VIII International Conference on Computational Methods for Coupled Problems in Science and Engineering
COUPLED PROBLEMS 2019
E. Oñate, M. Papadrakakis and B. Schrefler (Eds)

NUMERIC-ANALYTICAL METHODS OF THE COEFFICIENTS DEFINITION OF THE ROLLING FRICTION MODEL OF THE PNEUMATIC AVIATION TIRE

ALEXEY A. KIREENKOV[*†], SERGEY I. ZHAVORONOK[◊]

[*] Moscow Institute of Physics and Technology (State University),
Institutskiy per. 9, Dolgoprudny, Moscow Region, 141701, Russia

[†] Institute for Problems in Mechanics of Russian Academy of Sciences (IPMech RAS)
Prospekt Vernadskogo 101, korp.1, 119526, Moscow, Russia
Email: kireenk@ipmnet.ru, kireenk@mail.ru

[◊] Institute of Applied Mechanics of Russian Academy of Sciences (IPriM RAS)
Leningradskiy Prospekt 7, 125040, Moscow, Russia
e-mail: Zhavoronok@iam.ras.ru

Key words: Combined Dry Friction, Numeric-Analytical Methods, Pneumatic Aviation Tire.

Abstract. A new approximate models of the tire rolling accounting for coupled longitudinal and lateral sliding as well as the spinning and the deformed state resulting in elastic forces were proposed in previous works [1-7]. The main goal of this investigation consists in the development of the numeric-analytical methods of these models coefficients definition.

1 INTRODUCTION

The intensive oscillations of main landing gears were observed at initial stages of landings of some modern aircrafts. Some of the registered cases of oscillations resulted the destruction of torque links and the alarm conditions with hard damaging of aircraft's frame. Among the main features of this phenomena are the following:

- the high-intensive coupled longitudinal, lateral, and spinning oscillations of "shimmy" type observed directly after touchdown (i.e. at the stage of non-steady rolling with longitudinal sliding) as well as directly after the end of the unstably rolling stage;
- the high-amplitude oscillations are coupled with significant wheels' sliding that is proved by the tracks of wheels at runaways' surfaces.

The last specificity leads to the main conclusion: the classical shimmy models cannot be applied to the theoretical description of the investigated phenomenon. Indeed, the hypothesis of sliding and spinning absence is the background of these models. Thus, a qualitatively new model with no non-holonomic constraint and accounting the dry friction forces is needed. This kind of models was proposed by the Russian academicians D.M.Klimov and V.Ph.Zhuravlev in 2009 on the groundwork of the multi-component dry friction theory. Their dry friction theory was efficiently used to model the shimmy of the rigid wheel without tire, i.e. with vanishing deformations and significant sliding during the unstable rolling. It is to be noted that their solution of the rolling stability problems was obtained for the Hertzian contact pressure distribution that is allowed only for small strains and deformations of tires, i. e. the approximation of the pressured thick-walled shell by an elastic solid. Nevertheless, the pressure distributions obtained from the numerical simulation on the groundwork of 3D finite element models as well as on the known approximate analytical solutions for the nonlinear problem of the soft shell theory differ significantly from the Hertzian distribution if even the quasi-statics is investigated. The real contact pressure distribution or it approximation has to be taken into account to compute the friction parameters, primarily for the friction torque due to its significant effect on the stability of rolling [5, 6, 7, 12]. Such a distribution can be obtained on the background of the finite element simulation in dynamics or in quasi-statics. On the other hand the three-dimensional finite element modeling of such strongly nonlinear systems is very resource consuming. Thus, the numerical simulation of the dynamics of systems like aircraft landing gears become practicable only at the stage of final calculations if the transient contact interaction of a pneumatic wheel and a road is modeled in details; it remains almost meaningless at the stage of the preliminary engineering design. Roughly speaking, it is strictly required to know what effect will be studied before the detailed quantitative dynamics analysis is performed.

