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THE INFLUENCE OF THERMAL BARRIERS IN ANISOTROPIC MEDIA APPLIED TO PCB USING MEC

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Abstract. Many electronical components were developed during the last years and many efforts were devoted to the miniaturization of their components due to the global tendency. The matrix in which the components are mounted are made of composite materials which presented anisotropic behaviour. The main goal of this work relies on determining the influence of the thermal barriers position inside of a PCB. The plate has 168 thermal barriers inside the domain where each one has a 360° of freedom of rotation. A Dirichlet boundary condition was imposed to all corners of the plate to analysis. The heat flux was observed at the A, B, and C corners, as the internal barriers were rotated. A quadratic boundary element was used and The multipoint Genetic Algorithm was employed in order to maximize the objective function at the corner A and minimizing at the corners B and C. Despite the elevated number of variables classified this problem such as non-convex, the final results showed good convergence.

1 INTRODUCTION

Due to the trend of electronics miniaturization, some circuits' settings bring devices with a high power dissipation, which results in serious problems on the behavior of the product. Knowing the limitation for the temperature and the thermal flow inside a printed circuit board (PCB) is crucial to make the right decisions in order to improve a project.

In this sense, several researchers have focused their effort in studying how to improve thermal efficiency of electronic components, which can be done using numerical or experimental techniques [1]. It is common knowledge that experimental techniques are, a few times, extremely expensive due to the need of manufacturing numerous prototypes and considerable cost measuring equipment.

The Finite Element Method (FEM) is widely applied in the numerical techniques. The main characteristic of FEM is the need of an electrical grid to solve the differential equation ruler of the problem.

In cases of optimization, a strict control of the electrical grid is needed to check the convergence of the solution, which raises the computing cost in a considerable way [7]. On the last few decades, a new method, known as Boundary Element Method (BEM), has been catching the scientific community's attention, because of its characteristic of needing discretization only on its borders. This particularity makes this method be more attractive to optimization problems [2]. This unique characteristic brings an advantage, because during the iterative process of optimization, the geometry of the problem doesn't need rediscretization [3]. Another important factor refers to the matrix system, that, for being smaller than the one generated by the FEM, results in a significant reduction in computing costs for optimization problems. Despites the advantages of BEM, FEM is still the most used method, because the formulation of BEM is more complex. [6] employs FEM and Genetic Algorithm (GA) in the simulation and optimization of a thermal flow in anisotropic materials. In this paper, the thermal flow was controlled by a non-linear programming method, in which each part of the iteration process generated a convex problem of the approach of the solution of the problem. The optimization process employed in this paper was the mathematical method known as "Method of Moving Asymptotes" or MMA, that stabilizes or speeds up the convergence of the process of pursuit for great solution. The author focused his efforts in substantially reducing the maximum temperature of the optimized material and in concentrating the power density in the intended places.

The convergence method that will be employed in this paper is the "Non-dominated Sorting Genetic Algorithm II", mostly known as NSGA II. The algorithm chosen calculates a multi-objective function in which implements the concept of Dominance, in other words, ranks the total population according to the level of importance [9]. The individuals are divided into different levels through this criterion. Using this formulation, the best individuals are stored in the system, as the worst ones are excluded from the system [5]. These results have been assigned to a fundamental reduction in the thermal resistance of the material. Following the same line of investigation, the purpose of this paper is studying the best spatial configuration of the inner fiber of a manufactured PCB with anisotropic behavior to control the thermal flow, employing BEM linked to the GA through the NSGA II process.

2 BEM FOR POTENTIAL PROBLEMS

The heat transfer problem when there is no power supply is controlled by Laplace Equation $\Delta u = 0$. Applying the divergence theorem, Betti's theorem and the weighted residual method makes possible reduce a domain problem into a boundary problem [4]. In the weighted residual method, the function weight used is the fundamental solution of the ruler equation. The problem to be handled is established by a BIE (Boundary Integral Equation), presented as the Eq. (1).

$$\frac{1}{2}u^i\left(x\right)+\int_\Gamma u\left(x\right)\,q^*\left(x,x'\right)d\Gamma=\int_\Gamma q\left(x\right)\,u^*\left(x,x'\right)d\Gamma \tag{1}$$

When it comes to entirely anisotropic devices, the ruler equation is written in Cartesian coordinates [11], the equations can be represented according to the Eq. (2).

