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DAMPED FREE MOTION OF ROCKING SPECIMENS IN NON-LINEAR TIME HISTORY RESPONSE ANALYSIS

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Abstract. In dynamic soil-structure interaction (SSI), the structure such as a building, the sway of the foundation and the rocking on ground, during an earthquake, are usually analyzed without coupled system. In this paper, assuming a restoring force - displacement characteristics for rocking, while the structure and the sway on the assumption are rigid body, non - linear time history response analysis of rocking in the damped free motion of are carried out, with the comparison of the experimental values.

1 INTRODUCTION

In dynamic soil-structure interaction (SSI), the structure such as a building, the sway of the foundation and the rocking on ground, during an earthquake, are usually analysed without coupled system. But coupled system analyses are possible [1]. In the analysis of the SSI, the three relationships of the restoring force - displacement characteristics are required respectively. Therefore, in this paper, assuming a restoring force - displacement characteristics for rocking, while the structure and the sway on the assumption are rigid body, non - linear time history response analysis of rocking in the damped free motion are carried out, with the comparison of the experimental values.

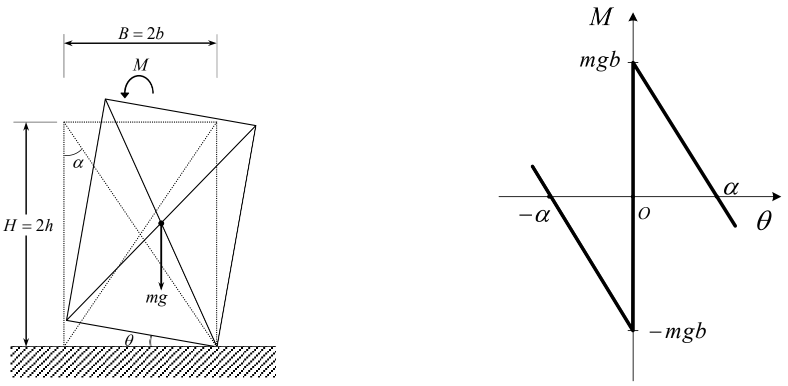
Restoring force – displacement characteristics of rocking are already analysed when the stiffness is negative and these complementary functions are known without the external forces. However, probably because the period on the ground is not taken into account, the time history response analysis are not so much made. Therefore, in this analysis, assuming a period on the ground in 0.16 seconds, etc. and the moment - rotation angle relationship as the bilinear model, non - linear time history response analysis are executed.

2 NON-LINEAR TIME HISTORY RESPONSE ANALYSIS FOR ROCKING

Generally, the equation of damped free motion of rocking in Figure 1 is described in equation (1). In Figure 1, *m* is the mass, *B*(=2*b*) is the width of the rigid body, *H* (=2*h*) is the height and

g is the gravity acceleration. In equation (1), I is the moment of inertia at the ground surface, c_I is the damping coefficient, k_I is the rocking stiffness, $\ddot{\theta}$ is the response acceleration of angle, $\dot{\theta}$ is the response velocity of angle, h_I is the length of the moment arm by external forces and θ is the angle. M (=$k_I\theta$) is the internal moment of the rigid body, the relationship between M and θ is shown in Figure 1 (b) [2]. When k_I is assumed to be too large at θ=0 and M is assumed to be 0, mgb or $-mgb$, the mathematical solutions for each θ are very difficult.

