VIII International Conference on Computational Methods for Coupled Problems in Science and Engineering
COUPLED PROBLEMS 2019
E. Oñate, M. Papadrakakis and B. Schrefler (Eds)

ON THE OPTIMIZATION OF A CAPSUBOT WITH A SPRING

ALEXANDER P. IVANOV

Moscow Institute of Physics and Technology
9 Institutskiy per., Dolgoprudny, Moscow Region,
141701, Russian Federation
Ishlinsky Institute for Problems in Mechanics RAS
e-mail: a-p-ivanov@inbox.ru

Key words: Capsule Robots, Friction Based Locomotion, Optimal Control

Abstract. A capsule robot (capsubot) without external drivers is considered. The device consists of a shell, an actuator, and certain equipment. It can move on a rough surface due to the controlled movement of an internal mass and external friction. The mass is driven by the actuator according to a given program, which ensures the movement of the shell due to the inertia forces. The system includes an elastic spring, connecting the shell with the internal mass. A control is proposed that allows to maximize the average velocity under given technological restrictions.

1 INTRODUCTION

A capsule robot (capsubot) is a type of compact mobile device which can explore fields inaccessible to humans. In recent years, such devices received intensive attraction mostly in connection with medical purposes: a tiny capsubot with camera can be swallowed by patient to diagnose diseases; another application is inspection of pipelines. Some methods of active locomotion were developed. The simplest approach is based on internal control forces and external static friction [1-5]. Comparatively to a legged design, it leads to more save interaction between the capsule and the explored area. Similar idea was used to study floating robots [6,7]. A capsubot without external moving parts contains an internal mass, being put in motion by an actuator. As the internal mass changes its position relative to the capsule, the center of gravity shifts. An appropriate control strategy is needed to provide periodic motion of the capsubot in desired direction. Due to limited resources (size and mass of the robot, power supply, etc.), the optimization problem (in a sense) is of great practical importance. A number of results in this direction are obtained [1,2,8]. Further modifications of the capsubot include elastic springs [9-11]. Numerical studies and experiments show that the addition of springs leads to improved technical characteristics such as the average velocity of the robot and energy consumption. Generally speaking, the use of springs allows to increase the maximum force during acceleration of the internal body, as well as to reduce the energy consumption during its braking. The present paper is devoted to the control of a capsubot with a linear spring. Taking as a basis the results of cited papers, we will look for the parameters of the spring and the control law, providing the maximal average velocity of the robot.

2 PROBLEM STATEMENT

The physical model of the capsubot is shown in Fig.1. It consists of a capsule shell m_1, interacting with an internal moving mass m_2 and with a horizontal support. Let y be the relative position of the inner body, x is the position of the shell with respect to an external fixed frame, and $v = \dot{x} = dx/dt$.The following forces will be taken in account: (i) a control force $F(t)$ is generated by an actuator, attached to the shell (not shown), and acts on the inner body; (ii) a force $G(y)$, generated by a linear) spring, acts on the inner body (the opposite force is applied to the shell); (iii) friction force $T(v)$ between the shell and support.

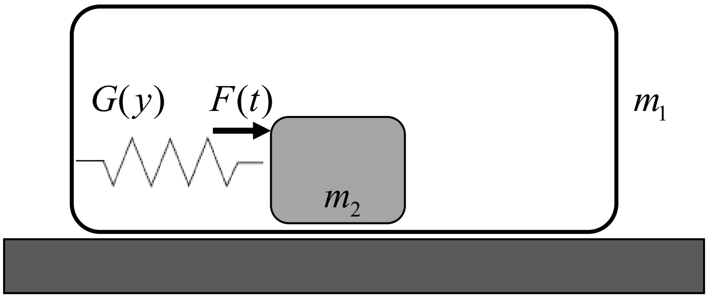

Fig.1. The capsubot model

Equations of motion in Newton form are

$$\left(m_1 + m_2\right)\dot{v}_0 = \left(m_1 + m_2\right)\dot{v} + m_2\ddot{y} = T(v), \quad m_2\left(\dot{v} + \ddot{y}\right) = F(t) + G(y) \tag{1}$$

where v_0 is velocity of the mass center. Define friction force by the Coulomb friction law

