VII International Conference on Computational Methods for Coupled Problems in Science and Engineering
COUPLED PROBLEMS 2017
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COUPLED DYNAMICS OF SOLID SYSTEM WITH SLIDER-CRANK MECHANISMS AS INTERNAL MOVERS ON ROUGH SURFACE WITH FRICTION

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Key words: Coupled Dynamics, Solid System, Internal Movers, Slider-Crank Mechanisms, Dry Friction, Mathematical Modeling.

Abstract. Equations of coupled dynamics of the solid system with slider-crank mechanisms as internal movers are obtained on rough surface with friction for cases of translational and rotational movement by the methods of mathematical modeling. Such class mechanism driven by inner movements of masses being isolated from surrounding space in shell can be used in conditions where traditional movers (wheels, tracks, legs) for some reasons are not applicable.

1 INTRODUCTION

An interest in solid systems driven by inner movements of masses without outer movers, such as wheels, chain tracks, or legs, arises in the last decade [1-4]. A new class of mechanisms (robots), able to move in a resisting medium without external movers due to movement of internal bodies, attracts attention and is studied. At constant outer shell by changing internal geometry of mass the movement of solid system can be carried out in an arbitrary point.

A solid system moving on three points of support is also very attractive for researchers [5-7]. They investigate: dynamics of a body sliding on a rough plane and supported at three points; exact normal forces and trajectories for a rotating tripod sliding on a smooth surface; problems on the motion of a disc with three supports on a rough plane.

This work is devoted to the same class mechanism. Studied solid system consists of main frame (supporting structure) that has three points of contact with rough surface and moves coplanar; and two nesting slider-crank mechanisms that move respectively two internal masses (sliders) relatively to the frame. The crank is rotated by direct-current motor, so in mathematical modelling it moves under assumption of a decreasing linear relationship between torque of motor and angular velocity of its shaft. In dry friction between frame and surface the local Amontons–Coulomb law is used. The movement of the system is studied with the help of mathematical modelling.

The equations of motion of the system with movable masses are obtained. Two types of the frame movement are considered: translational (sliding) and rotational (spinning). Preliminary experimental observations make it possible to assert that approximately periodic movement can be achieved in sliding or spinning of supporting frame on rough surface.

The movement can be explained qualitatively as follows. Slider-crank mechanism approximately periodically moves masses with different accelerations in different directions. Unequal pulses of the masses are converted into a non-uniform translational/rotational movement of the supporting frame, and hence there are unequal frame pulses differently compensated by friction forces between points of contact and rough surface.

Such class mechanism being isolated from surrounding space in shell can be used in conditions where traditional movers (wheels, tracks, legs) for some reasons are not applicable: corrosive environments or limited in size, on the outside plating of a spaceship or pipeline, in conditions of different planets, etc.

2 MECHANICAL MODEL

3D-concept of the mechanism is shown in Figure 1. As you can see, the solid system has three points of contact with the horizontal surface.

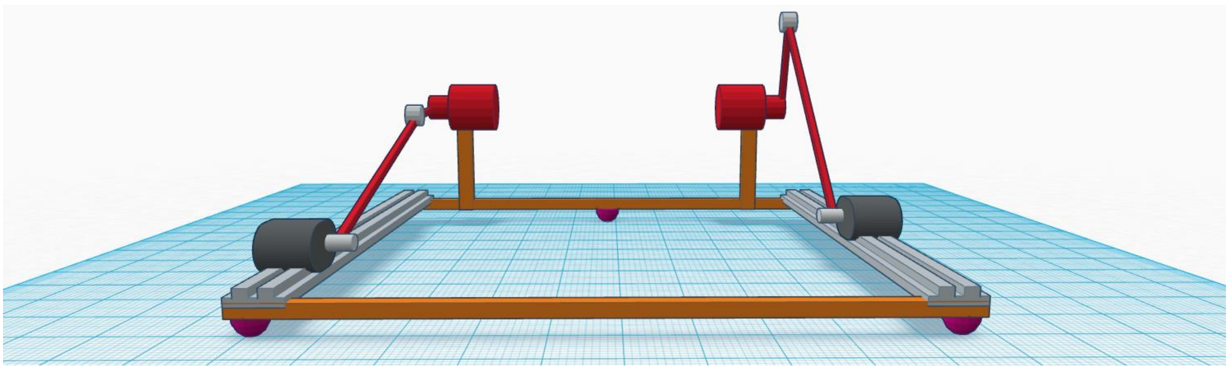

Figure 1: 3D-concept of the model

Two heavy sliders move along the guide rails on the supporting structure. Two motors transmit mechanical power through cranks and links to the sliders. When moving, mechanism does not bounce as well as sliders. It remains in full contact with the surface.