$$k_{11}\frac{\partial^2 u}{\partial x_1^2}+2k_{12}\frac{\partial^2 u}{\partial x_1 \partial x_2}+k_{22}\frac{\partial^2 u}{\partial x_2^2}=0 \tag{2}$$

Assuming that the tensile properties of the materials are symmetric, the equations can be written as temperature and flow fundamental solutions according to Eq. (3) and Eq. (4), respectively. The distance value between the point supply and the point field can be acquired according to the Eq. (5).

$$u^* = -\frac{1}{2\pi |k_{ij}|^{1/2}} ln(r) \tag{3}$$

$$q^* = \left(k_{11}\frac{\partial u^*}{\partial x_1} + k_{12}\frac{\partial u^*}{\partial x_2} \right) n_{x1} + \left(k_{12}\frac{\partial u^*}{\partial x_1} + k_{21}\frac{\partial u^*}{\partial x_2} \right) n_{x2} \tag{4}$$

$$r = \left\{ k_{11}\left(x_1^i - x_i\right)^2 + 2k_{12}\left(x_1^i - x_1\right)\left(x_2^i - x_2\right) + k_{22}\left(x_2^i - x_2\right)^2 \right\}^{1/2} \tag{5}$$

Where $|k_{ij}|$ is the determinant of the conductivity and s is the inverse of the matrix k, as shown in Eq. (6).

$$|k_{ij}| = k_{11}k_{22} - k_{12}^2 \quad ; \quad s = k^{-1} = \frac{1}{|k_{ij}|}\begin{bmatrix} k_{22} - k_{12} \\ -k_{12}\, k_{11} \end{bmatrix} \tag{6}$$

To turn the anisotropic solution into an isotropic solution, it is established that $k_{12} = 0$ e $k_{11} = k_{22}$ to the symmetric properties of the ruler equation of Cartesian coordinates in the Eq. (2). Isoparametric elements are those in which the same functions are used to approach both the geometry and the boundary variable. The values of u and q for any point belonging to the element can, this way, be written in terms of nodal values and interpolation function in accordance with Eq. (7).

$$\begin{aligned} u(\xi) &= N_1 u_1 + N_2 u_2 + N_3 u_3 = \left[N_1 N_2 N_3 \right] \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix} \\ q(\xi) &= N_1 q_1 + N_2 q_2 + N_3 q_3 = \left[N_1 N_2 N_3 \right] \begin{Bmatrix} q_1 \\ q_2 \\ q_3 \end{Bmatrix} \end{aligned} \tag{7}$$

Where ξ is the isometric coordinates location, determined between [-1,+1] and N_1, N_2 e N_3 are functions of continuous quadratic equations. Considering the discretizations, the integrals of equations can be written according to the Eq.(8) and Eq.(9).

$$\int_{\Gamma j} u^* q \, d\Gamma = \int_{\Gamma j} u^* \left[N_1 \, N_2 \, N_3 \right] q \, d\Gamma = \left[G_1^{ij} \, G_2^{ij} \, G_3^{ij} \right] \begin{Bmatrix} q_1 \\ q_2 \\ q_3 \end{Bmatrix} \tag{8}$$

$$\int_{\Gamma j} q^* u \, d\Gamma = \int_{\Gamma j} q^* \left[N_1 \, N_2 \, N_3 \right] u \, d\Gamma = \left[H_1^{ij} \, H_2^{ij} \, H_3^{ij} \right] \begin{Bmatrix} u_1 \\ u_2 \\ u_3 \end{Bmatrix} \tag{9}$$

The Eq. (8) e (9) are discretized to all the boundary and result as the Eq. (10).

$$c^i u^i + \sum_{j=1}^{N} H^{ij} u^j = \sum_{j=1}^{2N} G^{ij} q^j \tag{10}$$

For reordering of Eq. (10), the unknown variable are placed on the left side {X} and a vector on the right side {F} gotten by multiplying the matrix elements by known values of flow and potential. This procedure results in the system shown in the Eq. (11) that discovers all the boundary unknowns when solved.

$$\left[A \right] \left\{ X \right\} = \left\{ F \right\} \tag{11}$$

2.1 Continuous Quadratic Elements

In the discretization that uses continuous quadratic elements, the geometry is approached by a quadratic function along each element, therefore, three nodal dots per element are required, according to the Figure 1.

