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A REGULARIZED DAMAGE MODEL FOR STRUCTURAL ANALYSES OF CONCRETE DAMS IN THE PRESENCE OF ALKALI-SILICA REACTION

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Abstract. Alkali-silica reaction is a chemical phenomenon that affects concrete structures built some decades ago and subject to a very wet environment, e.g. dams. The starting point of this work is a bi-phase damage model present in the literature. In general, finite element solutions with damage models for material having a softening behaviuor exhibit a sensitivity to the element size and do not converge to physically meaningful solutions as the mesh is refined. In literature, some regularization techniques have been proposed and the fracture energy one has been implemented in the bi-phase chemo-damage model. The limit of this approach is that the solution remains mesh-dependent, so if the mesh is refined the damage localizes in a band of width fixed by the element size. In this work the nonlocal formulation of this damage model has been developed, validated with simple examples and applied to an existing concrete gravity dam, subject to service loading and affected by the ASR. A comparison between fracture energy regularization approach and nonlocal formulation is performed.

1 INTRODUCTION

Concrete is one of the most used materials in civil engineering, but its durability can be reduced by several chemical phenomena, among them the alkali-silica reaction (ASR) plays a fundamental role. During ASR amorphous silica of aggregates reacts with the high alkaline solution in concrete micro pores to form a hydrous alkali-calcium-silica gel, which expands and causes increase of displacements in concrete structures. Another key dissipative phenomenon related to ASR is micro-cracking, which results in non-symmetric,

progressive degradation of mechanical properties (strength and stiffness). In [1] a phenomenological two-phase isotropic damage model for the evaluation of the effects of ASR has been proposed. This model, which extends the one originally proposed in [11] takes into account the simultaneous influence of both humidity and temperature through two uncoupled diffusion analyses: the heat diffusion analysis and the moisture diffusion analysis. The solution of these two analyses are considered as input for a consequent mechanical analysis, used to define the response due to ASR.
The model in [1] has been implemented with fracture energy pseudo-regularization, hence, as damage develops, the boundary value problem may still become ill-posed and the damage pattern obtained in numerical analyses is mesh-dependent. Such difficulties can be solved implementing a real regularization technique, as proposed in the literature ([2]-[4]). In all regularized models the introduction of a material characteristic length fixes the width of the zone in which damage localizes, thus preventing strain localisation into a line with consequent zero energy dissipation. In this work a nonlocal formulation of the bi-phase damage model is proposed for the description of ASR-induced degradation. Nonlocality has been introduced replacing strain invariants with their nonlocal counterpart, obtained by weighted average. This approach has been validated on a simple example, then it has been applied to a real case of existing concrete gravity dam.

2 BI-PHASE CHEMO-MECHANICAL DAMAGE MODEL

At the mesoscale concrete affected by ASR is composed of two phases (the solid skeleton and the gel), so the macroscopic stress is written as the sum of the effective stress $\boldsymbol{\sigma'}$ (acting on the skeleton) and of the stress -$bp\mathbf{1}$ (acting on the gel)

$$\boldsymbol{\sigma} = (1 - D)\,[2G\,\mathbf{e} + K\,(tr\boldsymbol{\varepsilon} - \alpha\,\theta)\mathbf{1} - b\,p\,\mathbf{1}] \tag{1}$$

with

$$p = (1 - D)\,M\,(b\,tr\boldsymbol{\varepsilon} - \zeta_g - \alpha_g\,\theta) \tag{2}$$

where: G and K are respectively the shear and bulk moduli of the homogenized concrete skeleton; M and b are the Biot's modulus and the Biot's coefficient; α e α_g are respectively the volumetric coefficients of thermal expansion for the concrete skeleton and the gel; ζ_g is the gel volume content; D is the damage variable, governed by the activation function, written in terms of strain invariants ($tr\boldsymbol{\varepsilon}$ and J_ε) in the following form

