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A FINITE ELEMENT APPROACH FOR A COUPLED NUMERICAL SIMULATION OF FLUID-STRUCTURE-ELECTRIC INTERACTION IN MEMS

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Abstract. In this analyze, a novel finite element coupled algorithm using numerical methods to analyze the interaction between fluid-structure-electric fileds has been presented for piezoelectric actuators. Piezoelectricity is fundamentally an interaction between structure and electric fields. In this paper, at first we analyze the piezoelectric interaction using 3D solid elements and MITC4 shell elements. Solid elements are used for electric analysis and MITC4 shell elements are used for geometric nonlinear structural analysis. The induced electric forces and moment of forces are translated from 3D solid elements to MITC4 shell elements using a novel translation method, and displacements from MITC4 shell elements are translated to 3D solid elements using shell element displacement interpolation functions. A projection method is employed in order to solve the interaction between MITC4 shell structure and fluid field.

1 Introduction

Micro Electro Mechanical System (MEMS) based piezoelectric actuators can be used to actuate the thin and flexible wings of micro air vehicles (MAVs). The actuator produces large deformation at its resonance, which is located at the root of the wing, so that the flexible wing can flap and produce enough lift force to support the MAV weight during flight. In the mean time, MAV is surrounded by air and has a significant influence from it on the vibration characteristics of the flapping motion of the wing and hence its response. Therefore, the analysis of fluid-structure-electric interaction must be carefully taken into account during the MAV design process. The analytical solution to the model equations

are limited in the scope. From this view point, a novel finite element method for fluid-structure-electric interaction is imperative.
Block Gauss-Seidel (BGS) partitioned iterative coupling with Newton-Rapson equilibrium iteration is used to analyze the structure-electric interaction using 3D solid elements and MITC4 shell elements. A projection method for FSI [1] is employed to analyze the interaction between MITC4 shell structure and fluid field interaction. This paper treats the interaction between electric field using 3D solid elements and structural fields using MITC4 shell elements. MEMS structures consists of thin layers and undergoes large deformation at resonance, shell elements are more suitable to capture the nonlinear effect than 3D solid elements. That is the reason we employed MITC4 shell elements to perform geometric nonlinear structural analysis. A novel translation method is introduced to exchange the electrical and mechanical variable between solid elements and MITC4 shell elements. It follows from the comparison between the numerical and analytical results that our translation method accurately translates the induced nodal electric forces from 3D solids to MITC4 shell elements and displacements from any material point in shell elements are translated to corresponding nodes in 3D solid elements.

2 Methodology : Fluid-structure-electric interaction analysis

The proposed method to analyze fluid-structure-electric field interaction is schematically given in Figure 1 and Figure 2. At first we analyze the piezoelectric interaction, which is fundamentally an interaction between electrical and structural fields. Partitioned iterative coupling algorithm is used to analyze the structure-electric interaction. Using 3D solid elements, we obtain an induced electric forces due to the inverse piezoelectric effect causing the piezoelectric bimorph actuator to bend. Secondly, a novel translation method is implemented to translate the induced nodal electrical forces from 3D solid elements to the equivalent nodal forces and moments acting on MITC4 shell elements for the purpose to take into account the thin structure of the MEMS based piezoelectric actuators. Block Gauss-Seidel partitioned iterative method is employed to couple electric and mechanical variables [6]. Acceleration based Newmark's time β time integration scheme is employed to analyze structure-electric interaction. The full Newton-Rapson iteration is used for the nonlinear structural analysis in MITC4 shell elements.

