Doubly fed induction generator-based variable-speed wind turbine: Proposal of a simplified model under a faulty grid with short-duration faults

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Abstract
The aim of this study is to provide a simplified model of a variable-speed wind turbine (VSWT) with the technology of a doubly fed induction generator (DFIG), which operates under faulty grid conditions. A simplified model is proposed, which consists of a set of electrical and mechanical equations that can be easily modeled as simplistic electrical circuits. It makes it an excellent tool to achieve fault ride-through capability of grid-connected VSWT with DFIGs. Both symmetrical and unsymmetrical grid faults, which cause symmetrical and unsymmetrical voltage sags, have been applied to the system in order to validate the model. The proposed simplified model has been compared with the traditional full-order model under multiple sags (different durations and depths), and the results reveal that both models present similar accuracy. As the idea is to reduce the computational time required to simulate the machine behavior under faulty grid conditions, the proposed model becomes suitable for that purpose. The analytical study has been validated by simulations carried out with MATLAB.

KEYWORDS
doubly fed induction generator (DFIG), fault ride-through (FRT) capability, variable-speed wind turbine (VSWT), voltage sag

1 | INTRODUCTION

Variable-speed wind turbines (VSWTs) equipped with doubly fed induction generators (DFIGs) are the most common technology utilized nowadays in wind energy conversion systems (WECSs).1 Because of the increase of renewable energy units into the distribution grids during the last years, the grid operators have created grid codes in order to make these systems meet fault ride-through (FRT) capability; that is, VSWTs must keep connected to the main grid when disturbances occur.2 For this reason, the understanding of WTs equipped with DFIGs under faults in the main grid is the first step to ensure a continuous injection of electricity to improve power quality for electricity consumers.3

The most complete model when describing induction machines is the fifth-order (or full-order) model, which implies that five equations are needed: four electrical equations (two related to stator and two related to the rotor) plus the mechanical equation. The classical approach to reduce the complexity of this model is to neglect the derivatives of the stator fluxes, which results in the well-known third-order model.4 Regarding DFIGs, a comparison between the fifth-order model and the third-order model is detailed in Ekanayake et al.5 In Feijóo et al6 a variation of the third-order model considering the rotor-side converter, modeled as a voltage source, is given.

Regarding the study of DFIG under symmetrical voltage sags, examples of theoretical analyses are López et al7 and Lima et al.8 In the former study, the analytical expressions for the stator flux and for the rotor voltage are given. A simplification of the DFIG electrical equations can be found in the latter study under the assumption that the stator resistance is much smaller than the stator reactance. Regarding the behavior of DFIG under
unsymmetrical voltage sags, examples of theoretical analyses are López et al.9 and Xu et al.10 In López et al.9 the mathematical expressions for stator flux and rotor voltage are given using the sequence components (positive and negative components), while Xu et al.10 provide analytical expressions for currents, rotor voltage, and electromechanical torque. The limits of the converter of a DFIG under a faulty grid are considered in Chondrogiannis and Barnes11 and Hu and Ye,12 but in those studies, there is a lack of analytical approach. When considering a faulty grid with unsymmetrical voltage sags, the analytical model of the DFIG must include the sequence (positive and negative) components, which increases the number of equations to model the system. Moreover, if the control system is considered, the complexity is higher. Although some studies in the literature have dealt with this problem, they do not provide an analytical solution for the electrical transient, as their differential equations have no constant coefficients.

The present paper tries to fill the gap in the analytical models of DFIG-based VSWT considering the control of the power converter under faulty (balanced and unbalanced) grid conditions. To this end, the following assumptions are made: the control can maintain the rotor current constant in the synchronously rotating reference frame, and the mechanical speed is constant because the fault has a short duration (around 100 ms). These hypotheses make it possible to solve analytically the electrical transient of the machine. The authors have already dealt with this problem under symmetrical and unsymmetrical voltage sags.13,14 However, the present study goes a step further: a more simplified equations of the DFIG are obtained, and a mathematical model that includes the electrical system of the DFIG plus its control is given. Moreover, an algorithm for the detection of the positive- and negative-sequence components of the voltage sags is proposed. This is an excellent tool to understand the behavior of VSWTs equipped with DFIGs, as its equations can be solved analytically. This study provides insights into the understanding of FRT capability of WECSs.

### 2 ELECTRICAL MODEL AND CONTROL

The Ku variables15,16 (see Appendix B) are used for this study because they provide a more compact form of the mathematical model, as they are the complex form of the Park variables.17 Consequently, it eases the task of obtaining an analytical solution. The electrical model of the DFIG written in Ku variables in the synchronously rotating reference frame (motor-sign convention) is as follows:

\[
\begin{align*}
    v_{it} &= M(p + j\omega_s) i_t + [R_s + L_s(p + j\omega_s)] i_{it}, \\
    v_{rt} &= M(p + j\omega_s) i_t + [R_r + L_r(p + j\omega_s)] i_{rt}, \\
    T_m &= 2\rho M \text{Im} \{i_{it}^* i_{rt}\},
\end{align*}
\]

where \(p = d/dt\) is the derivative operator, \(\omega_s = 2\pi f_s\) is the grid pulsation (\(T = 1/f_s\) is the grid period), \(\rho\) is the DFIG pole pairs, \(T_m\) is the electromagnetic torque, the subscript \(s\) stands for the stator, the subscript \(r\) stands for the rotor, the subscript \(f\) stands for the Ku-transformed variable (forward component), and \(s = (\omega_s - \omega_{no})/\omega_s\) is the DFIG slip.