$$\begin{aligned} f =& (1 - D)^2\,4G^2\,J_\varepsilon - 9a_t\,(1 - D)^2\,[(K + Mb^2)tr\boldsymbol{\varepsilon} - Mb\zeta_g]^2 + \\ &+ 3b_t\,(1 - D)\,[(K + Mb^2)tr\boldsymbol{\varepsilon} - Mb\zeta_g]\,h(D) - k_t\,h(D)^2 \end{aligned} \tag{3}$$

where: $tr\boldsymbol{\varepsilon} = \varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz}$ is the first strain invariant and $J_\varepsilon = \frac{1}{2}\mathbf{e} : \mathbf{e}$ is the second one; a_t, b_t, k_t are material parameters governing the shape and dimensions of the elastic domain; h is the herdening-softening function

$$h(D) = \begin{cases} 1 - \left[1 - \left(\frac{\sigma_e}{\sigma_0}\right)\right]\left(\frac{D}{D_0}\right)^2 & if \;\; D < D_0 \\ \left[1 - \left(\frac{D-D_0}{1-D_0}\right)^c\right]^{0.75} & if \;\; D \geq D_0 \end{cases} \tag{4}$$

In the previous Equation $\sigma_e \sigma_0$ is the ratio between the stresses at the elastic limit and at peak, D_0 defines the damage level corresponding to the peak stress and c governs the negative slope of the softening part of the function $h(D)$. In finite element approach, the coefficient c is used to scale the fracture energy density of the material in such a way that each finite element can dissipate the correct amount of energy, independently of its size. This method, called as "fracture energy regularization", prevents the occurrence of spurious mesh dependency in the structural global response.
The evolution of ζ_g, which depends on temperature and humidity, is not reported here for brevity and reference is made to [1] for further details.

3 REGULARIZED MODELS

In the context of standard continuum theories, damage-induced softening constitutive models typically cause ill-posedness of the initial boundary value problem.
In order to introduce a remedy, various regularization techniques have been proposed in the literature, especially for damage models (as one used in this work). Among them, the following approaches are cited:

(i) Fracture energy pseudo-regularization: the parameters governing the material softening are modified with the mesh size in order to have a fixed value of the fracture energy associated with the finite element. It can not be considered a real regularization method as the length introduced is a mesh-dependent numerical parameter and not a material one.

(ii) Nonlocal integral models ([2]-[3]): the inelastic behaviour at a point is governed by a weighted average over a representative volume of the strains or strain invariants.

The nonlocal regularization methods introduce a characteristic material length in the formulation, which fixes the width of the zone in which damage localises, thus preventing strain localisation into a line with consequent zero energy dissipation. Usually, for material likes concrete this length depends on the aggregates size.
In 3.1 and 3.2 the two above cited methods are presented and in 3.3 a simple numerical test is performed.

3.1 Fracture energy approach

The simplest but crude remedy to pathological mash-dependence, popular in engineering applications, is to adjust the softening part of the stress-strain diagram as a function of the element size. When this is done properly, the energy dissipated in a band of cracking

elements does not depend on the width of the band. The fracture energy regularization introduces a modification of the softening parameters according to the mesh size, such that to impose the same energy dissipation per unit area even with mesh refinement. This can be considered as a numerical "trick" to obtain a phisically sense overall response, for this reason it is known as a pseudo-regularization method.
The method is based on the assumtion that dissipation always takes place in a band one element thick, irrespective of the element size. The constituive law is modified in such a way that the energy dissipated over a completely fractured finite element is equal to an assigned value depending on the fracture energy G_f (which is a material property, independent of the specimen size) and on the element size. The area below the softening part of the $\sigma - \varepsilon$ curve represents the energy dissipated per unit volume (or specific fracture energy g_f) in uniaxial test. This is related to the corresponding fracture energy G_f through the material characteristic length w (i.e., $G_t = g_t w$) representing the width of the crack band front. For each element, the "material characteristic length" w is substituted by the "element characteristic length" l^e, which depends on the mesh and measures the numerical width of the fracture process zone. The specific fracture energy g_f is then scaled for each element so that $g_f\, l^e = G_f$.
The fracture energy density g_f is defined as the area below the stress-strain curve and it depends on parameter c through Equation 4. For $c < 1$, the $\sigma - \varepsilon$ curve has a snap-back branch, while for $c = 1$ the slope is discontinuous at the peak. Values of $c > 1$ are adopted in all cases. Examples of fracture energy density scaling for varying c are shown in Figure 1.