3 MATHEMATICAL MODEL

For motion control on a predictable trajectory let's divide the movement into two basic types. The first – sliding or moving forward in a straight line; and the second – spinning or rotation around a fixed point. In this section we obtain the equations of motion for these cases.

3.1 Underlying assumptions

No friction is inside construction. Friction is only between points of contact and horizontal surface. The friction is described by Amonton-Coulombs law, we use dry friction model. Direct current motor is described by linear dependence between torque and angular velocity of shaft. Supporting structure movement occurs without jumping, as well as sliders. We lay in the model that the contact points do not lose contact with the surface, and the reaction forces are always opposite to direction of free fall acceleration.

3.2 Translational case (sliding)

Illustration for the translational case of mathematical model is presented in Figure 2.

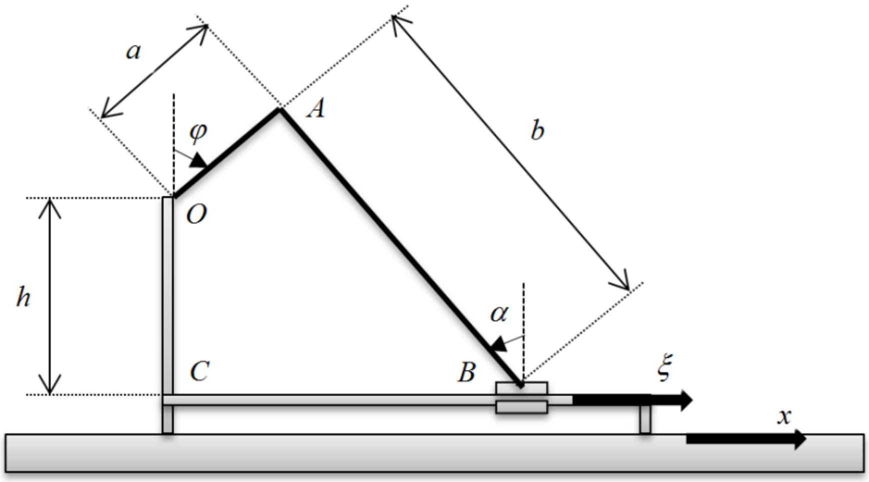

Figure 2: Mathematical model of the translational case (side view)

Mechanical coupling equation for slider-crank mechanism:

$$h + a\cos\varphi = b\cos\alpha \tag{1}$$

Kinetic energy of the system:

$$T = \frac{M\dot{x}_C^2}{2} + \frac{m\dot{x}_B^2}{2} \tag{2}$$

For absolute and relative coordinates we have:

$$x_B = x_C + \xi_B,\ \ \dot{x}_B = \dot{x}_C + \dot{\xi}_B \tag{3}$$

where:

$$\xi_B = a\sin\varphi + b\sin\alpha,\ \dot{\xi}_B = a\cos\varphi\dot{\varphi} + b\cos\alpha\dot{\alpha} \tag{4}$$

Taking into account the relation of the equation (1), we have:

$$a\sin\varphi\dot{\varphi} = b\sin\alpha\dot{\alpha} \tag{5}$$

Consequently:

$$\dot{\alpha} = \frac{a\sin\varphi}{b\sin\alpha}\dot{\varphi} \tag{6}$$

So for the relative velocity of slider *B*:

$$\dot{\xi}_B = a\cos\varphi\dot{\varphi} + (h + a\cos\varphi)\frac{a\sin\varphi}{b\sin\alpha}\dot{\varphi} \tag{7}$$

In our model $\sin\alpha > 0$, hence we obtain:

$$\sin\alpha = \left(1-\cos^2\alpha\right)^{1/2} = \left(1-\left(\frac{h+a\cos\varphi}{b}\right)^2\right)^{1/2} = \frac{1}{b}\left(b^2-\left(h+a\cos\varphi\right)^2\right)^{1/2} \tag{8}$$

With regard to the latter, we can write for (7):

$$\dot{\xi}_B = a\dot{\varphi}\left(\cos\varphi + \frac{\left(h+a\cos\varphi\right)\sin\varphi}{\left(b^2-\left(h+a\cos\varphi\right)^2\right)^{1/2}}\right) \tag{9}$$

For simplicity, we omit the index:

$$x_C \equiv x \tag{10}$$

Then finally for kinetic energy:

$$T = \frac{M\dot{x}^2}{2} + \frac{m}{2}\left(\dot{x} + a\dot{\varphi}\left(\cos\varphi + \frac{\left(h+a\cos\varphi\right)\sin\varphi}{\left(b^2-\left(h+a\cos\varphi\right)^2\right)^{1/2}}\right)\right)^2 \tag{11}$$

In our model, the potential energy is constant, so we assume it to be zero. The Lagrangian is equal to the kinetic energy.