$$I\ddot{\theta} + c_I\dot{\theta} + k_I\theta = -m\ddot{y}h_I \quad (1)$$

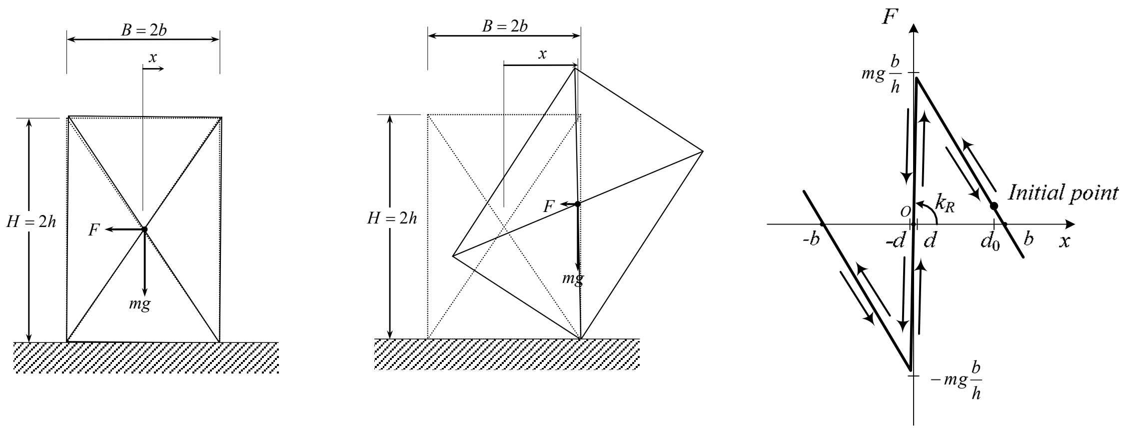


(a) Rigid Body in Motion (b) Relationship of M and θ

The ground is rigid. α is the angle of the width to the height (b/h).

Figure 1: Rigid Body in Motion and the Relationship between the internal moment M of the centre of the gravity and the rotation angle of rocking θ when the ground is rigid

In order to make analysis for rocking, a stiffness at θ(= 0) is needed and the relationship of $M-\theta$ is more difficult than the relationship between the horizontal restoring force F and the horizontal displacement x to understand the rocking motion. If the start of analysis has some horizontal displacement without velocity, and during the motion there are no external forces, the analysis should be simple.



(a) F is almost $mg(b/h)$ (b) x is almost b (c) Relationship of F and x

k_R is the rocking stiffness of the ground based on F and x.

Figure 2: Rigid Body in Motion and the Relationship between the horizontal force F and the horizontal displacement x of the centre of gravity to the ground when the rocking stiffness of the ground is k_R

Therefore, in Figure 2, in this non-linear time history response analysis for rocking, the relationship of $F - x$ is assumed, the rocking stiffness of the ground k_R (>0) is also assumed when x is around zero from $-d$ to d and F is decreased to be zero after F is almost $mg(b/h)$ until x is almost b. The initial point is when x is some value and the velocity is zero.

The relationship of $F - x$ and the equation of damped free motion of rocking are as follows;

$$d < x \quad : \quad F = mg\frac{b-x}{h} \tag{2}$$

$$-d \le x \le d \quad : \quad F = k_R\, x \tag{3}$$

$$x < -d \quad : \quad F = -mg\frac{b+x}{h} \tag{4}$$

$$m\ddot{x} + c\dot{x} + F = 0 \tag{5}$$

These complementary functions at time t are known. The undetermined coefficients, A_1, B_1 and others, are calculated by the initial values and the stiffness respectively as follows [3];

$$d < x \quad : \quad x = b + \mathrm{e}^{-h_1\omega_1 t}(A_1\cosh\omega_{n1}t + B_1\sinh\omega_{n1}t) \tag{6}$$

$$-d \le x \le d \quad : \quad x = \mathrm{e}^{-h_0\omega_0 t}(A_0\cos\omega_{n0}t + B_0\sin\omega_{n0}t) \tag{7}$$

$$x < -d \quad : \quad x = -b + \mathrm{e}^{-h_2\omega_2 t}(A_2\cosh\omega_{n2}t + B_2\sinh\omega_{n2}t) \tag{8}$$

The functions of velocity are defined as the differential equations at time t of the above functions of displacement.

In equation (6) and (8), because the stiffness (F/x) is negative to be ($-mg/h$), the complementary functions includes the hyperbolic functions (sinh, cosh) and constant b. The parameters, damping factor h_1, circular frequency ω_1, constant A_1, circular frequency ω_{n1}, constant B_1 and others are defined by the initial values $x = d_0$ and the velocity is zero. Some of the parameters, h_1 or ω_1, are defined by imaginary number i (= $\sqrt{-1}$) because the stiffness is negative. For example, according to equation (2) and (5), the stiffness k_1 (= $-mg/h$) is negative and the circular frequency ω_1 should be square root of (k_1/m) ($\omega_1 = \sqrt{k_1/m} = \sqrt{-(g/h)} = i\sqrt{g/h}$) which is defined by i. Not only ω_1 or ω_3 but also h_1 or h_3 are defined by i because the damping coefficient c in equation (5) is a constant real number ($h_1\ \omega_1 = c/(2m) = h_0\ \omega_0 = h_2\ \omega_2$), even if the stiffness was changed. But the product of h_1 and ω_1 canceled i .