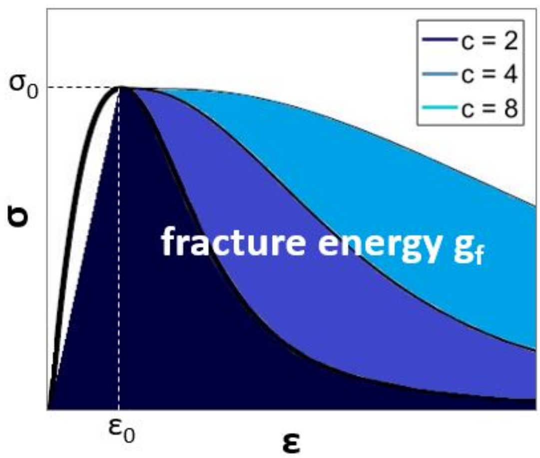

Figure 1: Specific fracture energy definition with different parameter c values.

3.2 Nonlocal model

A computationally efficient and theoretically sound localization limiter is provided by the nonlocal averaging, which is in principle applicable to any type of constitutive model. The idea of a nonlocal continuum originally appeared in elasticity [9].
The nonlocal approach replaces a certain variable by its nonlocal counterpart obtained by weighted averaging over a spatial neighborhood of each point under consideration. Thus the response at a point $\mathbf{x}$ depends not only on the state and internal variables at that point, but also on those of point $\mathbf{s}$, belonging to a proper neighborhood of the point. The characteristic length defines the size of this neighborhood.
If $\alpha(\mathbf{x})$ is some "local" field in a domain V, the corresponding nonlocal field is defined by

$$\langle a(\mathbf{x})\rangle = \int_V \alpha'(\mathbf{x},\mathbf{s})a(\mathbf{s})ds \tag{5}$$

where $\alpha'(\mathbf{x},\xi)$ is a given nonlocal weight function.
In this work the nonlocal model presented in [2], which defines the weighted average of a strain measure, has been developed, implemented and used. The basic non-local variables at a point $\mathbf{x}$ are assumed to be the average strain invariants, i.e. the weighted averages over the volume V of the local strain invariants

$$\langle J_\varepsilon(\mathbf{x})\rangle = \int_V W(\mathbf{x}-\mathbf{s})J_\varepsilon(\mathbf{s})d\mathbf{s} \tag{6}$$

$$\langle tr\boldsymbol{\varepsilon}(\mathbf{x})\rangle = \int_V W(\mathbf{x}-\mathbf{s})tr\boldsymbol{\varepsilon}(\mathbf{s})d\mathbf{s} \tag{7}$$

where $W(\mathbf{x}-\mathbf{s})$ is the weighting function, adequately defined to normalise the averaging. In Equations 6 and 7 and in what follows the symbol $\langle\bullet\rangle$ denotes the weighted average value of the quantity $\bullet$. $W(\mathbf{x}-\mathbf{s})$ is assumed as the normalised Gauss function and the average is extended to the whole body so that V coincides with the body volume:

$$W(\mathbf{x}-\mathbf{s}) = \frac{1}{W_0(\mathbf{x})}exp(\left(-\frac{\parallel \mathbf{x}-\mathbf{s}\parallel^2}{2l^2}\right) \tag{8}$$

with

$$W_0(\mathbf{x}) = \int_V exp\left(-\frac{\parallel \mathbf{x}-\mathbf{s}\parallel^2}{2l^2}\right)d\mathbf{s} \tag{9}$$