$$T(v) = -T_0 \operatorname{Sign} v, \quad T_0 = k\left(m_1 + m_2\right), k = fg, \ \operatorname{Sign} v = \begin{cases} 1, & v > 0 \\ -1, & v < 0 \\ [-1,1], & v = 0 \end{cases} \tag{2}$$

where g is acceleration of gravity and f is the coefficient of friction. If the capsule moves to the right, then $v > 0$ and $T = -T_0$; similarly, we have $v < 0$ and $T = T_0$ for sliding to the left. In the stick phase $v = 0$, then according to (1)

$$\alpha\ddot{y} \in [-k, k], \quad \alpha = \frac{m_2}{m_1 + m_2} \tag{3}$$

Assume that control function $F(t)$ is τ - periodic, piecewise continuous, and has zero mean value, i.e.

$$\left\langle F(t)\right\rangle = \frac{1}{\tau}\int_0^\tau F(t)dt = 0 \tag{4}$$

then $v(t)$ is continuous, and $y(t)$ is smooth. We look for such control that functions $v(t)$ and $y(t)$ are τ - periodic and maximize average velocity of the robot:

$$\langle v(t)\rangle = \frac{1}{\tau}\int_0^\tau v(t)dt \to \max \quad \text{s.t.} \quad |F(t)| \le M, \quad y(t)\in[0,L] \tag{5}$$

3 SIMPLEST CASE: NO SPRING

First discuss the case $G(y)\equiv 0$ [2]. Then in eqs (1) v does not depend on y directly:

$$m_1\dot v = -k\left(m_1+m_2\right) - F(t), \quad \alpha m_1\ddot y = km_2 + F(t) \tag{6}$$

In such statement, problem (5) has the optimal periodic solution (period τ being fixed) [2]:

$$F(t) = \begin{cases} -M, & t\in[0,\tau_1) \\ M, & t\in[\tau_1,\tau_2), \\ T_0, & t\in[\tau_2,\tau) \end{cases} \quad \tau_1 = \frac{\tau}{4}\left(1+\frac{T_0}{M}\right), \tau_2 = \frac{\tau}{2}$$

$$\langle v(t)\rangle_{\max} = \sqrt{\frac{\alpha LM}{m_1}\frac{1-\rho^2}{3+\rho^2}}, \quad \rho = \frac{T_0}{M} \tag{7}$$

According to (7), the three-step control profile is used here: acceleration of the shell, its deceleration, and rest. The conditions of Pontryagin's principle are satisfied, since the maximal admissible control is used at each phase of motion. A geometric interpretation of the optimal solution is presented in the phase plane $(y,\dot y)$ (Fig.2, a). Each of the three steps is depicted by a parabolic arc; these lines form a closed loop.

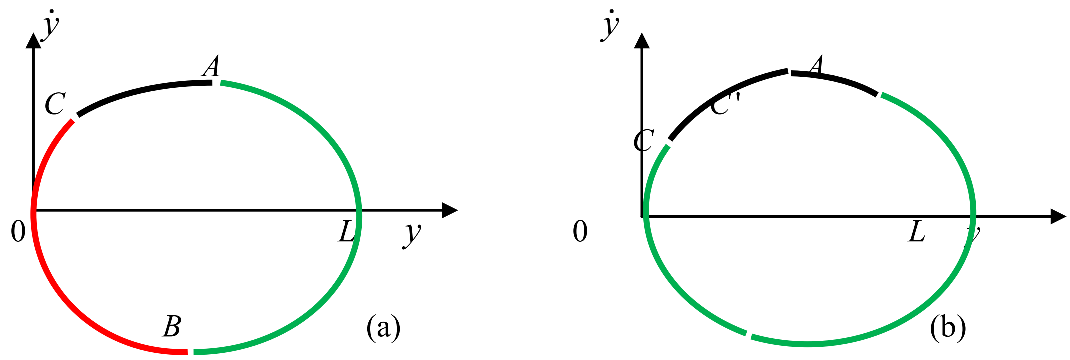

Fig.2. Periodic motions of the robot a) no spring: (i) acceleration AB (green); (ii) deceleration BC (red); (iii) rest phase CA (black); b) with spring: acceleration AC, two-step rest phase CC'A .