The following assumptions are made (the resulting model is shown in Figure 1A):

1. The control can maintain the rotor current constant in the synchronously rotating reference frame. To that purpose, the authors’ previous works on DFIG subject to voltage sags can be consulted: under symmetrical sags,13 under unsymmetrical sags,14 and considering the most severe sags,15 with all sag durations and depths. However, the focus of the present study is different, because a simplified model of the DFIG-based VSWT is proposed.
2. The voltage sag has a short duration (around 100 ms), so that the machine speed does not change considerably. Then, DFIG speed is assumed to be constant. It should be noted that faults in the transmission grid19,20 are usually cleared within 50 to 100 ms. It means that during this short time interval, the DFIG speed barely changes, because of the machine’s inertia: note from Table A1 (adapted from Slootweg et al.21 see Appendix A)
that the DFIG-based VSWT has an inertia of 0.5 seconds for the DFIG and 2.5 seconds for the whole system, while the faults are usually cleared between 0.05 and 0.1 second, as stated before. As a result, the assumption on constant speed is plausible for this study.

With the previous simplifications, from (1), it results in just one differential equation with constant coefficients, which can be solved analytically. In the author’s previous works,\textsuperscript{13,14} the solution for the differential equation (stator current’s forward component) is given under unsymmetrical and symmetrical sags. The second one is given now, as it is the most generic case:

\[ i_{sf} = \left( \frac{v_{sf} - v_{sf}^-}{R_s + j\omega_s L_s} - \frac{v_{sf}^-}{R_s - j\omega_s L_s} e^{-j2\omega_s t} \right) e^{\frac{R_s}{L_s} (t-t_i)} e^{-j\omega_s (t-t_i)} + \frac{v_{sf}^- - j\omega_s X_m h}{R_s + j\omega_s L_s} e^{-j2\omega_s t} + \frac{v_{sf}^- - j\omega_s X_m h}{R_s + j\omega_s L_s} e^{-j2\omega_s t} \]  

(2)

where \( t_i \) is the time instant when sag originates, \( i_r \) corresponds to the transformed rotor current (which is assumed constant), \( v_{sf} \) is the pre-sag steady-state-transformed voltage of the stator, \( v_{sf} = \sqrt{3}/2V^+ \), and \( v_{sf}^- = \sqrt{3}/2(V^-)^* \), where \( V^+ \) is the positive-sequence voltage and \( V^- \) is the negative-sequence voltage of the stator during the sag (see Appendix C for more details), which are shown in Table 1 for all sag types (see Section 5 for more details).

Now, the following assumptions are made:

1. The stator resistance is assumed to be much lower than the stator reactance \( R_s \ll X_s \) (it is usually an order of magnitude smaller), so \( R_s \approx \pm j\omega_s L_s \).

<table>
<thead>
<tr>
<th>Type</th>
<th>Phasors</th>
<th>Zero Seq.</th>
<th>Positive Seq.</th>
<th>Negative Seq.</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td></td>
<td>( V_0^a = 0 )</td>
<td>( V_1^a = hV )</td>
<td>( V_2^a = 0 )</td>
</tr>
<tr>
<td>B</td>
<td></td>
<td>( V_0^b = - \frac{1-h}{3}V )</td>
<td>( V_1^b = \frac{2+h}{3}V )</td>
<td>( V_2^b = - \frac{1-h}{3}V )</td>
</tr>
<tr>
<td>C</td>
<td></td>
<td>( V_0^c = 0 )</td>
<td>( V_1^c = \frac{1+h}{2}V )</td>
<td>( V_2^c = \frac{1-h}{2}V )</td>
</tr>
<tr>
<td>D</td>
<td></td>
<td>( V_0^d = 0 )</td>
<td>( V_1^d = \frac{1+h}{2}V )</td>
<td>( V_2^d = - \frac{1-h}{2}V )</td>
</tr>
<tr>
<td>E</td>
<td></td>
<td>( V_0^e = \frac{1-h}{3}V )</td>
<td>( V_1^e = \frac{1+2h}{3}V )</td>
<td>( V_2^e = \frac{1-h}{3}V )</td>
</tr>
<tr>
<td>F</td>
<td></td>
<td>( V_0^f = 0 )</td>
<td>( V_1^f = \frac{1+2h}{3}V )</td>
<td>( V_2^f = - \frac{1-h}{3}V )</td>
</tr>
<tr>
<td>G</td>
<td></td>
<td>( V_0^g = 0 )</td>
<td>( V_1^g = \frac{1+2h}{3}V )</td>
<td>( V_2^g = \frac{1-h}{3}V )</td>
</tr>
</tbody>
</table>

Abbreviation: Seq, sequence.
2. The voltage sag is assumed to start at \( t = 0 \), where the machine operates at its steady-state conditions, so the constant exponential terms with \( t \) are \( e^{-j2\omega_s t} = e^{-j\omega_s t} = 1 \).