The length l is a material parameter which can be related to the width of the zone in which damage phenomena localise. No particular provisions need to be introduced for points near the boundary of the body since $W_0(\mathbf{x})$ in Equation 9 already normalises the averaging.
The non-local model is then obtained by replacing in the loading functions the averages of the strain invariants. The resulting non-local loading functions F is

$$F = (1-D)^2\,4G^2\,\langle J_\varepsilon\rangle - 9a_t\,(1-D)^2\,[(K+Mb^2)\langle tr\boldsymbol{\varepsilon}\rangle - Mb\zeta_g]^2 + \\ + 3b_t\,(1-D)\,[(K+Mb^2)\langle tr\boldsymbol{\varepsilon}\rangle - Mb\zeta_g]\,h(D) - k_t\,h(D)^2 \tag{10}$$

Note that non-locality only intervenes in the damage activation function, while the stress-strain Equation 1 remains local.

3.3 Plane strain tension test

In order to clarify the limits of the fracture energy regularization and the effectiveness of the nonlocal regularization, these methods have been tested on a simple example subject to a linearly increasing displacement applied on one base. A bi-dimensional specimen in plane strain tension conditions has been considered, meshed with different finite element sizes (see Figure 2). To trigger localization, the strength is slightly reduced in the central part of the specimen. The analyses have been performed by a finite element Matlab code where the local and nonlocal models have been implemented.
Figure 3 (a) and (b) depict the global response in terms of reaction force as a function of axial displacements obtained with fracture energy regularization and nonlocal formulation. In both cases, as expected, the global response is quite similar for all considered meshes. On the contrary, the damage pattern, shown in Figure 4, is completely different: when using fracture energy regularization damage localizes in one element thick band and hence considerably changes by modifying the mesh (see Figure 4 (a)); when using nonlocal regularization instead the width of the localization band is fixed by the material lenght and thus does not change with the mesh size (see Figure 4 (b)).

4 STRUCTURAL ANALYSIS OF A GRAVITY DAM SUBJECT TO ASR

The comparison between fracture energy regularization approach and nonlocal formulation has been developed considering an existing concrete dam: Fontana dam is a gravity dam (maximum height, length, and thickness at the basis equal to 146, 720 and 114 m, respectively) located in Graham County, North Carolina (United States). Its construction was completed in 1946, but only 3 years later a pattern of cracking was first observed, together with an upstream movement of the structure. In late 1972, inspectors found a large longitudinal crack near the left abutment, in both the upstream and downstream walls of the foundation drainage gallery inside the dam (for more details see [10]).
The bi-phase model, both in local and nonlocal formulation, accounts for the simultaneous effect of the temperature and humidity. A weakly coupled approach has been followed: a preliminary heat diffusion analysis and moisture diffusion analysis allowed to compute the varying fields of temperature and humidity, which have been the input of the subsequent chemo-damage analysis. The preliminary thermal and humidity analyses have been performed using Abaqus software. Furthermore, with the same program the paraemter