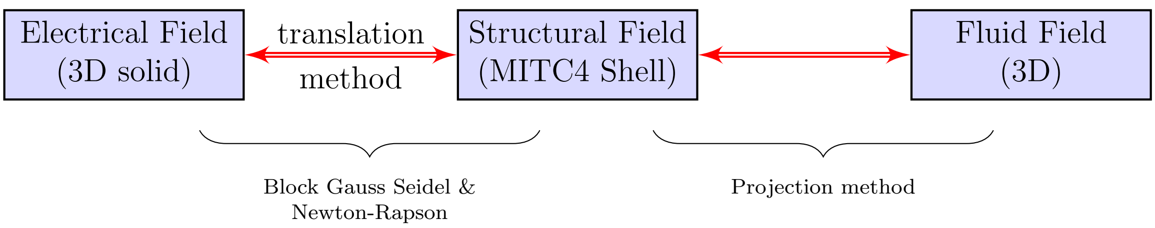

Figure 1: Schematic of fluid-structure-electric field interaction

And finally, a projection method for FSI [1] is employed in order to solve the interaction between MITC4 shell structure and fluid field interaction including the fluid incompressibilty, where the hierarchical decomposition approach [2] is used to couple the structure-electric and fluid-structure interaction analyses. Here structure-fluid interaction is split into the structure-fluid velocity field and pressure field using a kind of algebraic splitting as shown in Figure 2, which avoids larger DOF's of equations in comparison with monolithic coupling and uses the intermediate state variables. The incompressibity constrain is solved using pressure poisson equation (PPE). The predictor-multi corrector algorithm given by Newmark's β method is used for the time integration for the fluid-structure interaction analysis.

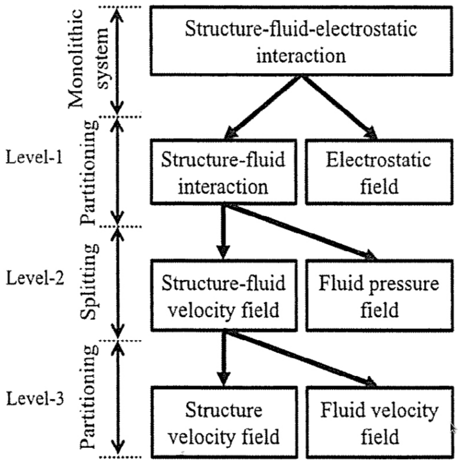

Figure 2: Schematic of hierarchical decomposition of fluid-structure-electric fields [2]

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We discuses about the interaction between solid and shell elements to analyze piezoelectric effect in detail in the following sections.

2.1 Translation method for interaction analysis between 3D solid and shell elements to analyze piezoelectric effect

The standard finite element equations of piezoelectricity is given as [3]

$$\mathbf{M}\ddot{\mathbf{u}} + \mathbf{K}_{\mathrm{uu}}\mathbf{u} + \mathbf{K}_{\mathrm{u}\phi}\phi = \mathbf{F}, \tag{1}$$

$$\mathbf{K}_{\mathrm{u}\phi}^{\mathrm{T}}\mathbf{u} + \mathbf{K}_{\phi\phi}\phi = \mathbf{q}, \tag{2}$$

where $\mathbf{M}$ is the mass matrix, $\mathbf{K}_{\mathrm{uu}}$ is the mechanical stiffness matrix, $\mathbf{K}_{\mathrm{u}\phi}$ is the piezoelectric stiffness matrix, $\mathbf{K}_{\phi\phi}$ is the dielectric stiffness matrix, $\mathbf{F}$ is the external force vector, superscript T stands for transpose matrix and $\mathbf{q}$ is the surface density charge vector. The term $\mathbf{K}_{\mathrm{u}\phi}\phi$ in Eq.(1) is a 3D nodal vector of electrical forces in solid elements. These forces are translated to the equivalent force and moment in shell elements. The transformation equation to translate the electrical forces from solid to shell is described as

$$ {}^{e}\mathbf{F}^{\text{shell}} = \mathbf{A}{}^{e}\mathbf{F}^{\text{solid}}, \tag{3} $$

where $\mathbf{A}$ is the transformation matrix, which connects electric forces from solid to shell. The components of this matrix are obtained from the equilibrium equations of the electric forces and their moment. After translating the electrical nodal forces and moments from solid to shell elements using the relation given in equation (3), displacements are obtained, and they transformed from shell to solid at any material point using the finite element interpolation as

$$ {}^{\text{t}}\mathbf{u}_i = \mathbf{h}_k {}^{\text{t}}\mathbf{u}_i^k + \frac{r_3}{2}\text{a}_k \mathbf{h}_k ({}^{\text{t}}\mathbf{V}_{ni}^k - {}^{0}\mathbf{V}_{ni}^k), \tag{4} $$

where ${}^{\text{t}}\mathbf{u}_i$ is the displacements of any material point with natural coordinates (r_1, r_2, r_3) of the shell element in the stationary Cartesian directions (x_1, x_2, x_3) at time t, ${}^{\text{t}}\mathbf{u}_i^k$ is the nodal displacements of shell elements into the Cartesian directions (x_1, x_2, x_3) at time t, $\mathbf{h}_k$ is the two dimensional shape functions corresponding to (r_1, r_2) at nodal point k, and a_k is shell thickness at node k along director vector $\mathbf{V}_n^k$. The displacements from any material point ${}^{\text{t}}\mathbf{u}_i$ with isoparametric coordinates (r_1, r_2, r_3) in shell elements are translated to the corresponding nodes in 3D shell elements using,

$$ \mathbf{u}^{\text{solid}} = \mathbf{B}\mathbf{u}^{\text{shell}}. \tag{5} $$

The components of $\mathbf{B}$ matrix are obtained from ${}^{\text{t}}\mathbf{u}_i$.