4 ADDING A LINEAR SPRING

Suppose now that function $G(y)$ in eq. (1) is linear: $G(y) = -c(y-y_0)$ and modify formula (7) for this case. In the acceleration phase we have $F(t) = -M$, and the equations of motion are

$$\dot{v} + \alpha \ddot{y} = -k, \quad m_2 \left(\dot{v} + \ddot{y} \right) = -M + G(y) \tag{8}$$

Therefore,

$$\begin{aligned} m_1 \dot{v} &= -k \left(m_1 + m_2 \right) + M - G(y), \\ \alpha m_1 \ddot{y} &= k m_2 - M + G(y) \end{aligned} \tag{9}$$

The second equation here can be considered separately. It admits one-parameter family of solutions, satisfying given length restriction:

$$\begin{aligned} & y(t) = \frac{L}{2}\left(1 + \cos \omega t\right), \ \dot{y}(t) = -\frac{\omega L}{2} \sin \omega t \\ & c = \frac{m_1 m_2}{m_1 + m_2} \omega^2, \quad y_0 = \frac{L}{2} + \frac{M - k m_2}{c} \end{aligned} \tag{10}$$

The presence of the spring allows to increase the acceleration of the shell, as well as eliminate the braking phase. Let A be such point at which the shell begins to move (Fig.2,b), its position on the curve (10) is defined by a value $t = t_A$. Substituting expressions (10) into the first formula (8), we obtain after integration:

$$v(t) = -k\left(t - t_A\right) + \alpha \omega \frac{L}{2}\left(\sin \omega t - \sin \omega t_A\right) \tag{11}$$

Now we can determine point C as first root of equation $v(t_C) = 0$. Thus, $t_C = t_A + 2\theta / \omega$, where θ is first positive root of equation

$$\theta = \sigma \sin \theta \cos\left(\varphi_A + \theta\right), \quad \theta = \frac{\varphi_C - \varphi_A}{2}, \quad \sigma = \frac{cL}{2km_1}, \quad \varphi = \omega t \tag{12}$$

Further, in the rest phase $v \equiv 0$, hence

$$m_2 \ddot{y} = T, \quad m_2 \ddot{y} = F(t) + G(y) \tag{13}$$

which is consistent provided

$$F(t) + G(y(t)) = T \tag{14}$$

In view of (5) we should minimize the duration of this part. This is known minimum time control problem: how to get point A starting from point C as fast as possible under restriction $\alpha\left|\ddot{y}\right| \le k$. The solution curve is either parabola $\alpha \ddot{y} = k$ or two parabolic arcs $\alpha \ddot{y} = \pm k$ depending on the coordinates of points C and A. Both arcs belong to the region $y \in [0, y_A]$. If $\alpha \ddot{y} = k$, then $T = T_0$ and to fulfil equality (14) for any $y \in [0, L]$ it is sufficient that

$$2M \ge k\left(m_1 + 2m_2\right) + \frac{cL}{2} \tag{15}$$

In fact, in this case we can impose condition (14) only on the interval $y \in [0, y_A]$ (which covers the arc CC'). Similarly, in the case $T = -T_0$ non-slip condition (14) must be satisfied for $y \in [y_{C'}, y_A]$, i.e.

$$y_{C'} \geq \frac{L}{2}\left(1 + \frac{1}{\sigma}\right) \tag{16}$$

which is rather restrictive. To satisfy both conditions (15) and (16), it is necessary that $M \geq T_0$.

To evaluate duration of the rest phase, note that equation $\alpha \ddot{y} = k$ is equivalent to

$$y = \frac{\alpha}{2k}\dot{y}^2 + \gamma, \quad \gamma = \text{const} \tag{17}$$

If points C and A belong to the same line (17), then the black line in Fig.2,b is single parabolic arc, and duration of the rest phase equals

$$\tau_0 = \alpha\left(\dot{y}_A - \dot{y}_C\right)/k$$

Otherwise, the fastest path from C to A consist of two arcs: $\alpha \ddot{y} = k$ and $\alpha \ddot{y} = -k$. The total duration of such path is

$$\tau_0 = \frac{\alpha}{k}\left(2\sqrt{\frac{k\left|\gamma_A - \gamma_C\right|}{\alpha} + \dot{y}_A^2} - \dot{y}_A - \dot{y}_C\right) \tag{18}$$

where the values γ_A and γ_C are defined by (14) for points A and C, correspondingly. With account of (10) and (12),

$$\gamma_A - \gamma_C = L\sin\left(\varphi_A + \theta\right)\left(\sin\theta + \theta\cos\theta\right) \tag{19}$$

The total period τ is the sum of τ_0 and the duration of the first phase $\tau_1 = t_C - t_A$.