By doing these simplifications in (2), it results in

\[
id_f = \frac{j}{X_s} [jX_m I_t - v_{st}^+ + v_{st}^- e^{-j2\omega_s t} + (v_{st}^+ - v_{st}^-) e^{-j\omega_s t}],
\]

where the stator reactance is \( X_s = \omega_s L_s \) and the mutual reactance is \( X_m = \omega_s M \). Note that the transformed stator current during the voltage sag can be rewritten as follows:

\[
i_{st} = i_{st} + i_{2w} + i_{w},
\]

where

\[
i_{st} = \frac{j}{X_s} (jX_m I_t - v_{st}^+), \quad i_{2w} = \frac{v_{st}^-}{X_s} e^{-j2\omega_s t}, \quad i_{w} = \frac{j}{X_s} (v_{st}^+ - v_{st}^-) e^{-j\omega_s t},
\]

where subscript st stands for the steady state of the voltage sag, \( \omega \) stands for the component that pulsates at the fundamental frequency, and \( 2\omega \) stands for the component that pulsates at twice the fundamental frequency. Equations (4) and (5) can be represented by means of the three voltage-controlled current sources depicted in Figure 1B.

3 | MECHANICAL MODEL

The drive train mechanical equations neglecting the shaft stiffness and the shaft damping, considering the one-mass model and the motor sign convention, are (note that in this study, the mechanical speed is constant so its derivative is null) as follows:

\[
(J_m + J_t) \frac{d\omega_m}{dt} = T_m - T_t \rightarrow \omega_m = \text{const}, \quad T_m = T_t,
\]

where \( J_m \) is the generator inertia, \( J_t = J_t + r_{gb} \) is the turbine inertia (\( r_{gb} \) is the gearbox ratio), \( \omega_m \) is the generator speed, \( T_m \) is the electromagnetic torque (see Equation 1), and \( T_t = T_t + r_{gb} \) is the torque caused on the blades by the wind, which is calculated as follows:

\[
T_t = (0.5C_p\rho A v_w^3) / \omega_t,
\]

where \( C_p \) is the WT power coefficient, \( \rho \) is the air density (which is 1.225 kg/m\(^3\) at sea level and at ambient temperature of 25°C), \( v_w \) is the speed of the wind, \( A_t \) is the swept rotor area, and \( \omega_t \) is the rotating speed of rotor blades.

The electromagnetic torque can be calculated by substituting (3) in the equation \( T_m \) from (1), which results in

\[
T_m = T_{st} + T_{2w} + T_{w},
\]

where

\[
T_{st} = \frac{-j2\omega M}{X_s} \left[ \Re \{v_{st}^+\} \Re \{I_t\} + \Im \{v_{st}^-\} \Im \{I_t\} \right],
\]

\[
T_{2w} = \frac{2\omega M |v_{st}^-| |I_t|}{X_s} \cos \left( 2\omega t - \varphi_{v_t} + \varphi_{t} \right),
\]

\[
T_{w} = \frac{2\omega M |I_t|}{X_s} \begin{bmatrix} |v_{st}^-| \cos (\omega t - \varphi_{v_t} + \varphi_{t}) \\ -|v_{st}^-| \cos (\omega t - \varphi_{v_t} + \varphi_{t}) \\ -|v_{st}^-| \cos (\omega t - \varphi_{v_t} + \varphi_{t}) \end{bmatrix},
\]

where
\[ |v_{a}^{+}| = \sqrt{\text{Re}(v_{a}^{+})^2 + \text{Im}(v_{a}^{+})^2}; \quad \phi_{a}^{+} = \arctan\left(\frac{\text{Im}(v_{a}^{+})}{\text{Re}(v_{a}^{+})}\right). \]

\[ |v_{a}^{-}| = \sqrt{\text{Re}(v_{a}^{-})^2 + \text{Im}(v_{a}^{-})^2}; \quad \phi_{a}^{-} = \arctan\left(\frac{\text{Im}(v_{a}^{-})}{\text{Re}(v_{a}^{-})}\right). \]

\[ |v_{d}| = \sqrt{\text{Re}(v_{d})^2 + \text{Im}(v_{d})^2}; \quad \phi_{d} = \arctan\left(\frac{\text{Im}(v_{d})}{\text{Re}(v_{d})}\right). \]

\[ |i_{d}| = \sqrt{\text{Re}(i_{d})^2 + \text{Im}(i_{d})^2}; \quad \phi_{i} = \arctan\left(\frac{\text{Im}(i_{d})}{\text{Re}(i_{d})}\right). \]

In order to obtain an equivalent electrical circuit for the electromechanical model, the following analogies must be taken into account: torque corresponds to a voltage; speed corresponds to a current; inertia corresponds to an inductor; stiffness corresponds to a capacitor; and damping corresponds to a resistor. Taking into account those analogies and considering Equations (6) to (10) (with the assumption of neglecting the shaft stiffness and damping), the electromechanical model of the DFIG-based WT (reduced to the high-speed shaft) with constant rotor current and constant speed is depicted by the analogous electrical circuit of Figure 1C.