2.2 Piezoelectric nonlinear dynamic analysis

The nonlinear piezoelectric dynamic analysis is carried out using the block Gauss-Seidel (BGS) partitioned iterative coupling scheme to exchange the mechanical and electrical variables and the full Newton-Rapson (NR) equilibrium iterations to perform nonlinear structural analysis. The total Lagrangian formulation is used to take into account the structural large deformation. Newmark's time integration is employed for the time marching. The linearized coupled equations for the nonlinear piezoelectricity is given as [5],

$$ {}^{\text{t+}\Delta\text{t}}_{\ 0}\mathbf{K}_{\phi\phi}\ {}^{\text{t+}\Delta\text{t}}\phi^{(\text{i})} = {}^{\text{t+}\Delta\text{t}}_{\ 0}\mathbf{q} - {}^{\text{t+}\Delta\text{t}}_{\ 0}\mathbf{K}_{\text{u}\phi}^{\text{T}}\ {}^{\text{t+}\Delta\text{t}}\mathbf{u}^{(\text{i}-1)}, \tag{6} $$

$$ {}^{\text{t+}\Delta\text{t}}_{\ 0}\hat{\mathbf{K}}_{\text{uu}}^{(\text{k}-1)}\boldsymbol{\Delta}\mathbf{u}^{(\text{k}),(\text{i})} = {}^{\text{t+}\Delta\text{t}}\tilde{\mathbf{R}}^{(\text{k}-1)} - {}^{\text{t+}\Delta\text{t}}_{\ 0}\mathbf{F}^{(\text{k}-1)} - {}^{\text{t+}\Delta\text{t}}_{\ 0}\mathbf{K}_{\text{u}\phi}{}^{\text{t+}\Delta\text{t}}\phi^{(\text{i})}, \tag{7} $$

$$ {}^{\text{t+}\Delta\text{t}}_{\ 0}\hat{\mathbf{K}}_{\text{uu}}^{(\text{k}-1)} = \mathbf{M}\frac{1}{\beta\Delta\text{t}^2} + {}^{\text{t+}\Delta\text{t}}_{\ 0}\mathbf{K}_{\text{uu}}^{(\text{k}-1)}, \tag{8} $$

where ${}^{\text{t+}\Delta\text{t}}_{\ 0}\mathbf{K}_{\text{uu}}^{(\text{k}-1)}$ is the tangent stiffness matrix in Newton-Rapson iteration $(\text{k}-1)$, ${}^{\text{t+}\Delta\text{t}}_{\ 0}\mathbf{K}_{\phi\phi}$, ${}^{\text{t+}\Delta\text{t}}_{\ 0}\mathbf{K}_{\text{u}\phi}$ and ${}^{\text{t+}\Delta\text{t}}_{\ 0}\mathbf{q}$ are dielectric stiffness matrix, piezoelectric stiffness matrix and vector of electric charge, respectively. ${}^{t+\Delta t}\boldsymbol{\phi}^{(i)}$ is the electrical potential evaluated from the previous BGS iteration displacement solution ${}^{t+\Delta t}\mathbf{u}^{(i-1)}$. And, $\mathbf{\Delta u}^{(k),(i)}$ is calculated using the updated solution of electric potential in Eq.(7). Then, the displacement is updated using

$$ {}^{t+\Delta t}\mathbf{u}^{(k),(i)} = {}^{t+\Delta t}\mathbf{u}^{(k-1),(i)} + \mathbf{\Delta u}^{(k),(i)}. \tag{9} $$

Here, we propose two approach for the full Newton-Rapson iteration and BGS iteration as follows :