Such a qualitative analysis must result in the knowledge about main specificities of the system and requires approximate models up to the simplest ones allowing analytical estimates construction. For instance, the study of the stability of the wheel rolling can use different three-dimensional models as well as the shell models or even the simplest approximations of the contact pressure to compute the dry friction forces and torque, etc. This approach was realized by the authors in the previous works [3, 4, 6, 7, 8, 9]. These results were obtained with the aid of theory of multi-component dry friction which makes it possible to correctly and qualitatively describe the effects of dry friction in the case of combined kinematics. The narrow field of theoretical mechanics that began in the early 2000's with a few publications has formed in integral scientific direction, now, and has effectively used not only in works of authors [3-10] but in different publications of researchers from different scientific groups [1, 2, 11].

Unfortunately, the application of the theory of combined dry friction was limited by the need to calculate the coefficients of the models based on the calculation of the distribution of normal tangential stresses within the contact patch. The technique of experimental investigations of these coefficients and its validation on the base of numerical experiment is presented below. The verification procedure consists in two main stages: at the beginning the models coefficients are calculated on the basis of analytical formulae [3,4,6,8] with aid of numerical simulation of the stress distribution inside contact areas, then these coefficients are defined from the numerical dependence for dry friction torque and force components.

2 NUMERICAL EVALUATION OF THE COEFFICIENTS OF THE COUPLED DRY FRICTION MODEL ON THE BACKGROUND OF THE FINITE ELEMENT SIMULATION OF TIRE-ROAD CONTACT INTERACTION

The normal reaction N for an arbitrary pneumatic tire could be defined as follows:

$$N = \int_0^{2\pi} \int_0^R \sigma_v (r, \varphi) r dr d\varphi \tag{1.1}$$

Here $\sigma_v(r,\varphi)$ denotes the contact pressure distribution over the contact spot.

Let us consider the static deformed state corresponding to the maximum vertical deformation equal to 29 mm. The corresponding distribution of the dimensionless contact pressure $\sigma_0(r,\phi)$ referred to the boost pressure $P^* = 200$ kPa is shown on the Figure 1:

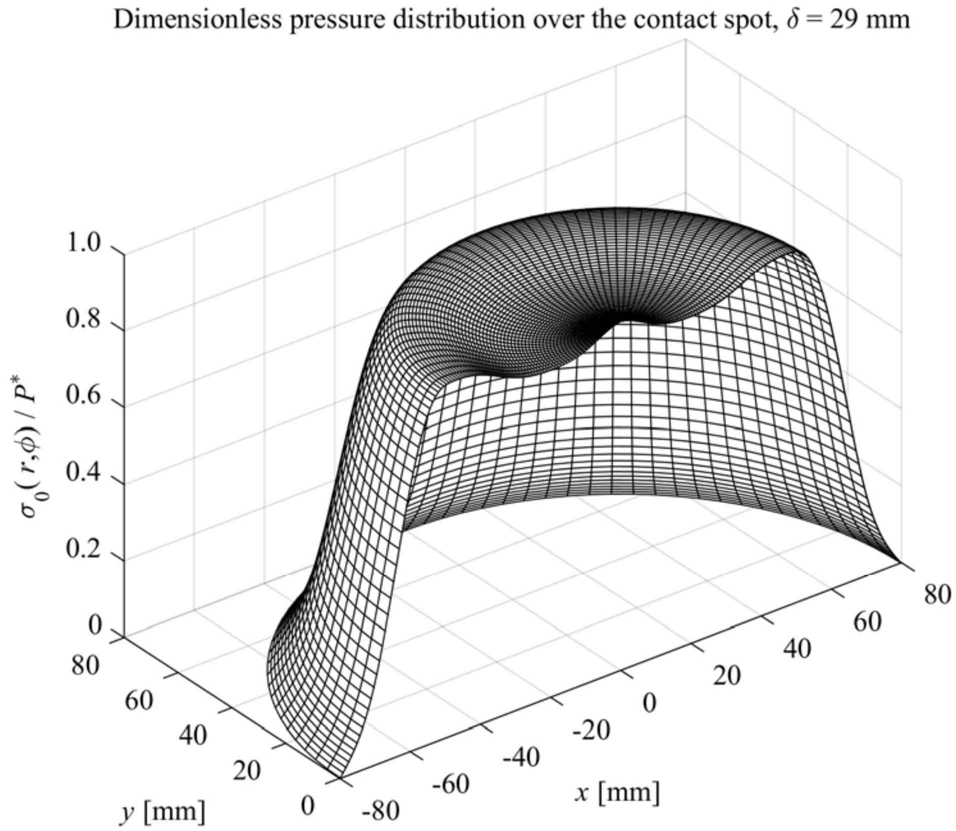

Figure 1. Spatial distribution of the static contact pressure in a typical tire

Now let us consider the distribution of the contact pressure for the rolling tire corresponding to the same boost pressure as well as to the same maximum vertical deformation before rolling initiation. The spatial distribution of the dimensionless pressure $\sigma_v(r,\phi)$ is shown below on the Figure 2:

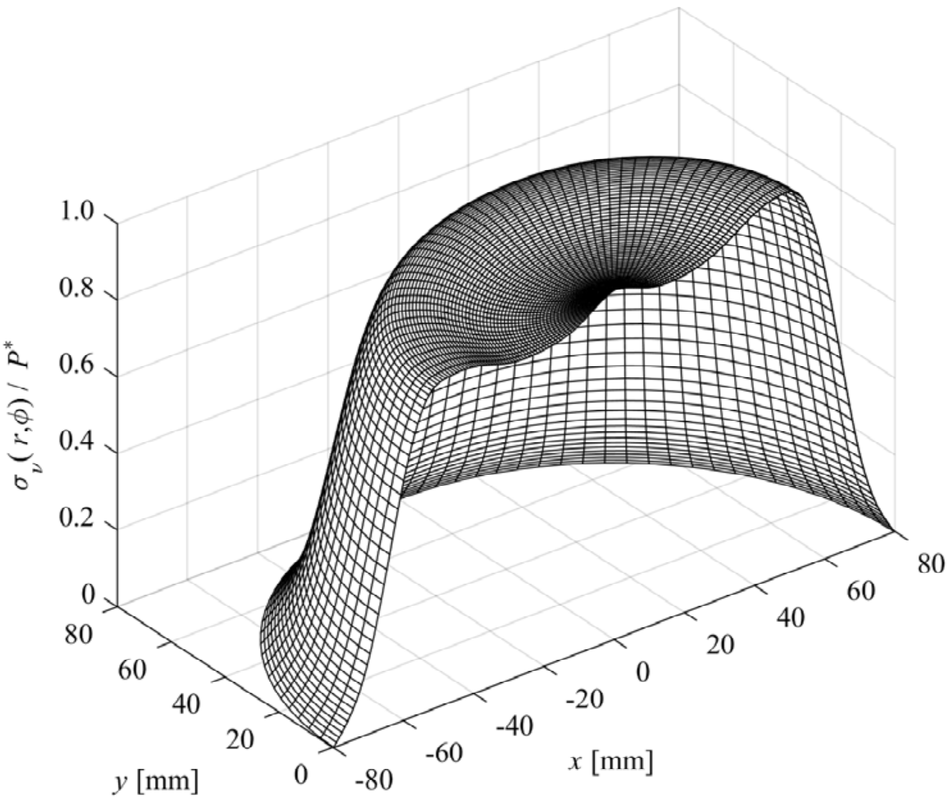

Figure 2. Spatial distribution of the static contact pressure in the rolling tire

For the diameter section of the contact spot we have the contact pressure distribution that is shown on the Figure 3.

Let us define hence the rolling correction factor k_x. In general, the pressure could be approximated as follows [3, 4, 6, 8]:

$$\sigma_\nu(r,\varphi) \approx \sigma_0(r,\varphi)\left(1 + k_x \frac{x}{R}\frac{\omega_y}{|\omega_y|}\right)$$

where R is the contact spot radius and k_x is interpreted as a shift of the gravity center of the contact pressure distribution due to the rolling. As a result, we obtain

$$s_x = \frac{\int_0^{2\pi}\int_0^R \sigma_\nu(r,\varphi) r^2 \cos\varphi dr d\varphi}{\int_0^{2\pi}\int_0^R \sigma_\nu(r,\varphi) r dr d\varphi}, \quad k_x = \frac{s_x}{s} R, \quad s = \pi\int_0^R \sigma_\nu(r,\varphi) r^3 dr \tag{1.2}$$

As the contact spot could be considered as a circle, the numerical evaluation of the formulae (1.2), (1.3) and (1.4) could be based on the trapezoid formula referred to the polar frame. As a result, we obtain $N = 2.41 \times 10^5$ N, $M_0 = 1.07 \times 10^4$ N×m, $m = 0.025$ m, $a = 0.052$ m$^{1/2}$, $s = k_x = 0.13$.