The path S, taken by the shell in one period, can be calculated by integration of (11):

$$S = \alpha L \sin(\varphi_A + \theta)\left(\sin\theta - \theta\cos\theta\right) \tag{20}$$

Finaly,

$$\left\langle v(t)\right\rangle = \frac{S}{\tau_0 + 2\theta/\omega} \tag{21}$$

5 RESULTS AND DISCUSSION

We discuss the feasibility of adding a spring for two cases: $\chi = M/T_0 < 1$ and $\chi \gg 1$. Take initial point A (see Fig.2,b) such that

$$c(y_A - y_0) = M + T_0 \tag{22}$$

In view of (9), the value of y_A is maximum possible to keep the shell at rest just before the acceleration phase (we set here $F = M$). To start the acceleration, we assume $F = -M$, so at this moment first eq. (9) becomes

$$m_1\dot{v} = 2M > 0$$

Note that the acceleration rate here is maximal regardless the coefficients of elasticity and friction. Therefore, the capsubot can move even for $\chi < 1$ in conrtast to a robot without spring. However, in this case motion in the rest phase will be rather slow. Indeed, in this case inequality (15) is violated. To ensure the rest phase, we should modify formulas (15), (16) by replacing k with such value $k' < k$ that they became true, i.e.

$$\beta + (1-\alpha)\sigma \le \min\{\alpha, 2\chi - \alpha\}, \quad \beta = \frac{k'}{k} < 1 \tag{23}$$

Note that the value k' will appear in denominator of (18) instead of k, thus there is a reason to increase β. In turn, the parameter σ is proportional to the spring elasticity, it determines the duration of the propulsion phase and the distance travelled. In view of physical restriction (25), it's impossible to increase both parameters. Hence, an optimal relation between them is to be found. For this purpose, the following algorithm can be implemented.

Suppose that the parameters m_1, m_2, L, k, and $\chi < 1$ are given. Explore admissible scope of variables β and σ in accordance with inequality (23). First we choose maximal value y_A with account of restriction (23). Unfortunately, the solution to (22)

$$\sigma(1-\alpha)\cos\varphi_A = 2\chi + \beta - \alpha \tag{24}$$

is not admissible, and we are to replace the control force M in equation (21) with some $M' < M$. This implies the relation

$$\sigma(1-\alpha)\cos\varphi_A = 2\chi' + \beta - \alpha, \quad \chi' = M'/T_0 < \chi \tag{25}$$

Then the conditions $\cos\varphi_A \le 1$ and (23) are equivalent to the system

$$\chi' + \beta \le \alpha, \quad \chi' + \beta \le \chi \tag{26}$$

Then find the first positive root of equation (12). At last, perform the calculations by formulas (18)-(21).

Example. Let $\alpha = \chi = 0.8, \varphi_A = 0$. To satisfy (23), put

$$\sigma = 4 - 5\beta, \quad \beta \in (0, 0.8)$$

then equation (12) has form

$$2\theta = \sigma \sin 2\theta \tag{27}$$

The interval $\sigma \in (0,4)$ correspond to the root of eq.(27) in the range $\theta \in (0,1.2)$. Numerical calculations by formulas (18) – (21) lead to the dependence $\langle v(t)\rangle(\theta)$, shown in Fig.3. A single maximum is situated at

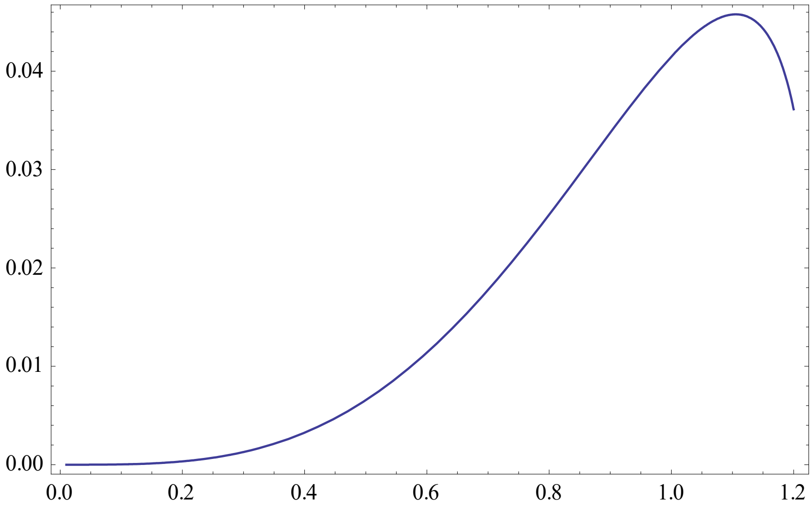

Fig.3. Typical dependence of normalized average velocity on the parameter θ (radians)