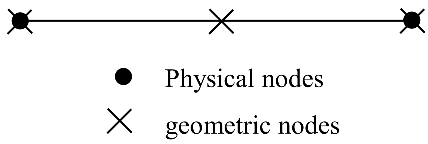

Figure 1: Discretization of the geometry of quadratic elements

The boundary elements Γ_i are considered parabolic, which means they are described by polynomials of 2nd order (parabola equation). This way, 3 points of Γ_i are required so that a parabola can be set. These points are determined by $(x_1, y_1), (x_2, y_2)$ e (x_3, y_3), that correspond, respectively, to the intrinsic coordinates $\xi = -1$, $\xi = 0$ e $\xi = 1$, as illustrated in the Figure 2. Creating a parabolic function to relate x to ξ, there is:

$$x = a\xi^2 + b\xi + c \tag{12}$$

Being:

$$x(\xi = -1) = x_1, \; x(\xi = 0) = x_2 \;\; e \;\; x(\xi = +1) = x_3 \tag{13}$$

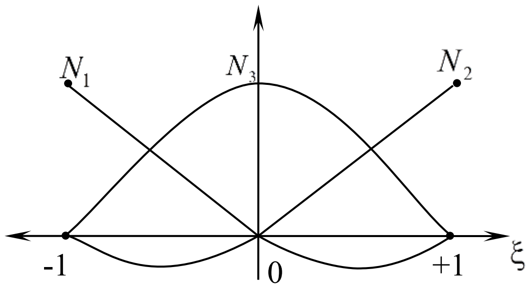

Figure 2: Continuous quadratic Interpolation Functions

The interpolation Functions can be written according to the Eq.(14).

$$x = \underbrace{\frac{\xi}{2}(\xi-1)x_1}_{N_1} + \underbrace{(1-\xi)(1+\xi)x_2}_{N_2} + \underbrace{\frac{\xi}{2}(\xi+1)x_3}_{N_3} \tag{14}$$

Being:

$$x = N_1x_1 + N_2x_2 + N_3x_3 \tag{15}$$

Where $N_{1,} N_2 \, \mathrm{e} \, N_3$ are the continuous quadratic interpolation Functions determined in accordance with Eq.(16).

$$N_1 = \frac{\xi}{2}(\xi-1),\ N_2 = (1-\xi)(1+\xi) = 1-\xi^2 \ and \ N_3 = \frac{\xi}{2}(\xi+1) \tag{16}$$

Likewise, there is:

$$y = N_1y_1 + N_2y_2 + N_3y_3 \tag{17}$$

In the discretization that uses BEM, the geometry is approximated by a quadratic function along each element, and needs three nodal points per element. This way, temperatures and flows are approximated according to the Eq. (18) and (19).

$$u = N_1u_1 + N_2u_2 + N_3u_3 \tag{18}$$

$$q = N_1q_1 + N_2q_2 + N_3q_3 \tag{19}$$

Where u_1 is the temperature in the local node 1, u_2 is the temperature in the local node 2, u_3 is the temperature in the local node 3, q_1 is the flow in the local node 1, q_2 is the flow in the local node 2, q_3 is the flow in the local node 3, N_1 is the Interpolation Function 1, N_2 is the interpolation function 2 and N_3 is the interpolation function 3. The continuous quadratic interpolation functions N_1, N_2 and N_3 are shown by the Figure 2 and the interpolation function formulation is shown by the Eq. (16). By writing in the matrix form, Eq. (20) and (21) can be found.

$$u = \left[N_1 N_2 N_3 \right] \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix} \tag{20}$$

$$q = \left[N_1 N_2 N_3 \right] \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix} \tag{21}$$

The integral equation is, then, written according to Eq.(22).

$$c\,u(\mathrm{d}) = \sum_{j=1}^{n_{elem}} \left\{ \left[H_1\, H_2\, H_3 \right]_j \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}_j - \left[G_1\, G_2\, G_3 \right]_j \begin{bmatrix} q_1 \\ q_2 \\ q_3 \end{bmatrix}_j \right\} \tag{22}$$

3 OPTIMIZATION USING GENETIC ALGORITHM

The optimization via GA is the act of discovering a result or a set of great solutions to some circumstances, in other words, it's the pursuit of a better result to a certain function or a set of functions [10]. The method copies the biological processes based on the evolution of the species, where the procedures are basically: natural selection, pairing and mutation.