4 ALGORITHM FOR THE DETECTION OF VOLTAGE SEQUENCE COMPONENTS

As seen from (5) and (9), the electromechanical model of a DFIG-based WT depends on the positive- and negative-sequence components of the transformed stator voltage, \(v_{a}^{+}\) and \(v_{a}^{-}\), respectively. In the literature, most of algorithms are based on the synchronization to the grid voltage by means of a phase-locked loop (PLL), which obtains the angle of the grid voltage. This method is valid for balanced three-phase systems (either in steady state or under three-phase faults), but when unbalanced conditions arise, such as the ones caused by unsymmetrical sags, some improvements need to be done. A possible solution is the use of second-order-generalized integrator (SOGI), which decouples positive- and negative-sequence components. The problem of these methods is that they are computing time-consuming, as they make use of a feedback loop control based on proportional integral (PI) controllers (sometimes with antiwindup, which increases even more the computing time). So, in order to provide a simplified model for DFIG-based WTs and reduce the time consumption in a computer simulation, the algorithm shown in Figure 2 is proposed. This algorithm is based on the combination of the Clarke and Ku transformation, and it consists on the following steps:

1. The abc components of the DFIG stator voltage are translated into a stationary reference frame by means of the Clarke transformation, which gives its \(\alpha\beta\) components (see Appendix D for more details).
2. The angle of the stator voltage, \(\Psi\), is obtained just by applying the arctangent function between \(\beta\) and \(\alpha\).
3. The angle \(\Psi\) is used to apply the Ku transformation (see Appendix B for more details) to obtain the positive-sequence components (with the positive value of \(\Psi\)) and the negative-sequence components (with the negative value of \(\Psi\)).

FIGURE 2 Algorithm for the detection of the stator voltage sequence components and its introduction to the proposed simplified doubly fed induction generator (DFIG)–based wind turbine (WT) model
4. The Ku transformation gives both forward (f) and backward (b) components. As they are complex conjugate, only the forward components are used. Then, the positive- and negative-sequence components of the transformed forward stator voltage, \( v_{sf} \) and \( v_{sf}^{-} \), are introduced in the proposed simplified model of the DFIG-based WT (Equations 5 and 9) in order to obtain the transformed stator current, \( i_{sf} \), and the electromagnetic torque, \( T_m \).

5 | DFIG UNDER FAULTY GRID CONDITIONS

In this paper, the dynamic behavior of a 2-MW DFIG-based WT, whose parameters are shown in Table A1 (Appendix A), is studied under voltage sags. The system is assumed to deliver to the grid its rated power (2 MW), which corresponds to a wind speed \( v_w = 12 \text{ m/s} \) and a DFIG mechanical slip \( s = -0.27 \).

5.1 | Voltage sag characterization

A voltage sag\(^{25} \) is a reduction in the root-mean-square voltage between 0.1 and 0.9 pu of the pre-fault voltage between 10 ms (or 0.5 cycles assuming a frequency of 50 Hz) and 1 minute. This paper focuses on short-duration voltage sags that are provoked by faults in transmission grid.

Voltage sags can be characterized by four parameters: depth (\( h \)), duration (\( \Delta t \)), phase-angle jump, and fault current angle (\( \psi_f \)).\(^{19} \) The sag depth is the remaining voltage for symmetrical sags (with respect to the steady-state voltage in the pre-fault conditions), while for unsymmetrical sags, it is calculated by applying a voltage divider of positive components and negative components in radial feeders.\(^{19} \) The duration is the time interval from the origin of the sag to the instant where the sag ends. The sag is assumed to finish abruptly, ie, with no voltage recovery process (for more information about the fault-clearing process, the work of Bollen\(^{26} \) can be consulted). The fault current angle corresponds to the instant where the fault current passes through 0, so it indicates the origin of the voltage recovery. This angle varies from 75° to 85° for transmission grids and from 45° to 60° for distributed grids.\(^{26} \) The DFIG-based WT is assumed to be connected to the transmission grid, so a value of 80° for the fault current angle is considered. Finally, for transmission grids, the phase-angle jump can be neglected.\(^{19} \)

Voltage sags can be classified into unsymmetrical or symmetrical types. If the abc components of the voltages have the same modulus and a phase shift of 120°, the sag is called symmetrical. Otherwise, the sag is unsymmetrical. Table 1 (adapted from Bollen\(^{19} \)) shows the sag classification with the phasor diagrams and the positive and negative components of all sag types (see Appendix C for more details). As shown in this table, there exist one symmetrical sag (namely, sag type A) and six unsymmetrical sags (namely, sag type B...G).