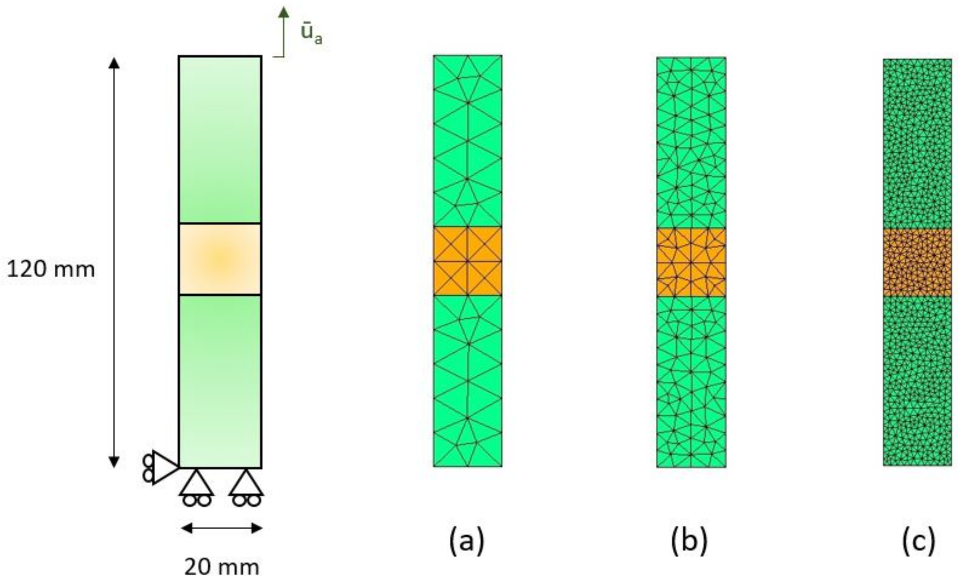

Figure 2: Specimen geometry for the simple tension test at imposed displacemnets; typical elements dimensions: (a) 10 mm, (b) 5 mm and (c) 2 mm.

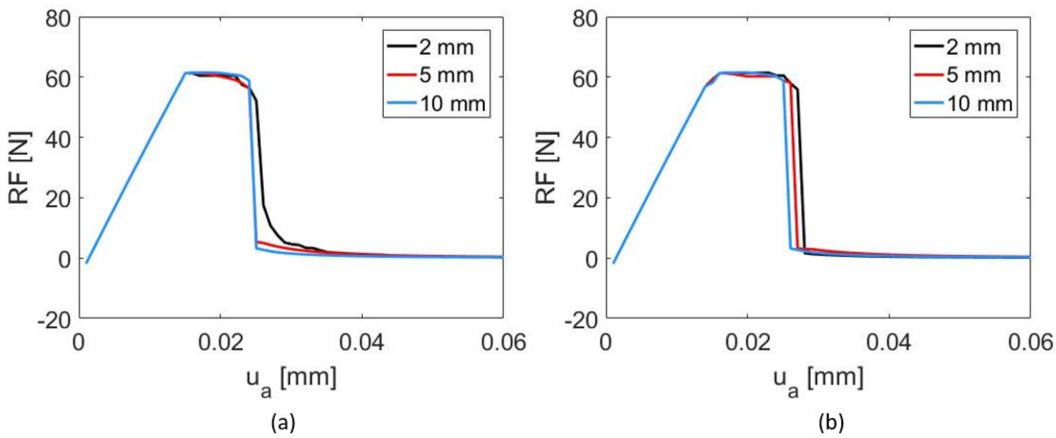

Figure 3: Comparison of the global response in terms reaction force vs of axial displacements between (a) local and (b) nonlocal approach.