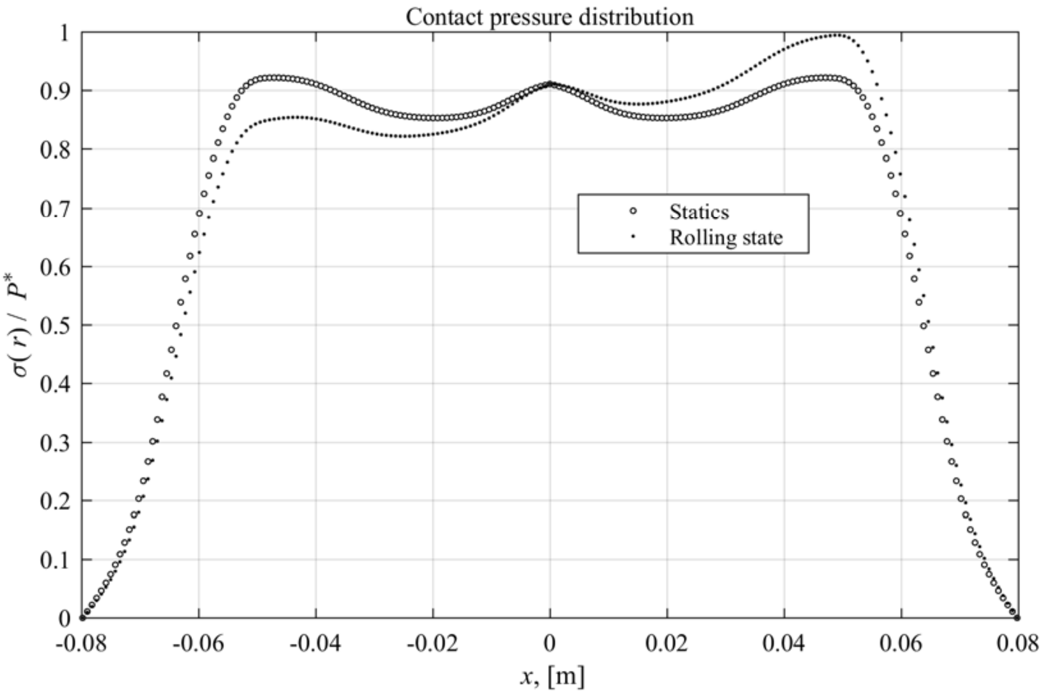

Figure 3. Distribution of the dimensionless static contact pressure over the diameter section of the contact spot, finite element simulation

The approximate formulae for the longitudinal dry friction force as well as for the dry friction torque could be written as follows [3, 6, 8]:

$$F_{\parallel} = \frac{F_0 v}{\sqrt{v^2 + au^2}}, \quad F_{\perp} = \frac{k_x b_1 u}{\sqrt{u^2 + bv^2}}, \quad M_C = \frac{M_0 u}{\sqrt{u^2 + mv^2}} z \tag{1.3}$$

Here the following coefficients are introduced:

$$F_0 = 2\pi f R^2 A^1, \quad M_0 = 2\pi f R^3 A^2 \tag{1.4}$$

$$\frac{1}{\sqrt{a}} = \frac{1}{F_0}\pi f R^2 A^0 = \frac{1}{2}\frac{A^0}{A^1}; \quad \frac{1}{\sqrt{m}} = \frac{1}{M_0}\pi f R^3 A^3 = \frac{1}{2}\frac{A^3}{A^2}. \tag{1.5}$$