A ramdom initial population was created in the numerical implementation, containing enough diversity so that it can combine characteristics in order to produce new populations. The selection operator employed was NSGA II, where an objective function is calculated in order to give the algorithm standards to select the best individuals. The objective function is essential to spread to future generation the best results in each generation. In this case of NSGA II application only the best solutions will keep on existing to the next generations. All the GA procedure can be seen in the Figure 3.

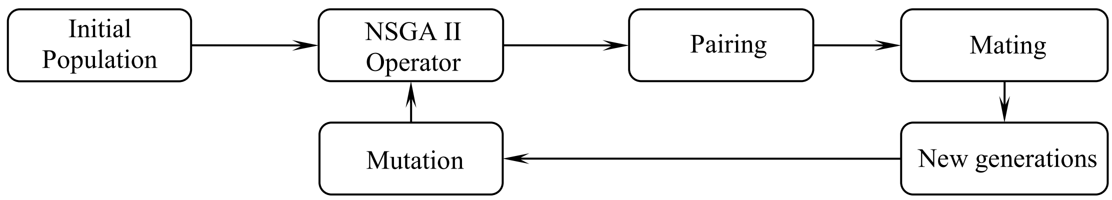

Figure 3: Genetic Algorithm

The pairing operator defines the pairs using the best characteristics from inviduals. The next step is the mating, that consists of creating new topologies (families). In this paper, a pairing based on Charles Darwin's *The Origin of Species*, a *crossover* kind (mating operator), was adopted. This procedure generates two children to each mating, where they will be members for the next generations. The *crossover* is determined with chromosomal percentages from the parents, where the process can be visualized from the scheme shown in the Figure 4. It is possible to observe that the first son received more chromosomal characteristics from the father, as the second son received more chromosomal characteristics from the mother. The same procedure is repeated, but with individuals carrying different characteristics from the first generation, until the individual who carries the best genetic information is found. The last operator is the mutation, which is responsible for randomly change a small amount of chromosomes. The chromosomes carry genetic information in form of binary numbers. The mutation is illustrated in the Figure 4, that consists in the random change of one of the chromosomes' position from 0 to 1 or vice versa, this simple procedure is one of the biggest GA attractives, because it avoids the algorithm stagnation in a minimum place, increasing the chances of finding the maximum global of the search field.

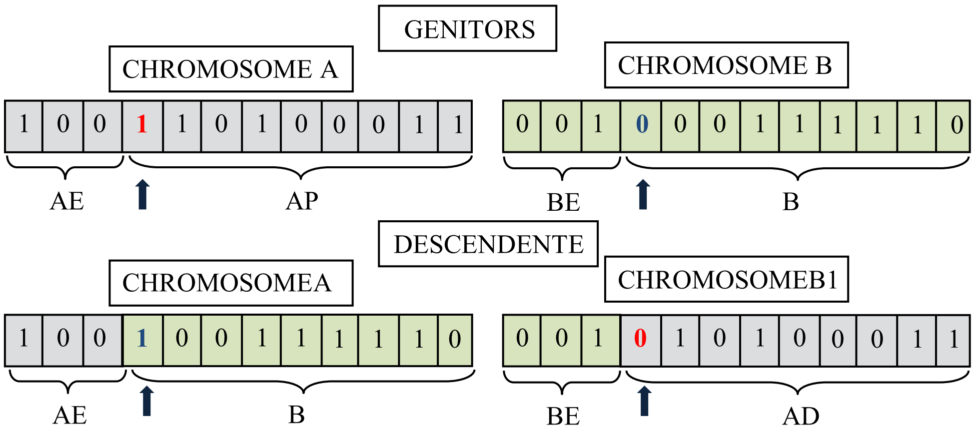

Figure 4: Details of the Pairing, Crossover and Mutation process.

The optimization process described continues until a stop command is reached. The stop command is usually defined as a maximum number of iterations or the maximum/minimum value of the objective function. The Figure 5 shows the algorithm scheme implemented in this paper, where the BEM/GA subroutines are included. In the optimization process the objective function is defined with the specific need of the project. For each individual there is an objective function value, where the best rated are selected and the worst ones are ruled out. In this sense, the population will converge to a best configuration that meets the project specification.