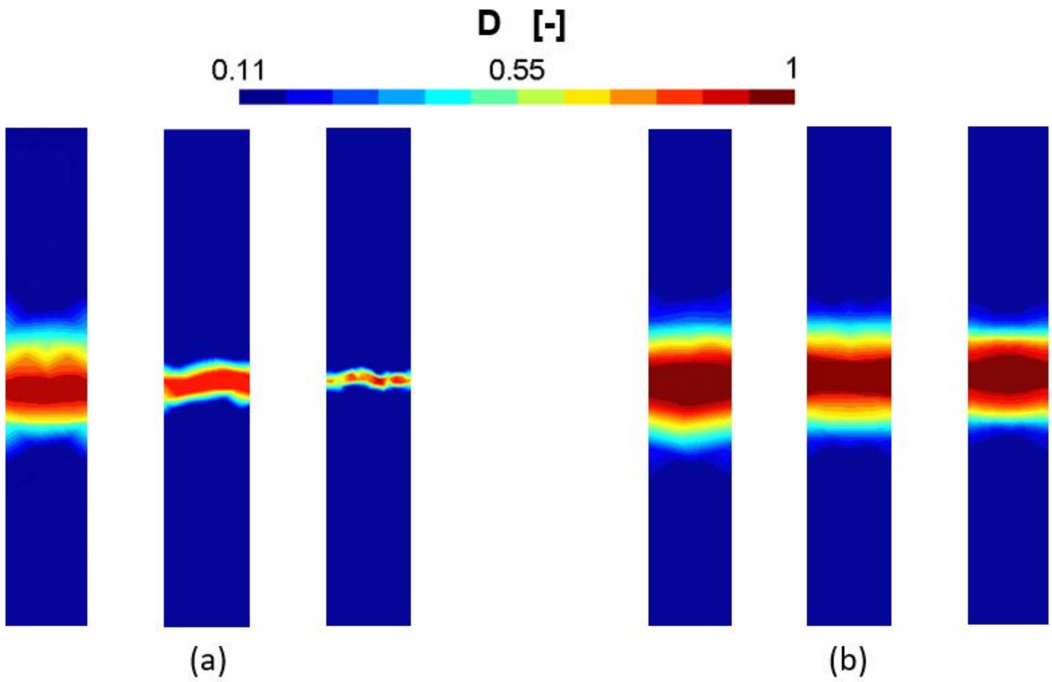

Figure 4: Contour plot of the damage, from the largest FE size (on the left) to the smallest one (on the right) for: (a) fracture energy regularization and (b) nonlocal formulation.

representing the reaction evolution has been calculated and used in Matlab in order to compare the local and nonlocal model results.

The 2D dam section has been discretized by plane strain 3-nodes element. To check the regularization properties of the two models we have considered two meshes with different refinements (with typical finite element dimension of 50 and 20 cm).

For this dam no detailed monitoring data are available, only the crack geometry due to ASR is known from [10]. For this reason, the damage material parameters have been calibrated considering this crack formation and direction.

For the annual variation of temperature, saturation degree for the water and reservoir level, in this work reference has been made to what reported in [11] (see Figure 5).

To determine the initial temperature and of saturation degree fields within the dam, a preliminary heat diffusion analysis and a preliminary moisture diffusion analysis have been performed respectively.

In these first steady state analyses the stabilized temperature and saturation degree in the internal nodes, starting from the initial uniform field of the two variables, have been evaluated by assuming as boundary conditions the mean values of temperature, saturation degree and reservoir level. The second step consisted of transient analyses, in which the assumed sinusoidal annual variation of air temperature and saturation degree, shown in Figure 5 (b) and (c) respectively, have been applied. Furthermore, a sinusoidal reservoir level variation (Figure 5 (a)) has been considered. In Figure 6 (a) the histories of tem-

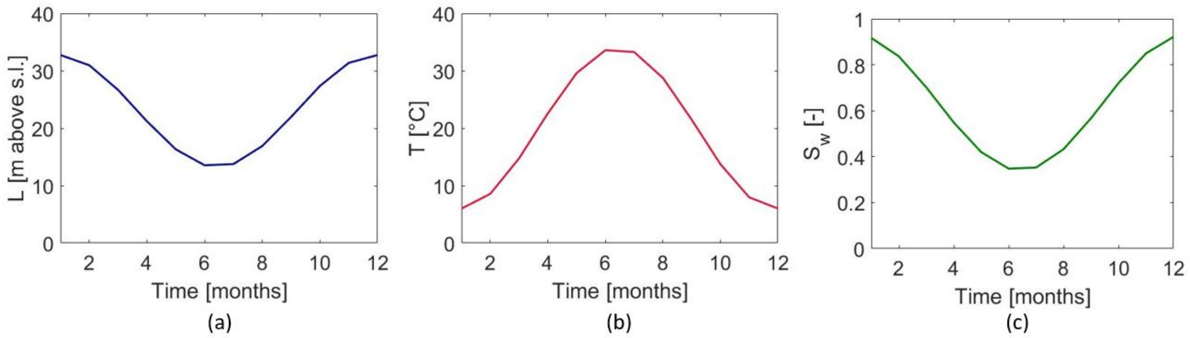

Figure 5: History of Fontana (a) reservoir level, (b) air temperature and (c) saturation degree for the water.

perature and (b) of saturation degree at different points of the dam obtained with these second analyses analyses are depicted.