1. Approach 1: For each BGS iteration, only one Newton-Rapson iteration is performed as shown in Figure 3.

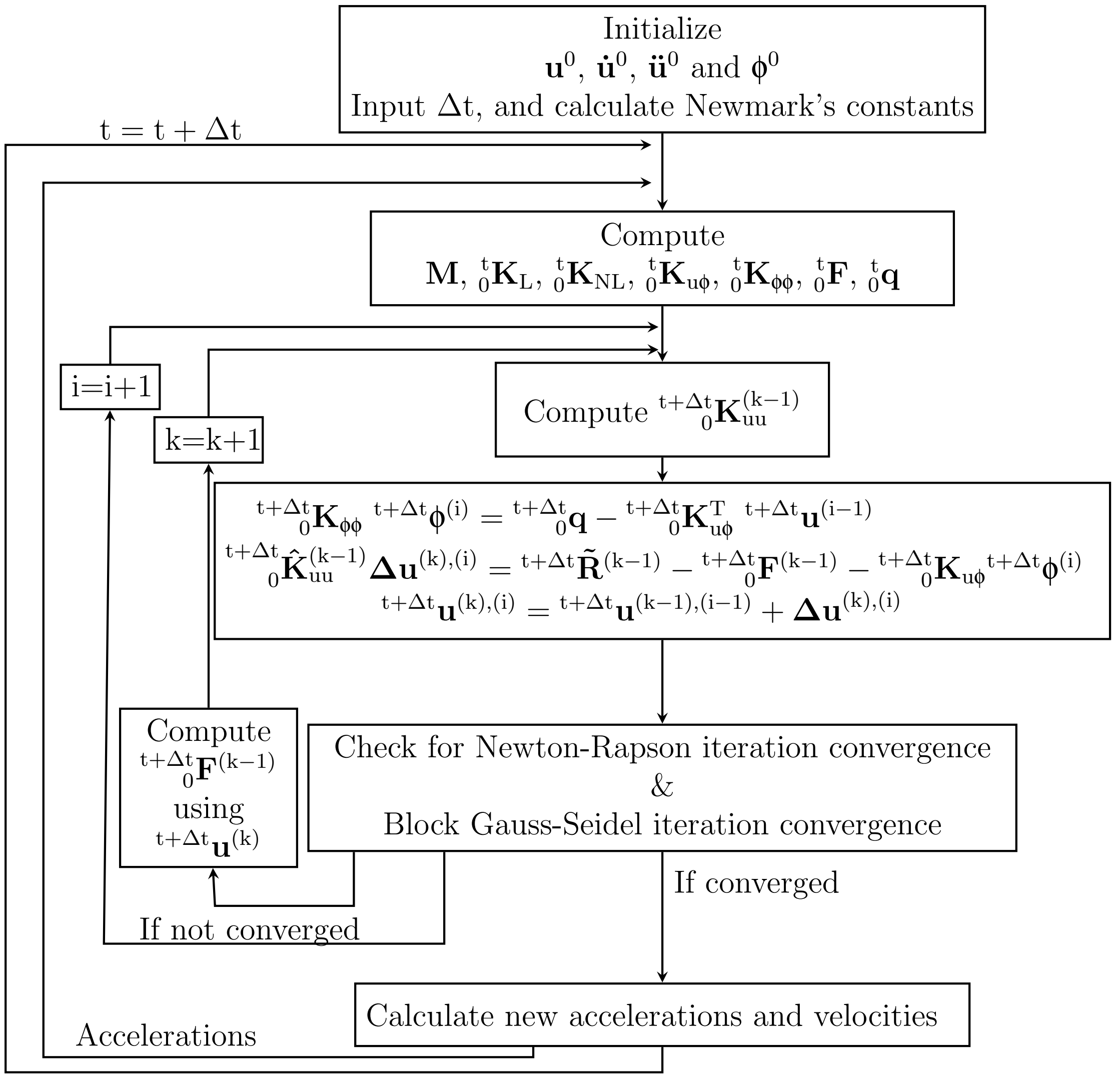


Figure 3: Nonlinear dynamic piezoelectric analysis: Approach (1)-Unified algorithm

2. Approach 2: In this approach, for each BGS iterations, the full Newton-Rapson equilibrium is evaluated until the energy tolerance is satisfied. There are several NR iterations for each BGS iterations in a time step depending on deformation. This approach may be computationally expensive, but the solution accuracy is much better than Approach 1.