Generalized force, taking into account the friction forces with the local Amontons–Coulomb law, will be equal to:

$$Q_x = \begin{cases} -\mu N, \text{ } if \ \dot{x} > 0 \\ 0, \text{ } if \ \dot{x} = 0 \\ +\mu N, \text{ } if \ \dot{x} < 0 \end{cases} \equiv -\mu N \operatorname{sgn}\dot{x} \tag{12}$$

And for the generalized force applied to the crank we consider direct-current motor, so in mathematical modelling it moves under assumption of a decreasing linear relationship between torque of motor and its shaft angular velocity:

$$Q_\varphi = \mathrm{A} - \mathrm{B}\dot{\varphi};\quad \dot{\varphi}, \mathrm{A}, \mathrm{B} > 0 \tag{13}$$

The first equation of motion in the Lagrangian form is:

$$\frac{d}{dt}\frac{\partial T}{\partial \dot{x}} - \frac{\partial T}{\partial x} = Q_x \tag{14}$$

Let’s denote the following functions:

$$f \equiv \left(h+a\cos\varphi\right)\sin\varphi \tag{15}$$

$$g \equiv \left(b^2-\left(h+a\cos\varphi\right)^2\right)^{1/2} \tag{16}$$

Then rewrite the expression for the kinetic energy in the form:

$$T = \frac{M\dot{x}^2}{2} + \frac{m}{2}\left(\dot{x} + a\dot{\varphi}\left(\cos\varphi + \frac{f}{g}\right)\right)^2 \tag{17}$$

So:

$$\frac{\partial T}{\partial \dot{x}} = M\dot{x} + m\left(\dot{x} + a\dot{\varphi}\left(\cos\varphi + \frac{f}{g}\right)\right); \quad \frac{\partial T}{\partial x} = 0 \tag{18}$$

$$\frac{d}{dt}\frac{\partial T}{\partial \dot{x}} = M\ddot{x} + m\left(\ddot{x} + a\ddot{\varphi}\left(\cos\varphi + \frac{f}{g}\right) + a\dot{\varphi}\left(-\sin\varphi\dot{\varphi} + \left(\frac{f}{g}\right)'\right)\right) \tag{19}$$

It is well known that:

$$\left(\frac{f}{g}\right)' = \frac{f'g - g'f}{g^2} \tag{20}$$

We calculate the derivatives:

$$f' = \left[(h + a\cos\varphi)\sin\varphi\right]' = -a\sin^2\varphi\dot{\varphi} + (h + a\cos\varphi)\cos\varphi\dot{\varphi} = \tag{21}$$

$$= \dot{\varphi}(a\cos 2\varphi + h\cos\varphi)$$

$$g' = \left[\left(b^2 - (h + a\cos\varphi)^2\right)^{1/2}\right]' = \frac{1}{2g}\left(b^2 - (h + a\cos\varphi)^2\right)' = \tag{22}$$

$$= \frac{1}{2g}2(h + a\cos\varphi)a\sin\varphi\dot{\varphi} = \frac{f}{g}a\dot{\varphi}$$

So, for (20) we have:

$$\left(\frac{f}{g}\right)' = \frac{\dot{\varphi}(a\cos 2\varphi + h\cos\varphi)g - \frac{f}{g}a\dot{\varphi}f}{g^2} = \tag{23}$$

$$= \frac{\dot{\varphi}}{g^3}\left((a\cos 2\varphi + h\cos\varphi)g^2 - af^2\right)$$

In this way for (19) we obtain:

$$\frac{d}{dt}\frac{\partial T}{\partial \dot{x}} = (M + m)\ddot{x} + ma\left(\ddot{\varphi}\left(\cos\varphi + \frac{f}{g}\right) + \dot{\varphi}^2\left(-\sin\varphi + \frac{1}{g^3}\left((a\cos 2\varphi + h\cos\varphi)g^2 - af^2\right)\right)\right) \tag{24}$$

