify(addrow(% ,2,3,(a+1)/2));
```

$$\begin{bmatrix} 1 & 1 & a & b \\ 0 & 2 & a-2-a^2 & b-3-ab \\ 0 & 0 & -\frac{3}{2}a-\frac{1}{2}a^3 & -\frac{3}{2}a-\frac{1}{2}a^2b-\frac{1}{2}b+\frac{5}{2} \end{bmatrix}$$

```
> mulrow(% ,3,2);
```

$$\begin{bmatrix} 1 & 1 & a & b \\ 0 & 2 & a-2-a^2 & b-3-ab \\ 0 & 0 & -3a-a^3 & -3a-a^2b-b+5 \end{bmatrix}$$

```
> solve(-3*a-a^3,a);
```

$$0, I\sqrt{3}, -I\sqrt{3}$$

```
> a:=0:
```

```
matrix([[1, 1, a, b], [0, 2, -2+a-a^2, b-3-a*b], [0, 0, -3*a-a^3, -3*a-a^2*b-b+5]]);
```

$$\begin{bmatrix} 1 & 1 & 0 & b \\ 0 & 2 & -2 & b-3 \\ 0 & 0 & 0 & 5-b \end{bmatrix}$$

```
>
```

Apartat (d)

$$\begin{aligned} 3ax+4y-2az &= b+4 \\ -ax-y+az &= -1 \\ -ax-2y+az &= -2 \end{aligned}$$

```
> unassign('a'); unassign('b');
```

```
> A:=matrix(3,3,[3*a, 4, -2*a,-a, -1, a,-a, -2, a]);
```

```
B:=matrix(3,1,[b+4, -1, -2]);
```

```
> AB:=matrix([[3*a, 4, -2*a, b+4], [-a, -1, a, -1], [-a, -2, a, -2]]);
```

$$AB := \begin{bmatrix} 3a & 4 & -2a & b+4 \\ -a & -1 & a & -1 \\ -a & -2 & a & -2 \end{bmatrix}$$

```
> swapcol(% , 1, 2);
```

$$\begin{bmatrix} 4 & 3a & -2a & b+4 \\ -1 & -a & a & -1 \\ -2 & -a & a & -2 \end{bmatrix}$$

```
> swaprow(% , 1, 2);
```

$$\begin{bmatrix} -1 & -a & a & -1 \\ 4 & 3a & -2a & b+4 \\ -2 & -a & a & -2 \end{bmatrix}$$

```
> addrow(% , 1, 2, 4);
```

$$\begin{bmatrix} -1 & -a & a & -1 \\ 0 & -a & 2a & b \\ -2 & -a & a & -2 \end{bmatrix}$$

```
> addrow(% , 1, 3, -2);
```

$$\begin{bmatrix} -1 & -a & a & -1 \\ 0 & -a & 2a & b \\ 0 & a & -a & 0 \end{bmatrix}$$

```
> addrow(% , 2, 3, 1);
```

$$\begin{bmatrix} -1 & -a & a & -1 \\ 0 & -a & 2a & b \\ 0 & 0 & a & b \end{bmatrix}$$

```
> addrow(% , 2, 1, -1);
```

$$\begin{bmatrix} -1 & 0 & -a & -b-1 \\ 0 & -a & 2a & b \\ 0 & 0 & a & b \end{bmatrix}$$

```
> addrow(% , 3, 1, 1);
```

$$\begin{bmatrix} -1 & 0 & 0 & -1 \\ 0 & -a & 2a & b \\ 0 & 0 & a & b \end{bmatrix}$$

```
> addrow(% , 3, 2, -2);
```

$$\begin{bmatrix} -1 & 0 & 0 & -1 \\ 0 & -a & 0 & -b \\ 0 & 0 & a & b \end{bmatrix}$$

```
> a:=0;
```

```
matrix([[ -1, 0, 0, -1], [0, -a, 0, -b], [0, 0, a, b]]);
```

$$\begin{bmatrix} -1 & 0 & 0 & -1 \\ 0 & 0 & 0 & -b \\ 0 & 0 & 0 & b \end{bmatrix}$$

```
> addrow(% , 2, 3, 1);
```

$$\begin{bmatrix} -1 & 0 & 0 & -1 \\ 0 & 0 & 0 & -b \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

```
>
```