3 Numerical problem : Piezoelectric bimorph actuator connected in series

Piezoelectric bimorph actuators are widely used to MEMS. Bimorph actuators consist of a double layer of piezoelectric ceramics as shown in Fig.4. The electrostatic forces across the ceramic layers causes one top layer to contract and bottom layer to expand. The bimorph actuator is connected in series type has dimensions length L=100 mm, width w=1mm and thickness t_p=0.5mm. The mesh for electric analysis consists of 1343 nodes and 160 hexahedron 20 nodes elements and the mesh for structural analysis consists of 82 nodes and 40 MITC4 shell elements. The theoretical solution for deflection in thickness direction along length for this numerical problem is given as [7],

$$\mathrm{u}_3(\mathrm{x}_1) = \frac{3\mathrm{x}_1^2}{4t_p} d_{31} E_3, \tag{10}$$

where d_{31} is piezoelectric constant, E_3 is an applied electric field and t_p is the thickness of each piezoelectric layers. The tip deflection is given as,

$$\delta = \frac{3L^2}{4t_p} d_{31} E_3. \tag{11}$$

The material properties used in the analysis are given in Table.1.

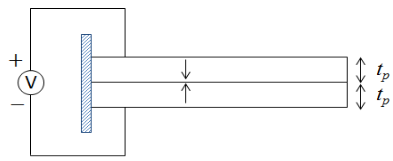


Figure 4: Piezoelectric bimorph actuator connected in series type, ↑ is polarization direction

Table 1: Material properties used for calculation

materials	e_{31} (C/m^2)	d_{31} (C/N)	Young's modulus (N/m^2 $\times 10^9$)	Density (kg/m^3)	ν
PVDF	0.046	2.30×10^{-11}	2.0	1800	0.29

Substituting the actuator dimensions and material properties into Eq. 11 for an applied voltage 1 V, we get tip deflection in thickness direction analytically,

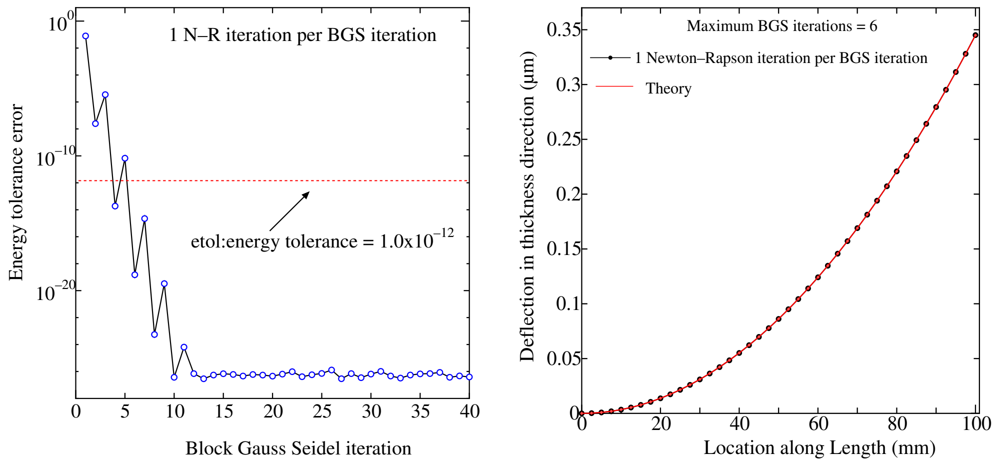
$$\delta = 0.3450\mu m$$

4 Results and Discussion

The numerical problem described in Figure 4 is examined here using both Approach 1 and Approach 2 described in Section 2.2 to analyze the static and dynamic behaviors.

4.1 Static analysis of bimorph actuator connected in series type

At first, the convergence properties and the solution accuracy of the unified algorithm (Approach 1) is presented. In the Figure 5(a) it shows that 6 BGS iterations and 6 NR iterations are necessary to satisfy the convergence tolerance. The energy tolerance value is set as 1.0×10^{-12}. Figure 5(b) shows the transverse deflection of the bimorph actuator at the interface between two PVDF layers compared with the theoretical solution from Eq. 10. The transverse static deflection using Approach 1 along the length of the bimorph actuator coincides with the theoretical solution. The numerical solution for the deflection at the tip of the actuator is $0.3451\mu m$, and theoretical solution for the same using Eq. 11 is $0.3450\mu m$. The % error between numerical and theoretical solution is less than 0.02%.



(a) Iteration convergence analyze (b) Static deflection: Accuracy

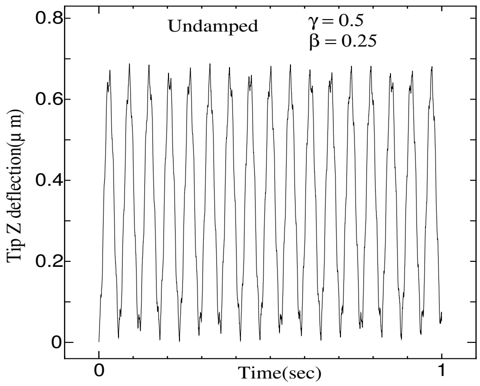
Figure 5: Numerical analyses using unified algorithm : static deflection

In Approach 2, two NR equilibrium iterations for the first BGS iteration and one NR iteration for successive BGS iterations is sufficient. Approach 2 also has a same level accuracy of solution with Approach 1.