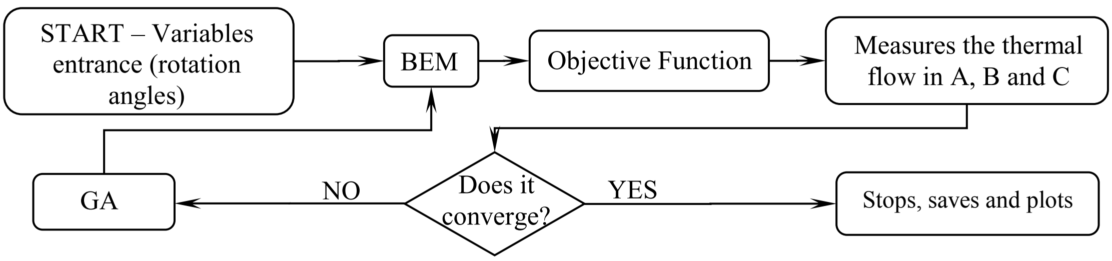

Figure 5: Genetic Algorithm routine associated with BEM

4 NUMERICAL RESULTS

In order to evaluate the methodology proposed to solve a PCB, a rectangular area of 2x1 units is considered, subject to Dirichlet boundary condition in three of its edges (in A, B, C and D) and subject to Neumann boundary condition in others (check Figure 2). All the area was discretized with 1464 boundary continuous quadratic elements. From these 1464 elements, 792 were used discretize the external boundary and the remaining ones were utilized to discretize the holes. Each hole was discretized with 8 elements. In the inner of the domain were inserted 168 holes, which represent the fibers with characteristics of low thermal conductivity materials (insulating). In this sense, was prescribed Neumann condition equals to zero and the numerical integration was accomplished with six Gauss points. To the optimization process for better arrangement of fibers inside the matrix, the GA with theory of evolution was employed. The Table 1 shows the parameters utilized in this process.

Table 1: Optimization Parameters

GA Parameters	
Initial Population	50
Final Population	500
Crossover	5%
Mutation	1%

The initial arrangement of fibers was defined initially as random (iteration 1), and for the optimization process the project variables are the rotation angles of fibers themselves, in which each one of them is able to, idenpendently of each other, rotate 360 degrees. For the optimization process, the objective function aims to maximize the flow at the edge A and minimize it at the edges B and C. Boundary conditions were added to the point D (100º C) and 25º C to the points A, B and C. The purpose consists in measuring the flow fields at the edges displayed as A, B and C in order to meet the multi-objective function, required by the GA.

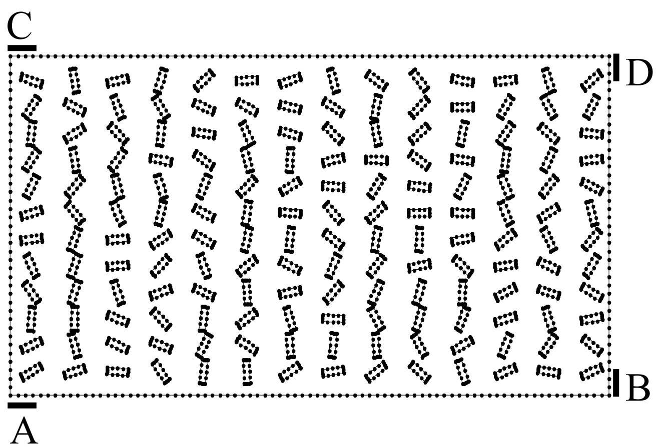

Figure 6: Details of boundary conditions and initial PCB lay-out

In this paper two cases will be analyzed, both isotropic and anisotropic behavior matrix. The thermal flux behavior, as well as the fibers orientation will be presented and discussed for both cases.

4.1 Isotropic Matrix

In this paper the Eq.(2) is reduced to the isotropic problem when established that $k_{11} = k_{22} = k$ e $k_{12} = 0$ and rewritten according to Eq. (23).

$$k\left(\frac{\partial^2 u}{\partial y_1^2} + \frac{\partial^2 u}{\partial y_2^2}\right) = 0 \tag{23}$$

Where k is the material thermal conductivity for isotropic environments. The isotropic thermal conductivity tensor for the 2D problem studied in this paper is shown in Eq.(24).

$$k = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \tag{24}$$

The Figure 7 refers to the iteration 478, which presents the best project configuration after the optimization process reaching the stop command established. The total number of iterations was adjusted in 500, where the population was evaluated until the acquisition of the biggest flow value on the edge A, from the best fiber angles distribution. The Figure 7 makes possible to observe the fibers disposal in the sense of prioritizing the thermal flow from the edge D to the edge A.