The DFIG-based WT has been simulated under the most unfavorable voltage sags, ie, under the values of sag depths and durations that define the limit of the controllability of the machine. The most unfavorable depths and durations for each sag for a VSWT equipped with DFIG with constant rotor current and constant mechanical speed are obtained from the authors’ previous work.\(^{18} \) They are summarized in Table 2, where \( n = 0, 1, 2, \ldots \) and \( T \) is the period. Short-duration voltage sags are considered in this study: specifically, sags of five cycles are simulated, ie, sags with duration of 100 ms (assuming a frequency of 50 Hz), so \( n = 5 \) in Table 2.

5.2 | DFIG under unbalanced grid conditions

Equations (4) and (5) and (8) to (10) define the simplified electromechanical model of a DFIG-based VSWT under unsymmetrical sags. The comparison of the dynamic behavior with the fifth (or full)-order model, under the assumptions of constant rotor current and constant mechanical speed for unsymmetrical sags, is shown in Figure 3. This figure shows the time evolution of the forward stator current (real and imaginary parts, according to B.4 from Appendix B) and the electromagnetic torque, considering the unsymmetrical voltage sags with the most unfavorable depths and durations.

The variables are represented with their per unit (pu) values, considering the following base values: \( U_b = 690 \text{ V}, S_b = 2 \text{ MW}, I_b = S_b/\left(\sqrt{3}U_b\right) = 1673.5 \text{ A}, \omega_b = 2\pi f_b = 100\pi \text{ rad/s}, \text{ and } T_b = S_b/(\omega_b/\psi) = 12.73 \text{ kNm}: \)

<table>
<thead>
<tr>
<th>Table 2</th>
<th>Most unfavorable depths and durations of a sag in a variable-speed wind turbine (VSWT) equipped with doubly fed induction generator (DFIG) with constant rotor current (from Rolán et al(^{15} ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sag Type</td>
<td>Depth</td>
</tr>
<tr>
<td>A</td>
<td>( h \geq 0.45 )</td>
</tr>
<tr>
<td>B, C, D</td>
<td>( h \geq 0.2 )</td>
</tr>
<tr>
<td>E, F, G</td>
<td>( h \geq 0.2 )</td>
</tr>
</tbody>
</table>
FIGURE 3 Behavior of a variable-speed wind turbine (VSWT) equipped with a doubly fed induction generator (DFIG) under unsymmetrical sags, with constant rotor current and constant speed. Comparison between the fifth (or full)-order model (solid blue line) and the simplified model presented in this study (dashed red line). All sags have been simulated with their most unfavorable parameters (sag types B, C, and D: \( h = 0.2 \) and \( \Delta t = 5.7 \) T; sag types E, F, and G: \( h = 0.2 \) and \( \Delta t = 5.3 \) T). Fault current angle \( \psi = 80^\circ \). WT operating point: rated power \( (P_n) \) and maximum slip \( (s = 0.27) \). (A) Real part of the stator current (forward component), (B) imaginary part of the stator current (forward component), and (C) electromagnetic torque [Colour figure can be viewed at wileyonlinelibrary.com]
\[ i_{st \text{ pu}}(t) = \frac{i_{sd \text{ pu}}}{\sqrt{3/2} I_b} \quad ; \quad T_m \text{ pu}(t) = \frac{T_m}{T_b} . \tag{11} \]

The results of Figure 3 lead to the following remarks:

1. The simplified model has a good accuracy, as the evolution in time of the represented variables is quite similar to the variables of the fifth (or full)-order model.

2. As the simplified model has no exponential term that depends on \( R_s \) and \( L_s \), there is no damping in the time evolution of the DFIG variables (compare Equation 3 with Equation 1). It is clearly observed in the details of Figure 3: when the sag originates, both models present similar behavior, but as the time goes by, the full-order model shows a damping in the variable, but the simplified model does not. However, note that the difference between both models is really small, as short-duration voltage sags are considered. This is not a problem to validate the simplified model, as this is a realistic approach: faults in the transmission grid\textsuperscript{19,20} are usually cleared within 50 to 100 ms, as stated in Section 2.

3. When unbalanced faults are originated in the grid, pulsations appear in the machine variables, which correspond to two times the fundamental pulsation. These are defined by the term \( 2\omega_s t \) for the transformed stator current \( (5) \) and for the electromagnetic torque \( (9) \).

4. Voltage sag types B and D, and E and G, cause similar time evolution on the DFIG variables. This is because of the fact that the windings (stator and rotor) of the DFIG are isolated Y or \( \Delta \) connected so the zero-sequence of the grid voltages has no influence on the machine’s behavior when subjected to voltage sags. Then, according to Table 1, sag type B is a specific case of type D: sag type D with a depth from 1/3 to 1 has the same sequence components as type B with a depth from 0 to 1. Moreover, note from Table 1 that when there is no zero-sequence voltage, sag types E and G have the same sequence components.