Consequently for (14) we have equation:

$$(M + m)\ddot{x} + ma\left(\ddot{\varphi}\left(\cos\varphi + \frac{f}{g}\right) + \dot{\varphi}^2(-\sin\varphi + q)\right) = -\mu N \operatorname{sgn}\dot{x} \tag{25}$$

where:

$$q \equiv \frac{1}{g^3}\left(\left(a\cos 2\varphi + h\cos\varphi\right)g^2 - af^2\right) \tag{26}$$

As for normal reaction *N*, we can find it from projection of Newton's law on the vertical direction (opposite to the free fall acceleration g_{FF}), taking into account that the common center of mass does not move vertically:

$$\left((M+m)\vec{a},\vec{n}\right) = \left(\vec{R}^{ext},\vec{n}\right) \;\Rightarrow\; 0 = -(M+m)g_{FF} + N \tag{27}$$

$$N = (M+m)g_{FF} \tag{28}$$

Finally the first equation of motion:

$$(M+m)\ddot{x} + ma\left(\ddot{\varphi}\left(\cos\varphi + \frac{f}{g}\right) + \dot{\varphi}^2\left(-\sin\varphi + q\right)\right) = -\mu(M+m)g_{FF}\operatorname{sgn}\dot{x} \tag{29}$$

Now let’s consider the second equation of motion in the Lagrangian form:

$$\frac{d}{dt}\frac{\partial T}{\partial\dot{\varphi}} - \frac{\partial T}{\partial\varphi} = Q_\varphi \tag{30}$$

Calculation of derivative gives us:

$$\frac{\partial T}{\partial\dot{\varphi}} = ma\left(\dot{x} + a\dot{\varphi}\left(\cos\varphi + \frac{f}{g}\right)\right)\left(\cos\varphi + \frac{f}{g}\right) \equiv ma\left(\dot{x} + a\dot{\varphi}u\right)u \tag{31}$$

where:

$$u \equiv \cos\varphi + \frac{f}{g} \tag{32}$$

We continue calculation:

$$\frac{d}{dt}\frac{\partial T}{\partial\dot{\varphi}} = \left[ma\left(\dot{x} + a\dot{\varphi}u\right)u\right]' = ma\left(\left(\ddot{x} + a\ddot{\varphi}u + a\dot{\varphi}u'\right)u + \left(\dot{x} + a\dot{\varphi}u\right)u'\right) = \tag{33}$$

$$= ma\left(\left(\ddot{x} + a\ddot{\varphi}u\right)u + \left(\dot{x} + 2a\dot{\varphi}u\right)u'\right)$$

where, taking into account also (23) and (26):

$$u' = \left[\cos\varphi + \frac{f}{g}\right]' = \dot{\varphi}\left(-\sin\varphi + q\right) \tag{34}$$

For simplicity we may assign:

$$w \equiv -\sin\varphi + q \tag{35}$$

Then:

$$\frac{d}{dt}\frac{\partial T}{\partial\dot{\varphi}} = ma\left(\left(\ddot{x} + a\ddot{\varphi}u\right)u + \left(\dot{x} + 2a\dot{\varphi}u\right)\dot{\varphi}w\right) \tag{36}$$

$$\frac{\partial T}{\partial \varphi}=\left[\frac{M\dot{x}^2}{2}+\frac{m}{2}\left(\dot{x}+a\dot{\varphi}u\right)^2\right]'_{\varphi}=m\left(\dot{x}+a\dot{\varphi}u\right)a\dot{\varphi}u'_{\varphi}=m\left(\dot{x}+a\dot{\varphi}u\right)a\dot{\varphi}w \tag{37}$$

Finally the second equation of motion:

$$ma\left\{\left(\ddot{x}+a\ddot{\varphi}u\right)u+auw\dot{\varphi}^2\right\}=\mathrm{A}-\mathrm{B}\dot{\varphi} \tag{38}$$

In (29) and (38) let’s express higher derivatives through the lower order:

$$\begin{cases}\left(M+m\right)\ddot{x}+mau\ddot{\varphi}=-\mu\left(M+m\right)g_{FF}\operatorname{sgn}\dot{x}-maw\dot{\varphi}^2\\ u\ddot{x}+au^2\ddot{\varphi}=\dfrac{\mathrm{A}-\mathrm{B}\dot{\varphi}}{ma}-auw\dot{\varphi}^2\end{cases} \tag{39}$$