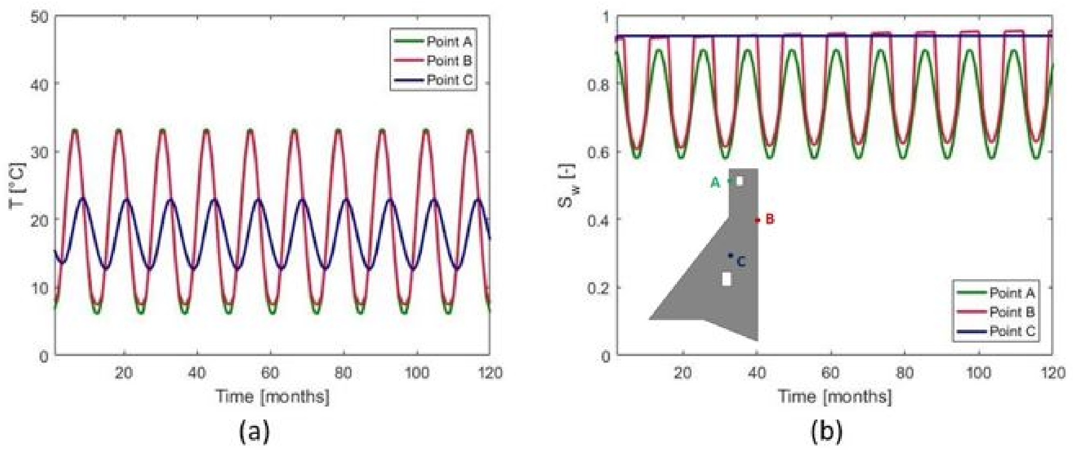

Figure 6: Histories of (a) temperature and (b) saturation degree of some nodes (A, B and C) of Fontana dam (10 years of the full analysis).

The diffusion analyses have been performed in Abaqus and the reaction extent evolution could then be computed. Then, the chemo-mechanical analysis has been implemented in the Matlab code considering both the local and nonlocal models. In Figure 7 the damage contour plot after 15 years with different meshes is shown: as discussed in the section 3.1, with fracture energy regularization (green outline) the band of the damaged area depends strongly on the finite element dimension (it becomes smaller when the mesh size is reduced). On the contrary, instead, with nonlocal formulation (orange outline), the damage pattern remains almost unchanged passing from a mesh to another. The same Figure emphasizes that using the pseudo-regularization technique the results in terms of the global response remain quite good: the same vertical crest displacements have been obtained for the two meshes, similarly to what happened with nonlocal formulation.