### 5.3 | DFIG under balanced grid conditions

When considering balanced grid conditions caused by symmetrical voltage sags, there exists no negative-sequence voltage (it only appears under unbalanced conditions). Therefore, by neglecting the term \( v_{sd} \) in \( (5) \), we obtain the following expression for the transformed stator current:

\[ i_{st} = i_{st \text{ st}} + i_{st \omega} , \tag{12} \]

where

\[ i_{st \text{ st}} = \frac{j}{X_s} (|X_t| i_{st \text{ t}} - v_{st}^{-}) \quad ; \quad i_{st \omega} = \frac{j}{X_s} (v_{st}^{-} - v_{st}) e^{-j\omega_s t} . \tag{13} \]

And by neglecting the term \( v_{sd} \) in \( (9) \), we obtain the following expression for the electromagnetic torque:

\[ T_m = T_m \text{ st} + T_m \omega . \tag{14} \]

where

\[
T_m \text{ st} = -\frac{2\omega_s M}{X_s} |\text{Re}\{v_{sd}^{-}\}| |\text{Re}\{i_{st}\}| + |\text{Im}\{v_{sd}^{-}\}| |\text{Im}\{i_{st}\}| ,
\]

\[
T_m \omega = \frac{2\omega_s M}{X_s} |v_{sd}^{-}| \left[ |\text{Re}\{\omega_s t + \varphi_{v_{sd}} + \varphi_i\} - |v_{sd}^{-}| \cos(\omega_s t - \varphi_{v_{sd}} + \varphi_i) \right] . \tag{15}
\]

Equations (12) to (15) define the simplified electromechanical model of a DFIG-based WT under balanced grid conditions caused by symmetrical voltage sags. The comparison of the dynamic behavior with the full-order model, under the assumptions of constant rotor current and constant speed for the most unfavorable symmetrical sags, is shown in Figure 4.

The same observations as for the case of unsymmetrical sags can be made, with the exception that the pulsation of the DFIG variables under symmetrical sags is not twice the fundamental frequency but the fundamental frequency itself. This fact is shown in the term \( \omega_s t \) in \( (13) \) for the transformed stator current and in \( (15) \) for the electromagnetic torque.

### 5.4 | Sag duration and sag depth influence

It should be noted that the simplified model has been validated for the most unfavorable voltage sags, whose parameters are given in Table 2. Then, for other less severe sag conditions, the simplified model will also be valid. In order to prove this statement, the comparison of the peak value of the stator current and the peak value of the electromagnetic torque has been considered:

\[
i_{s \text{ peak pu}} = \frac{i_{s \text{ peak}}}{\sqrt{2} I_b} = \frac{\max(|i_{sa}(t)|, |i_{sb}(t)|, |i_{sc}(t)|)}{\sqrt{2} I_b} ,
\]

\[
T_{m \text{ peak pu}} = \frac{T_{m \text{ peak}}}{T_b} = \frac{\max(|T_m(t)|)}{T_b} . \tag{16}
\]
and the evolution of these magnitudes for different sag characteristics (depth and duration) has been simulated. The results are shown in Figures 5 and 6, where the following sags have been considered: symmetrical sags (type A) and unsymmetrical sag types D and G (note that the effects of these sag types on the machine are similar to the ones caused by sag types B and E, respectively, as shown in Subsection 5.2).

Figure 5 shows the peak value of the variables for different sag depths, considering the most unfavorable sag duration for each sag type (according to Table 2). It is observed that there is a clear resemblance between the full-order model and the simplified model proposed in this paper. This resemblance is clearer for sag depths $0.5 \leq h \leq 1$, while for sag depths $h \leq 0.5$, there exists a slight difference from both models. Truly, it should be noted that the difference between the full-order model and the proposed simplified model is really small and it is almost 0 for sag type D. Moreover, note that real measurements in power systems show that when faults occur in a power system, most of the originated sags have a depth $h \geq 0.5$, which is exactly the region where Figure 5 shows the good resemblance between the full-order model and the simplified model.

Figure 6 shows the peak value of the variables for different sag durations, considering the most unfavorable sag depths for each sag type (according to Table 2). This figure also shows a good similarity between the full-order model and the simplified model proposed in this paper. Note, however, that this similarity is reduced when the sag duration is higher. This is a logical consequence, because the simplified model has no damping because of the fact that the stator resistor has been neglected when compared with the stator reactance. However, it should be noted that real
measurements in power systems show that faults in the transmission grid are usually cleared within 50 to 100 ms, i.e., between 2.5 and 5 cycles, assuming a frequency of 50 Hz. Note also that the present study considers short-duration faults so the comparison that Figure 6 shows between the full-order model and the simplified model for short durations has good accuracy.

6 | CONCLUSIONS

This work has dealt with the modeling of VSWTs equipped with a DFIG under faulty grids with symmetrical and unsymmetrical voltage sags. A simplified model, which includes the DFIG plus its control, has been given by means of a proper calculation of the stator voltage sequence components through the proposed algorithm. Short-duration voltage sags have been considered, and the resistance of the stator has been neglected compared with the reactance of the stator. A simplified expression for the solution of the stator current in the complex form of Park components has been given. A compact and understandable equation of the electromagnetic torque has also been obtained, which helps in the prediction and explanation of the DFIG behavior under voltage sags.