Or in vector-matrix form:

$$\begin{pmatrix}\beta_{11} & \beta_{12}\\ \beta_{21} & \beta_{22}\end{pmatrix}\begin{pmatrix}\ddot{x}\\ \ddot{\varphi}\end{pmatrix}=\begin{pmatrix}\gamma_1\\ \gamma_2\end{pmatrix} \tag{40}$$

where for coefficients:

$$\begin{aligned}&\beta_{11}=M+m,\quad \beta_{12}=mau,\quad \beta_{21}=u,\quad \beta_{22}=au^2\\ &\gamma_1=-\mu\left(M+m\right)g_{FF}\operatorname{sgn}\dot{x}-maw\dot{\varphi}^2\\ &\gamma_2=\frac{\mathrm{A}-\mathrm{B}\dot{\varphi}}{ma}-auw\dot{\varphi}^2\end{aligned} \tag{41}$$

We can solve the system (40) using Cramer's rule:

$$\begin{pmatrix}\ddot{x}\\ \ddot{\varphi}\end{pmatrix}=\frac{1}{\begin{vmatrix}\beta_{11} & \beta_{12}\\ \beta_{21} & \beta_{22}\end{vmatrix}}\begin{pmatrix}\begin{vmatrix}\gamma_1 & \beta_{12}\\ \gamma_2 & \beta_{22}\end{vmatrix}\\ \begin{vmatrix}\beta_{11} & \gamma_1\\ \beta_{21} & \gamma_2\end{vmatrix}\end{pmatrix}=\frac{1}{\beta_{11}\beta_{22}-\beta_{12}\beta_{21}}\begin{pmatrix}\gamma_1\beta_{22}-\gamma_2\beta_{12}\\ \gamma_2\beta_{11}-\gamma_1\beta_{21}\end{pmatrix} \tag{42}$$

Then we reduce the problem to the Cauchy problem with variables:

$$\begin{pmatrix}\dot{x}\\ \dot{\varphi}\end{pmatrix}=\begin{pmatrix}v\\ \omega\end{pmatrix} \tag{43}$$

$$\begin{pmatrix}\dot{v}\\ \dot{\omega}\end{pmatrix}=\frac{1}{Mau^2}\begin{pmatrix}-\mu\left(M+m\right)g_{FF}au\operatorname{sgn}v-\left(\mathrm{A}-\mathrm{B}\omega\right)u\\ \left(M+m\right)\left(\dfrac{\mathrm{A}-\mathrm{B}\omega}{ma}+\mu g_{FF}u\operatorname{sgn}v\right)-Mauw\omega^2\end{pmatrix}$$

where, taking into account (32) and (35):

$$u=\cos\varphi+\frac{\left(h+a\cos\varphi\right)\sin\varphi}{\left(b^2-\left(h+a\cos\varphi\right)^2\right)^{1/2}} \tag{44}$$

$$w = -\sin\varphi + \frac{(a\cos 2\varphi + h\cos\varphi)\left(b^2 - (h + a\cos\varphi)^2\right) - a(h + a\cos\varphi)^2 \sin^2\varphi}{\left(b^2 - (h + a\cos\varphi)^2\right)^{3/2}} \tag{45}$$

As we can see the system (43) is valid when $u \neq 0$, but in case of $u = 0$ we should return to the main system of motion (39). This case of $u = 0$ corresponds to the extreme points that the slider may occupy in relative motion on guide rail. It is at these points (or near) that a pulse is transmitted from the slider to the carrier platform. As a result, the platform is moving. Let us assume in equation (39) that $u = 0$, so we obtain the angle corresponding to this value from equation (44):

$$\cos\varphi = \left\{ -\frac{h}{a+b}, \frac{h}{b-a} \right\} \tag{46}$$

and substitute in the system (39):

$$\begin{cases} \ddot{x} = -\mu g_{FF} \operatorname{sgn}\dot{x} - \dfrac{ma}{M+m} w \left(\dfrac{\mathrm{A}}{\mathrm{B}}\right)^2 \\ \dot{\varphi} = \dfrac{\mathrm{A}}{\mathrm{B}} \end{cases} \tag{47}$$

The latter relations determine the conditions for the onset of motion (note that $\dot{x} = \ddot{x} = 0$ in state of rest), in view of the overcoming of frictional forces.

3.3 Rotational case (spinning)

Illustration for the rotational case of mathematical model is presented in Figure 3.