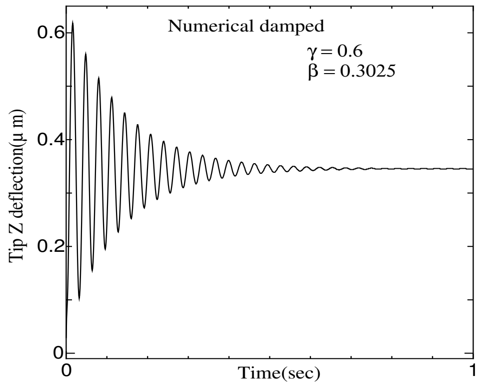
4.2 Dynamic analysis of bimorph actuator: Step response

Here, the vibration characteristics of the numerical problem in Figure 4 are presented for Approach 2. The actuator is driven by a step voltage of 1 V for undamped and numerically damped conditions. Δt is chosen as 1.0×10^{-4} sec for Approach 2, based on the previous convergence, the number of BGS iterations per time step is set as 4. The purpose to analyze the numerical damped case is to find the steady state deflection and to compare the static deflection in order to check the accuracy of the solution. The

Newmark's parameters β and and γ for damped condition are chosen as $\gamma = 0.6$ and $\beta = 0.25 \times (\gamma + 0.5)^2 = 0.3025$. As can be seen in Figure 6(b), the deflection at steady state is close to that of the static theoretical tip deflection.



(a) Numerically undamped condition



(b) Numerically damped condition

Figure 6: Step response of bimorph actuator

4.3 Dynamic analysis of bimorph actuator: AC response

The piezoelectric bimorph actuator described in Figure bimorph is examined here. The actuator is excited with input voltage $V sinw$t, where V is the amplitude and w is the angular frequency of the charge q. Although Newmark's time integration is unconditionally stable, the accuracy of the solution of a coupled problem is dependent on the choice of the time increment Δt [8]. The Newmark's parameters β and and γ for AC analysis are chosen as $\gamma = 0.5$ and $\beta = 0.25$. A bias voltage V=1V is applied to both PVDF layers.

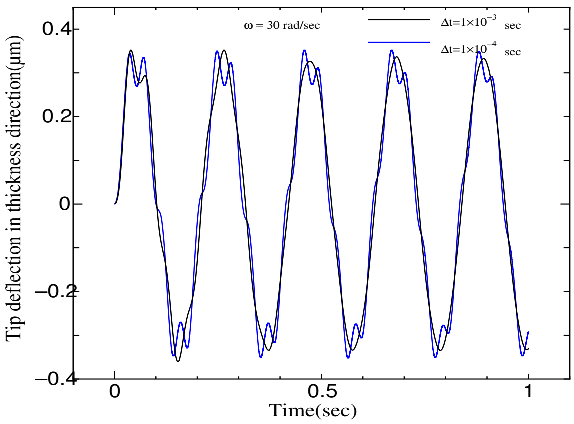


Figure 7: AC response for input frequency ω=30 rad/sec Approach 2

The Figure 7 shows the vibration characteristics of piezoelectric bimorph actuator excited with a bias voltage V=1V at a frequency ω = 30rad/sec using Approach 2. There are 4 BGS iterations per time steps are employed and NR iterations at each BGS iterations varies depending on the energy tolerance and deformation. The actual structural resonance frequency of the numerical problem is ω_r=107 rad/sec [6]. The input voltage frequency used in this study is far away from the resonance, therefore the maximum deflection is around the steady state. For input voltage frequency of ω_r=107 rad/sec, resonance can be obtained using a very fine time increment for this proposed methods.

5 CONCLUSIONS

The proposed coupled algorithms to analyze the interaction between electric field by employing 3D solid elements and structural field by using MITC4 shell elements accurately take into account the piezoelectric interaction. The translation of nodal electrical forces in solid elements to the equivalent force and moment in MITC4 shell elements, and translation of displacements from shell to solid is accurately done using our translation method. Both Approach 1 and Approach 2 posses a same level accuracy of the solution. As a future work, shell structure and fluid field interaction will be performed using projection method.

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