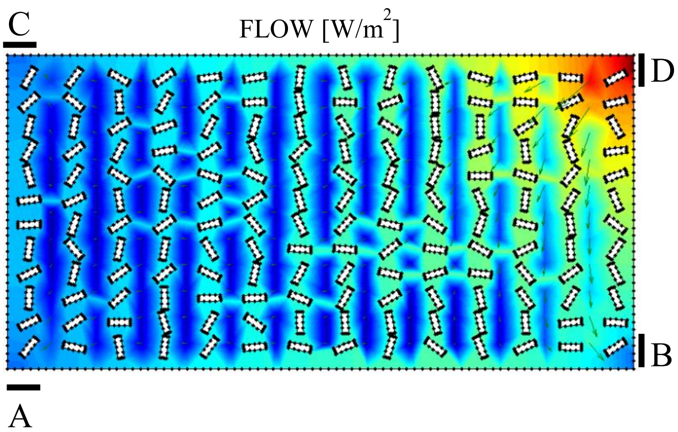

Figure 7: Iteration 478 with better flow distribution

The Figure 8 shows the flow evolution on the edges A, B and C as the iterative process evolves. The disposition of the fibers inside the PCB makes the thermal flows on the three edges oscillate. It happens because the optimization is multi-objective and at the same time that it maximizes on the edge A, it minimizes on the edges B and C. Observing specifically the point A, the iteration 1 was started in 131,29 W/m² and, in the end of the iteration process the flow is maximized to 145,41 W/m². On the edges B and D the thermal flow of the first iteration was measured in 360,54 W/m² and 132,23 W/m², respectively. Considering the last iteration, the thermal flows values to these edges were minimezed to 132,23 W/m² and 127,54 W/m². By answering the multi-objective function is possible to verify, simultaneously on Figure 7 and Figure 8 that the fibers were rotated in a way to direct the flow field to the edge A.

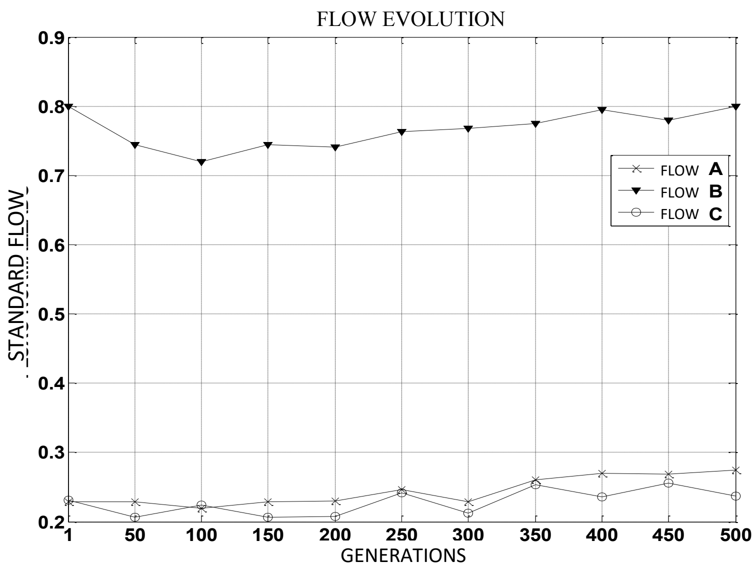

Figure 8: Thermal flow evolution on the edges A, B and C

4.2 Anisotropic Matrix

To materials in a totally anisotropic environment, the ruler equation presented in Eq. (2) is used in its complete form and rewritten again for convenience, as shown in Eq. (25).

$$k_{11}\frac{\partial^2 u}{\partial x_1^2}+2k_{12}\frac{\partial^2 u}{\partial x_1 \partial x_2}+k_{22}\frac{\partial^2 u}{\partial x_2^2}=0 \tag{25}$$

Where k_{ij} is the coefficient that represents the property terms of the tensors in directions i e j. In this paper the values of the thermal conductivity tensor to the anisotropic matrix were established in accordance with Eq.(26).

$$k=\begin{bmatrix} 1 & 0,5 \\ 0,5 & 2 \end{bmatrix} \tag{26}$$

The Figure 9 presents the best project configuration, which was reached in the iteration 473 of the optimization process. Despite the anisotropic behavior of the matrix, it is possible to verify the tendency of the fibers in directing their thermal flow to the edge A.