The study has been validated by comparing the simplified model with the full-order model for the most unfavorable symmetrical and unsymmetrical voltage sags caused by faults, i.e., considering the values of sag depths and durations that define the limit of the controllability of the machine. Moreover, the most unfavorable operating point of the DFIG-based WT has been taken into account. The model has been also validated for all sag conditions, i.e., considering all the possible sag depths and durations.

This study helps in the prediction of the dynamic behavior of VSWTs with DFIGs when short-duration faults are caused in the grid, which enables the improvement of the FRT capability of WECSs.

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REFERENCES


APPENDIX A.

DATA OF THE STUDIED VARIABLE-SPEED WIND TURBINE EQUIPPED WITH DOUBLY FED INDUCTION GENERATOR

The data of the studied 2-MW variable-speed wind turbine (VSWT) equipped with doubly fed induction generator (DFIG) are shown in Table A1 (adapted from Slootweg et al.19). The curve of the extracted power from the wind vs the WT speed is given in Figure A1. The most unfavorable operating point of the WT is considered (point A of Figure A1): the WT delivers its rated power (2 MW) to the grid with its maximum admissible rotating speed (19 rpm), corresponding to a mechanical slip of $s = -0.27$ for the DFIG. This is obtained when the wind speed is $v_w = 12$ m/s.
APPENDIX B.

PARK VARIABLES WRITTEN IN COMPLEX FORM

The complex form of the Park variables can be given by the Ku transformation. The power-invariant (or normalized) form of the Ku transformation is defined as follows:\textsuperscript{15,16}:

\[ [K(\Psi)] = \frac{1}{\sqrt{3}} \begin{bmatrix} 1 & e^{i\Psi} & e^{-i\Psi} \\ a^2 & ae^{i\Psi} & ae^{-i\Psi} \\ a & e^{i\Psi} & ae^{-i\Psi} \end{bmatrix} \]  

where \( \Psi \) is the transformation angle and \( a \) corresponds to \( e^{j2\pi/3} \). By means of this matrix, it is possible to obtain the zero component (0), the forward component (f), and the backward component (b), which are the transformed components of the abc components of a variable. If the synchronously rotating reference frame is assumed, the angle that is used to transform the stator variables is \( \Psi_s = \omega_st \), while the angle that is used to transform the rotor variables is \( \Psi_r = \Psi_s - \varphi_\theta_m = s\omega_s t - \varphi_\theta_m \), where \( \varphi_\theta_m \) is the initial mechanical angle (at \( t = 0 \)).

Assuming that the windings (stator and rotor) of the DFIG are isolated Y or \( \Delta \) connected, the zero components of the transformation are not considered. Moreover, the backward component is the same as the complex conjugate of the forward component, so it is enough to consider one of these two components.\textsuperscript{16} The chosen component is the forward one. Then, the abc components of the DFIG variables (stator and rotor) are given by the following expressions:

\[ x_{sa} = \frac{2}{\sqrt{3}} \text{Re}\left( e^{j\omega_st} x_{sf} \right), \quad x_{sb} = \frac{2}{\sqrt{3}} \text{Re}\left( a^2 e^{j\omega_st} x_{sf} \right), \quad x_{sc} = \frac{2}{\sqrt{3}} \text{Re}\left( ae^{j\omega_st} x_{sf} \right) \]

\[ x_{ra} = \frac{2}{\sqrt{3}} \text{Re}\left( e^{j(\omega_st-\varphi_\theta_m)} x_{rf} \right), \quad x_{rb} = \frac{2}{\sqrt{3}} \text{Re}\left( a^2 e^{j(\omega_st-\varphi_\theta_m)} x_{rf} \right), \quad x_{rc} = \frac{2}{\sqrt{3}} \text{Re}\left( ae^{j(\omega_st-\varphi_\theta_m)} x_{rf} \right) \]  

and the forward components in the synchronously rotating reference frame of the DFIG variables (stator and rotor) are given by the following expressions:

\[ x_{sf} = e^{-j\omega_st} \left( x_{sa} + ax_{sb} + a^2 x_{sc} \right), \quad x_{rf} = e^{-j(\omega_st-\varphi_\theta_m)} \frac{1}{\sqrt{3}} \left( x_{ra} + ax_{rb} + a^2 x_{rc} \right) \]  

TABLE A1  Characteristics of the variable-speed wind turbine (VSWT) equipped with DFIG (adapted from Slootweg et al\textsuperscript{21})