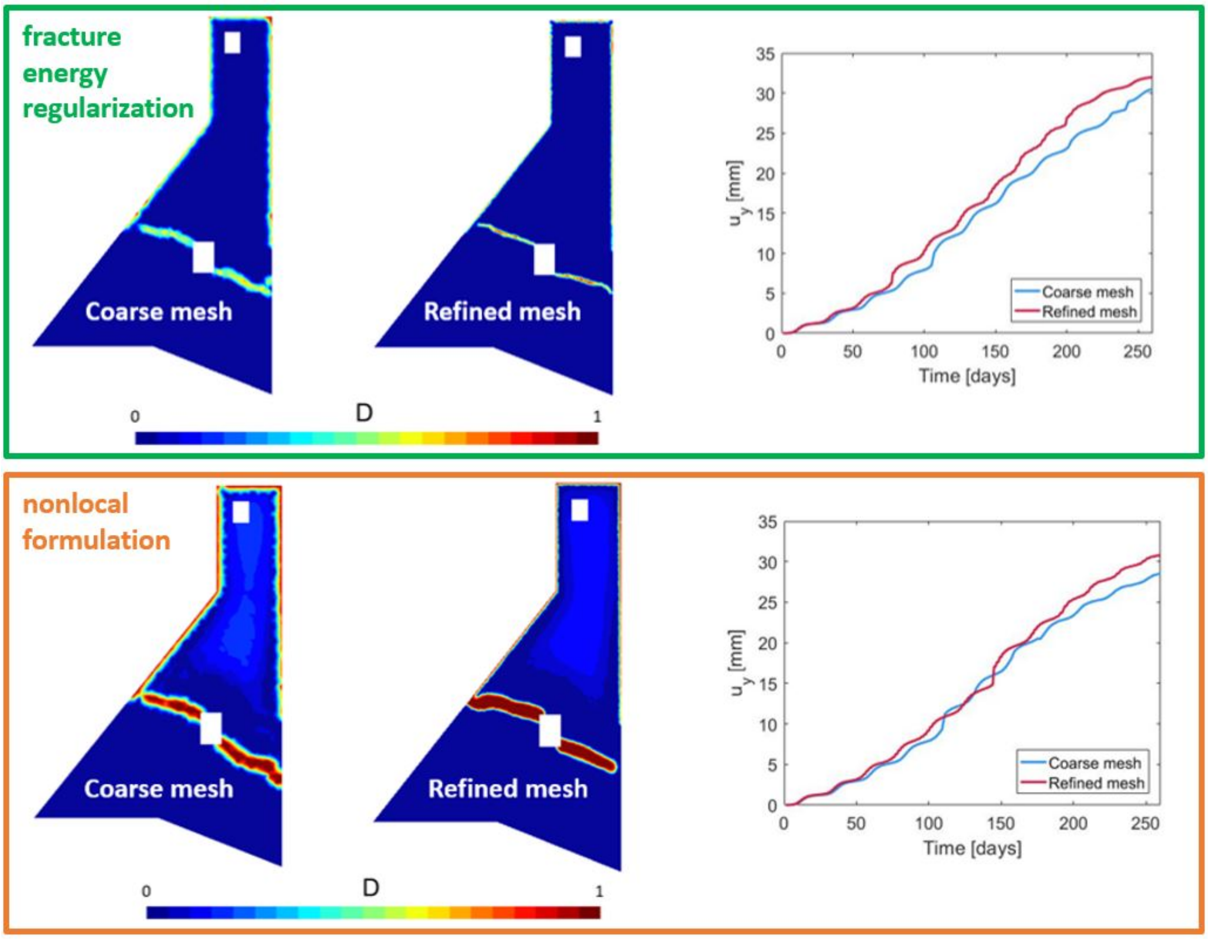

Figure 7: Damage contour plot after 15 years and crest vertical displacement evolution with coarse and refined meshe; fracture energy regularization at the top and nonlocal formulation at the bottom.

5 Conclusions

Concrete dams are strategic structures that may be subject to many degradation phenomena, such as alkali-silica reaction, which causes displacements increase, cracks and material expansion. These effects can be studied through a bi-phase damage model, present in the literature, which has been implemented using a fracture eenrgy regularization technique in order to avoid sensitivity to the element size and non-convergence to physically meaningful solutions as the mesh is refined. This method is a pseudo-regularization approach since the characteristich length introduced for scaling the material fracture energy depends on the element size. For this reason, in this work the nonlocal formulation of the bi-phase chemo-damage model has been developed. It consists in replacing the strain invariants with their nonlocal counterpart obtained by weighted averaging. The new version of the model has been validated with simple tensile testsin plane strain conditions. It has been demonstrated that with nonlocal model the damage localization band remains almost unchanged when the mesh is refined, as opposed to what happens with fracture energy regularization (the damage concentrates in a band one element thick). With both the methods, the global response in terms of reaction forces vs axial displacements is similar for all the considered meshes. Then, the comparison between the two approaches has been developed for an existing gravity dam, thus showing the effectiveness of the model also in structural analyses of real structures.

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