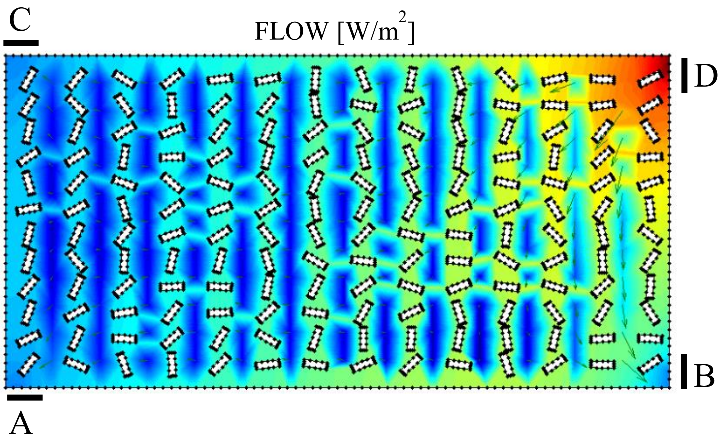

Figure 9: Iteration 473 com better flow distribution

The Figure 10 shows the thermal flow behavior during the evolution of the optimization accomplished by the GA. In the first iteration the thermal flow on the edge A was calculated in 201.67 W/m² being maximized to 217.06 W/m². To the edge B and C the thermal flow in the first iteration was measured in 445.35 W/m² and 135.73 W/m², being minimized at the end of the iterative process to 395.92 W/m² and 122.63 W/m², respectively. Despite not being as evident as in the isotropic case, it is also possible to verify a fiber alignment in the diagonal direction from the edge D to A, with the purpose of prioritizing the thermal flow between these two extremes.

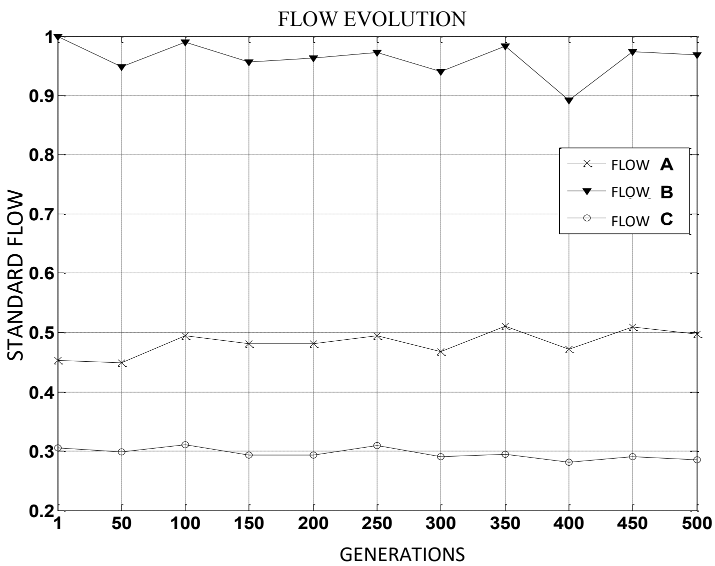

Figure 10: Flow developments on the edges A, B and C

5 CONCLUSIONS

The primary goal of this paper was to control the thermal flow of a Printed Circuit Board (PCB) with anisotropic behavior in three control points, which were defined under the edges A, B, and C. The BEM was used to solve Laplace's equation and, due to its characteristics, proved to be an efficient method to find the solution to the problem studied. Problems of nearly singularity, particular from BEM, were controlled by keeping a minimum distance between inclusions when rotated. The big number of project variables considered in this project rated the optimization problem as nonconvex, which means there is a huge possibility of great solutions. In this sense, as an alternative to the gradient methods, GA was used successfully in pursuing a possible global maximum point. The final results presented coherence with the objective of the multi-objective optimization proposal, which consisted in maximizing the thermal flow on the edge A and minimizing it on the edges B and C. In this meaning, the methodology employed was accepted as viable and can be extended to problems that consider thermal flows inside their domain.

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