By the way, ω_{n0}, ω_{n1} and ω_{n2} are real numbers because the complementary functions are solved by the characteristic equation using p of equation (5), $p^2 + 2h_1\omega_1 p - \omega_1^2 = 0$, $p^2 + 2h_0\omega_0 \times p + \omega_0^2 = 0$ or others, where the coefficient in the square of constant is negative or positive, when all parameters, h_0, h_1, ω_0, ω_1, ω_{n0}, ω_{n1} and others, are real numbers.

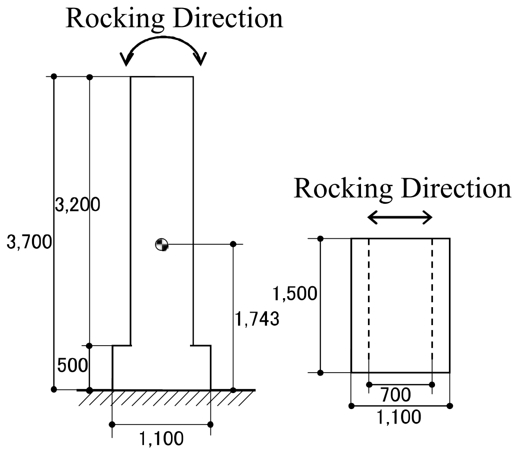
Finally the results of equation (6), (7) and (8) give the real numbers.

When x is close to d, after the initial point, the interval time in the analysis was 0.0005 (sec) (=2,000Hz) in equation (6). After the time, when x was d, was defined, the initial values of x and the velocity $\dot{x}$ in equation (7) were defined to continue the analysis.

These analyses were executed to be compared with the test results of the following section.

3 ROCKING SPECIMEN

In recent years, the experiment project of free standing concrete column specimens by placing some situations of ground was produced, when the damped free motion given an initial value of the horizontal displacement was performed by the several corporations [4]. Specimens, height 3.7m, width 1.1m, the thickness 1.5m and the weight 104.9 kN in the free end. The height of the centre of mass (◕) is 1.743m. The specimens were placed on the ground, were inclined to the top displacement horizontally 0.375m and so on.



(a) Elevation (b) Plan

Figure 3: Rocking Specimen (Reinforced concrete column on concrete slab)

(a) Rocking Specimen under loading (b) Photo of Rocking Specimen under loading

Figure 4: Rocking Specimen under loading (Reinforced concrete column on concrete slab)

After the horizontal tension force was released, the rocking damped free motions were conducted (Figure 3 and Figure 4). The release points for one loading were at the top displacement horizontally 0.125m for 2 times, 0.250m for 2 times, 0.375m for 2 times and 0.125m for 1 time. The total number of loadings was 7 times. In the experiment, the acceleration and the displacement of some parts were measured, when these data were compared to this analysis for the horizontal force and the horizontal displacement of the centre of mass.

4 ANALYSIS AND RESULTS

In the analysis, while the damping ratio was a parameter, the response horizontal force F and the response horizontal displacement x of the centre of mass were calculated and compared with the experimental values. In Figure 5, Idealized F/mg and x shows the typical points which are the angle of the width to the height (b/h)(=550/1,743=0.316) in y-axis and the width b (=55cm) in x-axis. Experimental F/mg and x shows the proposed points which are the Uplift point (F/mg, x)=(0.273-, 0.09cm) and the Initial point (0.202-, 19.7cm).

These points are exchanged to the relationship $M-\theta$. In Idealized relationship, $M=mgb$= 57.7kN･m is in y-axis and θ=0.316rad. is in x-axis. In experimental relationship, the Uplift point (M,θ)=(50kN･m, 0.0005rad.) and the Initial point (37kN･m, 0.113rad.) were proposed. The Initial point is located under the line of the Idealized relationship.

According to the elastic stiffness, k_R in equation (3) or ω_0 in equation (7), the period of the contact of the specimen to the ground was calculated to be 0.11 second. Outside of the elastic, the response functions include the hyperbolic functions (sinh, cosh) without trigonometric functions in equation (6) and (8) and don't have any period.