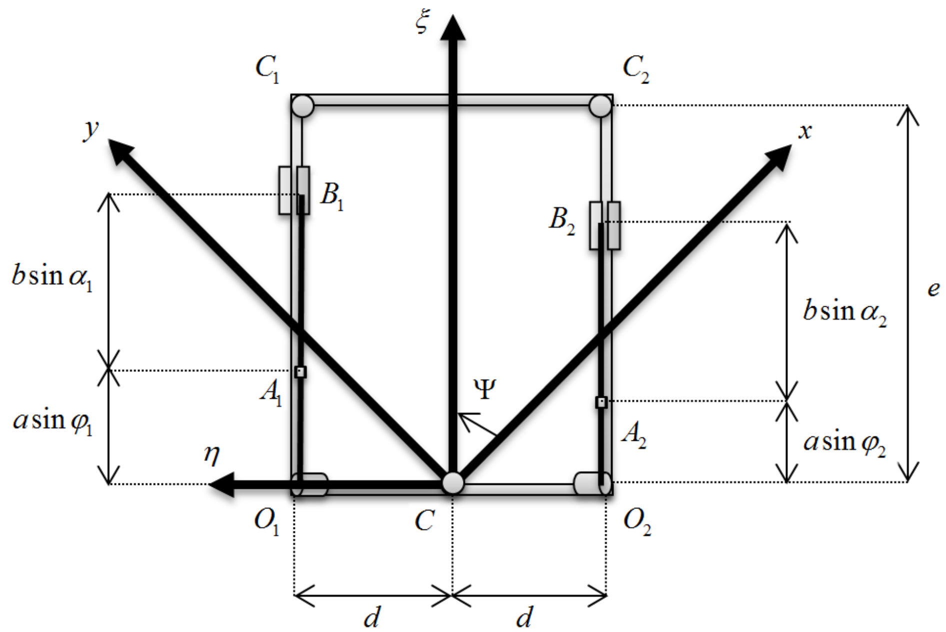

Figure 3: Mathematical model of the rotational case (view from above)

While in the case of the translational motion the sliders moved synchronously, in the case of rotational motion of supporting structure we must consider the movement of the sliders separately. This means that it has to be written for each slider mechanical coupling equation for slider-crank mechanism:

$$h + a\cos\varphi_i = b\cos\alpha_i, \quad i = 1,2 \tag{48}$$

Here, since point C is fixed (center of mass of the support structure coincides with the middle point of contact in the projection onto a plane), the kinetic energy of the system can be written as:

$$T = \sum_{i=1}^{2} \frac{m}{2}\left(\dot{x}_i^2 + \dot{y}_i^2\right) \tag{49}$$

The absolute coordinates of each slider:

$$\vec{r}_{B_i} = \begin{pmatrix} x_i \\ y_i \end{pmatrix} \tag{50}$$

We can link the absolute and relative coordinates through rotation matrix:

$$\vec{r}_{B_i} = A_\Psi \vec{\rho}_{B_i} \tag{51}$$

where:

$$A_\Psi = \begin{pmatrix} \cos\Psi & -\sin\Psi \\ \sin\Psi & \cos\Psi \end{pmatrix} \tag{52}$$

And for relative coordinates of the sliders:

$$\vec{\rho}_{B_1} = \begin{pmatrix} \xi_1 \\ \eta_1 \end{pmatrix} = \begin{pmatrix} a\sin\varphi_1 + b\sin\alpha_1 \\ d \end{pmatrix} \equiv \begin{pmatrix} a\sin\varphi_1 + g_1 \\ d \end{pmatrix} \tag{53}$$

$$\vec{\rho}_{B_2} = \begin{pmatrix} \xi_2 \\ \eta_2 \end{pmatrix} = \begin{pmatrix} a\sin\varphi_2 + b\sin\alpha_2 \\ -d \end{pmatrix} \equiv \begin{pmatrix} a\sin\varphi_2 + g_2 \\ -d \end{pmatrix}$$

where:

$$g_i \equiv \left(b^2 - \left(h + a\cos\varphi_i\right)^2\right)^{1/2} \tag{54}$$

Hence, considering (51) and (52):

$$\vec{r}_{B_i} = \begin{pmatrix} \xi_i\cos\Psi - \eta_i\sin\Psi \\ \xi_i\sin\Psi + \eta_i\cos\Psi \end{pmatrix} \tag{55}$$

For the absolute velocities of the sliders:

$$\vec{v}_{B_i} = \frac{d}{dt}\vec{r}_{B_i} = \begin{pmatrix} \dot{\xi}_i\cos\Psi - \xi_i\sin\Psi\dot{\Psi} - \eta_i\cos\Psi\dot{\Psi} \\ \dot{\xi}_i\sin\Psi + \xi_i\cos\Psi\dot{\Psi} - \eta_i\sin\Psi\dot{\Psi} \end{pmatrix} \tag{56}$$