The case $\chi \gg 1$ can be considered qualitatively. In view of (10), (21) we assume

$$\sigma = \frac{km_1 \cos\varphi_A}{km_1 + 2M} = O(\chi^{-1}), \quad c = \frac{2km_1}{\sigma L} = \frac{2km_1 + 4M}{L\cos\varphi_A} = O(\gamma) \tag{26}$$

We are free to choose $\varphi_A \in (-\pi/2, 0)$, then we determine θ as first root of eq. (12):

$$\theta = \left(\frac{\pi}{2} - \varphi_A\right)(1-\sigma) + O(\chi^{-2}) \tag{27}$$

Keeping in mind that the differnce $\gamma_A - \gamma_C$ is bounded, we obtain

$$\langle v(t)\rangle \approx \frac{\alpha L\sqrt{c}}{4\sqrt{\alpha m_1}}\left(\cos\varphi_A - (\pi/2 - \varphi_A)\sin\varphi_A\right) \approx 2\sqrt{\frac{\alpha LM}{m_1}}\,\frac{\left(\cos\varphi_A - (\pi/2-\varphi_A)\sin\varphi_A\right)}{\sqrt{\cos\varphi_A}} \tag{28}$$

As $\chi \to \infty$, the value c is large, and in theory the last fraction in (28) can be arbitrary big. We conclude that the use of springs is advisable both to increase the speed of the mobile device, and to ensure its movement with a low-power motor.

ACKNOWLEGEMENTS

This work is partially supported by the Russian Foundation for Basic Research, project No. 18-01-00335.

REFERENCES

[1] Li, H., Furuta, K. and Chernousko, F.L. Motion generation of the capsubot using internal force and static friction. *Proc.45-th IEEE Conf. on Decision and Control* (2006): 6575-6580.

[2] Figurina, T. Optimal motion control for a system of two bodies on a straight line. *J. Comput. Syst. Sci. Int.* (2007) **46**: 227-233.

[3] Semendyaev, S. and Tsyganov, A. Model and investigation of dynamics of solid system with two massive eccentrics on a rough plane. *Proc. ECCOMAS Congr* (2016) Vol. 3. 4572-4583.

[4] Semendyaev, S. Coupled dynamics of solid system with slider-crank mechanisms as internal movers on rough surface with friction. *Proc. Int. Conf. Coupled Probl. Sci. Eng.*, (2017) 185-196.

[5] Munitsyn, A. and Munitsyna, M. Oscillations of a rigid block on supported base. *Vibroeng. Procedia* (2016) **8**: 63-67.

[6] Ramodanov, S., and Kireenkov, A. Controllability of a rigid body in a perfect fluid in the presence of friction. *Proc. 7th Int. Conf. Coup. Probl. Sci .Engng* (2017) 179-184

[7] Borisov, A., Ryabov, P. And Sokolov, S. Bifurcation analysis of the motion of a cylinder and a point vortex in an ideal fluid. *Math. Notes* (2016) **99**: 834-839.

[8] Ivanov, G. Convex hull deviation and contractibility. *Topol. Meth. Nonlin. Anal.* (2017) **50**: 9-25.

[9] Yan, Y., Liu, Y., and Liao, M. A comparative study of the vibro-impact capsule systems with one-sided and two-sided constraints. *Nonlin.Dyns* (2017) **89**: 1063-1087

[10] Nunuparov, A., Becker, F., Bolotnik, N., Zeidis, I. and Zimmerman, K. Vibration-Driven Capsubot with an Opposing Spring: An Experimental Study. *ROMANSY 22–Robot Design, Dynamics and Control.* Springer (2019) 126-133.

[11] Liu, Y., Wiercigroch, M., Pavlovskaia, E. and Peng, Z.K.: Forward and backward motion control of a vibro-impact capsule system. *Int. J. Non-Lin. Mech.* (2015) **70**: 30-46.