<table>
<thead>
<tr>
<th>DFIG Rated Values</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( U_n ) (phase to phase) = 690 V</td>
<td>( f_n = 50 \text{ Hz} )</td>
</tr>
<tr>
<td>( \omega_m = 900-1900 \text{ rpm} )</td>
<td>( \varphi = 2 )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>DFIG Parameters in Per Unit (( U_b = 690 \text{ V} ), ( S_b = 2 \text{ MW} ), and ( f_b = 50 \text{ Hz} ))</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( R_s = 0.01 )</td>
<td>( R_r = 0.01 )</td>
</tr>
<tr>
<td>( X_{sl} = 0.1 )</td>
<td>( X_{rl} = 0.08 )</td>
</tr>
<tr>
<td>( X_m = 3.0 )</td>
<td>( H_m = 0.5 \text{ s} )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>WT Parameters</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_n = 2 \text{ MW} )</td>
<td>( \omega_{t,n} = 18 \text{ rpm} )</td>
</tr>
<tr>
<td>( \omega_{t,\text{min}} = 9 \text{ rpm} )</td>
<td>( \omega_{t,\text{max}} = 19 \text{ rpm} )</td>
</tr>
<tr>
<td>( r = 37.5 \text{ m} ) ( \text{ (radius) } )</td>
<td>( V_w,n = 12 \text{ m/s} ) ( \text{ (rated wind speed) } )</td>
</tr>
<tr>
<td>( \varphi_\theta_m = 1:100 \text{ (gearbox ratio) } )</td>
<td></td>
</tr>
</tbody>
</table>

Abbreviations: DFIG, doubly fed induction generator; WT, wind turbine.
Finally, the relation between the forward (Ku) component and the dq (Park) components is given by the following expressions\textsuperscript{15-17}:

\[
x_d = \sqrt{2 \Re\{x_f\}} \quad ; \quad x_q = \sqrt{2 \Im\{x_f\}},
\]  

where subscript d stands for the direct component and subscript q stands for the quadrature component (Park’s transformation).

\section*{APPENDIX C. SEQUENCE COMPONENTS OF A THREE-PHASE VARIABLE}

Given a three-phase variable under unbalanced conditions:

\[
\begin{align*}
X_a &= X_0 e^{j\phi_0} \rightarrow x_a = \sqrt{2}X_0 \cos(\omega t + \phi_0), \\
X_b &= X_0 e^{j\phi_0} \rightarrow x_b = \sqrt{2}X_0 \cos(\omega t + \phi_0), \\
X_c &= X_0 e^{j\phi_0} \rightarrow x_c = \sqrt{2}X_0 \cos(\omega t + \phi_0).
\end{align*}
\]  

Substituting (C.1) in (B.3) and considering the trigonometric relation \(\cos(\alpha) = (e^{j\phi} + e^{-j\phi})/2\), we obtain the following:

\[
x_i = x_i^+ + x_i^- e^{-j2\omega t},
\]  

where \(x_i^+\) and \(x_i^-\) are sequence components (positive and negative) of the transformed variable (forward component):

\[
x_i^+ = \frac{1}{\sqrt{6}} (X_a e^{j\phi_0} + aX_b e^{j\phi_0} + a^2X_c e^{j\phi_0}),
\]

\[
x_i^- = \frac{1}{\sqrt{6}} (X_b e^{-j\phi_0} + aX_a e^{-j\phi_0} + a^2X_c e^{-j\phi_0}).
\]  

The Fortescue transformation\textsuperscript{27} is used to obtain the sequence components (zero, positive, and negative components) of variables under unbalanced conditions:

\[
\begin{bmatrix}
X^0 \\
X^+ \\
X^-
\end{bmatrix} = \frac{1}{3} \begin{bmatrix}
1 & 1 & 1 \\
1 & a & a^2 \\
1 & a^2 & a
\end{bmatrix} \begin{bmatrix}
X_a \\
X_b \\
X_c
\end{bmatrix},
\]  

where the superscript 0 stands for the zero sequence, the subscript + stands for the positive sequence, the subscript – stands for the negative sequence, and the subscripts a, b, and c stand for the abc components of the variable \(X\).

Substituting (C.1) in (C.4), we obtain the following:

\[
\begin{align*}
X^0 &= \frac{1}{3} (X_a e^{j\phi_0} + X_b e^{j\phi_0} + X_c e^{j\phi_0}), \\
X^+ &= \frac{1}{3} (X_a e^{j\phi_0} + aX_b e^{j\phi_0} + a^2X_c e^{j\phi_0}), \\
X^- &= \frac{1}{3} (X_b e^{-j\phi_0} + aX_a e^{-j\phi_0} + a^2X_c e^{-j\phi_0}).
\end{align*}
\]  

Finally, comparing (C.3) with (C.5), we obtain the following:

\[
x_i^+ = \sqrt{3/2}X_i^+, \quad x_i^- = \sqrt{3/2}(X_i^-)^*.
\]  

\section*{APPENDIX D. CLARKE TRANSFORMATION}

The Clarke transformation relates the abc components of a given variable to its stationary components or \(0\alpha\beta\) components. The power-invariant (or normalized) form of the Clarke transformation is defined as follows\textsuperscript{28}:

\[
[C] = \frac{1}{\sqrt{2}} \begin{bmatrix}
1 & \sqrt{2}/2 & \sqrt{2}/2 \\
1 & 1/2 & -1/2 \\
0 & \sqrt{3}/2 & \sqrt{3}/2
\end{bmatrix}.
\]