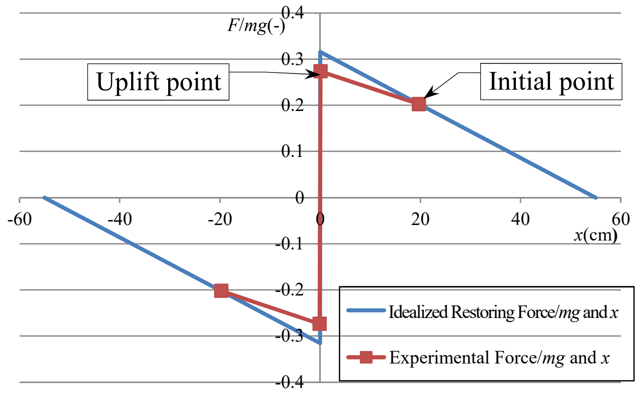
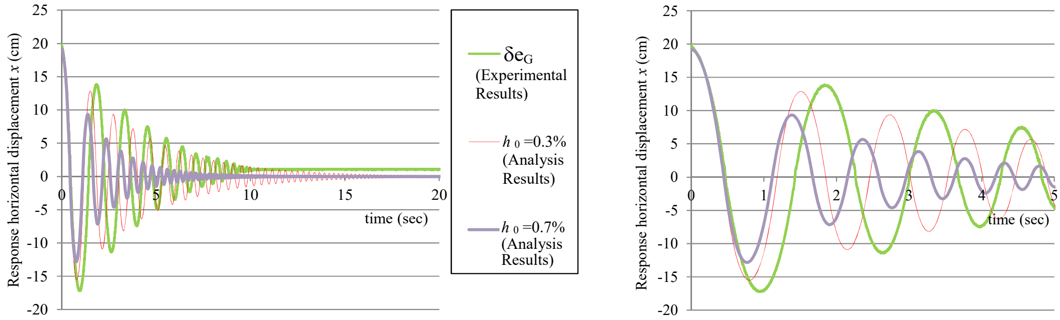


Figure 5: Relationship of F and x of the centre of mass

As the results, Figure 6 shows the non-linear time history response analysis results of the damped free motion of the Rocking Specimen. In Figure 6, δe_G is the horizontal displacement of the centre of gravity to the ground and started at 19.7cm when the top displacement horizontally was 0.375 m in the first time among the 2 times loadings. Until 10 (sec), δe_G has some amplitude and after 10 (sec), it has some residual displacement.

According to the relationship of Figure 5 and equations (2) – (8), the analysis results of x are also shown in Figure 6. They are 2 results when h_0 is 0.3% and h_1 is 0.1/i or h_0 is 0.7% and h_1 is 0.2/i .When h_0 is 0.3%, the amplitude of x are close to the experiment data until 10 (sec). When h_0 is 0.7%, the amplitudes of x are close to the experiment data after 10 (sec). Figure 6

(b) shows the same results of until 5 (sec) of 20 (sec) in Figure (6) (a), while the interval times between the peaks are not similar in the experimental data and the 2 results. In the analysis, the grounding times of the specimen were calculated to be about 0.002 (sec) or more.



Notes are same as the Notes of Figure 6 (a)

(a) Until 20 (sec) (b) Until 5 (sec)

Figure 6: Comparison of Experimental Result (δe_G) and Analysis Results (h_0=0.3% and h_0=0.7%)

5 CONCLUSIONS

In this paper, the damped free motions of rocking specimens were analyzed in non-linear time history response analysis. A comparison between these analysis and the experimental results were executed when the initial points were some horizontal displacement and the stiffness during the contact of the specimen to the ground were considered.

The main points of these results are as follows;

(1) When the stiffness is negative, the response horizontal force and the response horizontal displacement were calculated by the hyperbolic functions.

(2) The period by the stiffness during the contact of the specimen to the ground in this analysis was 0.11 (sec).

(3) The similar response displacements to the experimental results were when the damping factors at the displacement zero were 0.3% or 0.7%.

(4) The grounding times of the specimen in this analysis were calculated to be about 0.002 second or more.

(5) The comparison between the analysis and the experimental results were not so much similar, therefore, the coupled system like Soil-Structure Interaction should be necessary.

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