In (55) we take into account that for the relative coordinates:

$$\eta_i = const \tag{57}$$

By analogy with the translational case (9) for the sliders it can be written:

$$\dot{\xi}_i = a\dot{\varphi}_i \left(\cos\varphi_i + \frac{\left(h + a\cos\varphi_i\right)\sin\varphi_i}{\left(b^2 - \left(h + a\cos\varphi_i\right)^2\right)^{1/2}} \right) = a\dot{\varphi}_i \left(\cos\varphi_i + \frac{f_i}{g_i} \right) \tag{58}$$

where:

$$f_i \equiv \left(h + a\cos\varphi_i\right)\sin\varphi_i \tag{59}$$

On this basis of (56) – (59), we find for kinetic energy:

$$T = \frac{1}{2}\sum_{i=1}^{2}\left(\vec{v}_{B_i}\right)^2 = \frac{1}{2}\sum_{i=1}^{2}\left(\left(a\dot{\varphi}_i\left(\cos\varphi_i + \frac{f_i}{g_i}\right) - \dot{\Psi}\eta_i\right)^2 + \left(\dot{\Psi}\xi_i\right)^2\right) \tag{60}$$

As in the previous case, the potential energy is constant, so we assume it to be zero. The Lagrangian is equal to the kinetic energy.

And for the generalized forces applied to the cranks we consider direct-current motors, so in mathematical modelling they move under assumption of a decreasing linear relationship between torque of motor and its angular velocity:

$$Q_{\varphi i} = \mathrm{A} - \mathrm{B}\dot{\varphi}_i; \quad \dot{\varphi}_i, \mathrm{A}, \mathrm{B} > 0; \quad i = 1,2 \tag{61}$$

For the generalized force of friction we have:

$$Q_{\Psi} = -\mu\left(e^2 + d^2\right)^{1/2}\left(N_1 + N_2\right)\operatorname{sgn}\dot{\Psi} \tag{62}$$

Since the center of mass does not move relative to the vertical:

$$\left(\left(M + m + m\right)\vec{a}, \vec{n}\right) = \left(\vec{R}^{ext}, \vec{n}\right) \;\Rightarrow\; 0 = -\left(M + m + m\right)g_{FF} + N + N_1 + N_2 \tag{63}$$

Since the structure does not rotate relatively around its longitudinal axis passing through the point C:

$$\left(\vec{M}_C, \vec{e}_\xi\right) = 0 \quad \Rightarrow \quad N_1 = N_2 \tag{64}$$

And because the structure does not rotate relatively around its transverse axis passing through the point C:

$$\left(\vec{M}_C, \vec{e}_\eta\right) = 0 \quad \Rightarrow \quad -e\left(N_1 + N_2\right) + mg_{FF}\left(\xi_1 + \xi_2\right) = 0 \tag{65}$$

Considering (53), (64) – (65) we obtain:

$$N_1 = N_2 = \frac{mg_{FF}}{2e}\sum_{i=1}^{2}\left(a\sin\varphi_i + \left(b^2 - \left(h + a\cos\varphi_i\right)^2\right)^{1/2}\right) \tag{66}$$

So finally we get for (62):

$$Q_\Psi = -\mu\left(e^2+d^2\right)^{1/2}\frac{mg_{FF}}{e}\sum_{i=1}^{2}\left(a\sin\varphi_i+\left(b^2-\left(h+a\cos\varphi_i\right)^2\right)^{1/2}\right)\operatorname{sgn}\dot\Psi \tag{67}$$

The equations of motion in the Lagrangian form are:

$$\frac{d}{dt}\frac{\partial T}{\partial \dot q_i}-\frac{\partial T}{\partial q_i}=Q_i,\qquad \vec q=\left(\varphi_1,\varphi_2,\Psi\right)^T,\qquad \vec Q=\left(Q_{\varphi1},Q_{\varphi2},Q_\Psi\right)^T \tag{68}$$

Omitting details of derivatives calculation, we get after reduction to a first-order system:

$$\dot\omega_1=\frac{\Delta_1}{\Delta},\qquad \dot\omega_2=\frac{\Delta_2}{\Delta},\qquad \dot\Omega=\frac{\Delta_3}{\Delta} \tag{69}$$

where we have:

$$\omega_1\equiv\dot\varphi_1,\quad \omega_2\equiv\dot\varphi_2,\quad \Omega\equiv\dot\Psi \tag{70}$$

and where for determinants:

$$\Delta=\begin{vmatrix}\beta_{11}&\beta_{12}&\beta_{13}\\ \beta_{21}&\beta_{22}&\beta_{23}\\ \beta_{31}&\beta_{32}&\beta_{33}\end{vmatrix},\quad \Delta_1=\begin{vmatrix}\gamma_1&\beta_{12}&\beta_{13}\\ \gamma_2&\beta_{22}&\beta_{23}\\ \gamma_3&\beta_{32}&\beta_{33}\end{vmatrix},\quad \Delta_2=\begin{vmatrix}\beta_{11}&\gamma_1&\beta_{13}\\ \beta_{21}&\gamma_2&\beta_{23}\\ \beta_{31}&\gamma_3&\beta_{33}\end{vmatrix},\quad \Delta_3=\begin{vmatrix}\beta_{11}&\beta_{12}&\gamma_1\\ \beta_{21}&\beta_{22}&\gamma_2\\ \beta_{31}&\beta_{32}&\gamma_3\end{vmatrix} \tag{71}$$

for the coefficients in these determinants:

$$\begin{aligned}&\beta_{11}=au_1^2,\quad \beta_{12}=0,\quad \beta_{13}=-u_1d,\quad \beta_{21}=0,\quad \beta_{22}=au_2^2,\quad \beta_{23}=u_2d,\\ &\beta_{31}=-au_1d,\quad \beta_{32}=au_2d,\quad \beta_{33}=\xi_1^2+\xi_2^2+2d^2\\ &\gamma_1=\frac{\mathrm{A}-\mathrm{B}\omega_1}{ma}+\xi_1u_1\Omega-au_1w_1\omega_1^2,\quad \gamma_2=\frac{\mathrm{A}-\mathrm{B}\omega_2}{ma}+\xi_2u_2\Omega-au_2w_2\omega_2^2\\ &\gamma_3=ad\left(w_2\omega_2^2-w_1\omega_1^2\right)-2\Omega a\left(u_1\omega_1+u_2\omega_2\right)-\mu\left(e^2+d^2\right)^{1/2}\frac{g_{FF}}{e}\left(\xi_1+\xi_2\right)\operatorname{sgn}\Omega\end{aligned} \tag{72}$$

with the functions:

$$u_i=\cos\varphi_i+\frac{f_i}{g_i},\quad w_i=-\sin\varphi_i+\frac{1}{g_i^3}\left(\left(a\cos2\varphi_i+h\cos\varphi_i\right)g_i^2-af_i^2\right) \tag{73}$$

Just as in the case of sliding in the case of spinning there is a division by 0 in the system (69) in some conditions (extreme relative positions of the sliders on guide rails) when $\Delta=a^2u_1^2u_2^2\left(\xi_1^2+\xi_2^2\right)=0$, and we must return to the main (nondeterminant form) system for the case of rotation. If $\Delta\to0$ then a sharp increase in value of the angular acceleration of supporting structure is observed. If $\Delta=0$ we have:

$$\begin{cases}\ddot\Psi=\dfrac{ad\left(\dfrac{A}{B}\right)^2\left(w_1-w_2\right)-\mu\left(e^2+d^2\right)^{1/2}\dfrac{g_{FF}}{e}\left(\xi_1+\xi_2\right)\operatorname{sgn}\dot\Psi}{\xi_1^2+\xi_2^2+2d^2}\\ \dot\varphi_i=\dfrac{\mathrm{A}}{\mathrm{B}},\quad i=1,2\end{cases} \tag{74}$$

The latter relations determine the conditions for the onset of spinning (note that $\ddot\Psi=\dot\Psi=0$

at a rest), in view of the overcoming of frictional forces.

4 DISCUSSION

In the equations of translational/rotational motion (43)/(69) under certain conditions $(\Delta = 0 \;\; \Rightarrow \;\; u = 0 / u_i = 0)$, corresponding to the extreme position of the sliders on rail guides, certain functions tend to zero, which gives a sharp increase in the values of the accelerations/angular accelerations. When these functions are equal to zero, it is necessary to go to the basic equations of motion, where there is no division by zero and where we can obtain the conditions for the beginning of the movement, meaning the overcoming of frictional forces. The system has periodic functions with respect to the shaft's angle of rotation. In the case of steady motion, this suggests a periodic character of the motion.

5 CONCLUSIONS

- Equations for coupled dynamics of the solid system with slider-crank mechanisms as internal movers are obtained on rough surface with friction for cases of translational and rotational movement.
- Such class mechanism driven by inner movements of masses being isolated from surrounding space in shell can be used in conditions where traditional movers (wheels, tracks, legs) for some reasons are not applicable.

ACKNOWLEDGEMENTS

I would like to thank my colleagues encouraged me to explore new ways of movement.

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