$$b_1 = \pi f R^3 A^2 = \frac{M_0}{2}; \quad \frac{b_1}{\sqrt{b}} = \pi f R^3 A^3 = \frac{M_0}{\sqrt{m}} \quad \Rightarrow \quad \frac{1}{\sqrt{b}} = \frac{2}{\sqrt{m}} = \frac{A^3}{A^2}. \tag{1.6}$$

$$A^k = \int_0^1 \sigma_0(\rho)\rho^k d\rho, \quad \rho = \frac{r}{R}.$$

Thus, the formula for $F_\perp$ can be rewritten as follows:

$$F_\perp = \frac{k_x M_0 u}{2\sqrt{u^2 + \frac{1}{4} m v^2}} \tag{1.7}$$

Let us introduce dimensionless variables, $\upsilon = v / u$ and $\psi = u / v$. Thus, the one-dimensional dependencies for both dry friction force and torque could be derived from (1.1):

$$F_\parallel = \frac{F_0}{\sqrt{1 + a\psi^2}}, \quad F_\perp = \frac{k_x M_0}{2\sqrt{1 + \frac{1}{4} m\upsilon^2}}, \quad M_C = \frac{M_0}{\sqrt{1 + m\upsilon^2}}. \tag{1.8}$$

The corresponding diagrams are shown on Figures 4, 5, 6 as solid lines for $f = 0.3$.

We could consider these diagrams as test data; thus, the factors F_0, M_0, k_x, a, and m of the model (1.8) could be obtained from these diagrams, therefore the model could be interpreted as a rheological one. Indeed, let us perturb the diagrams by applying the random distribution with amplitude equal to $0.2\max(F_\parallel)$ and $0.2\max(M_v)$, respectively. In the other words, the 20% measurement error level is assumed. The corresponding "test values" are shown on Figures 4-6 as dotted lines.

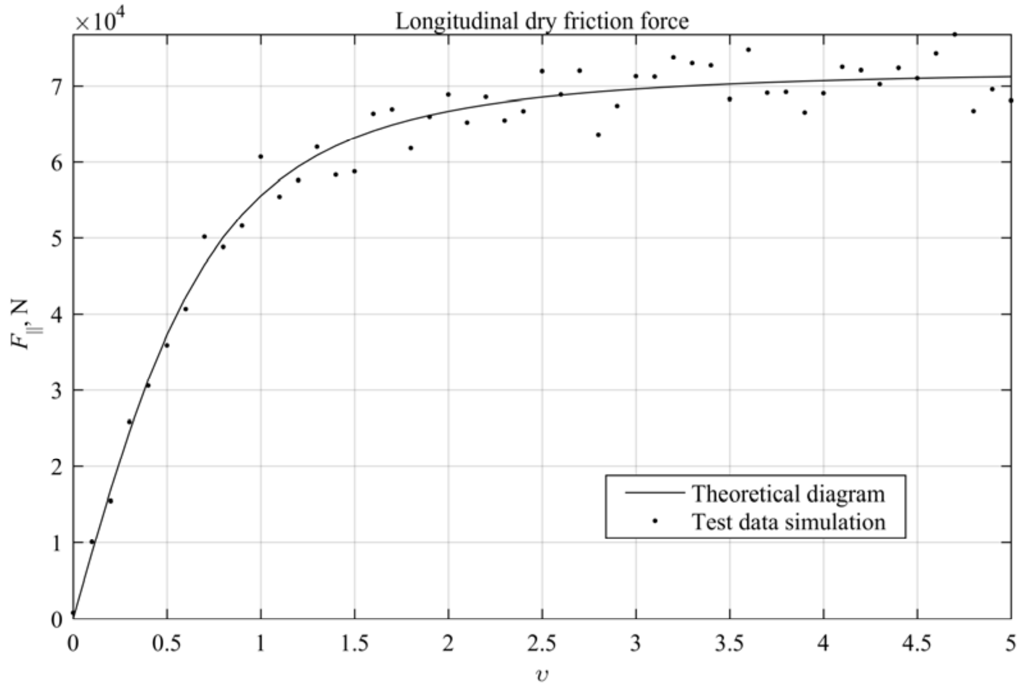

Figure 4. Dependence of the longitudinal friction force on the ratio between the sliding velocity and spinning angular velocity, υ

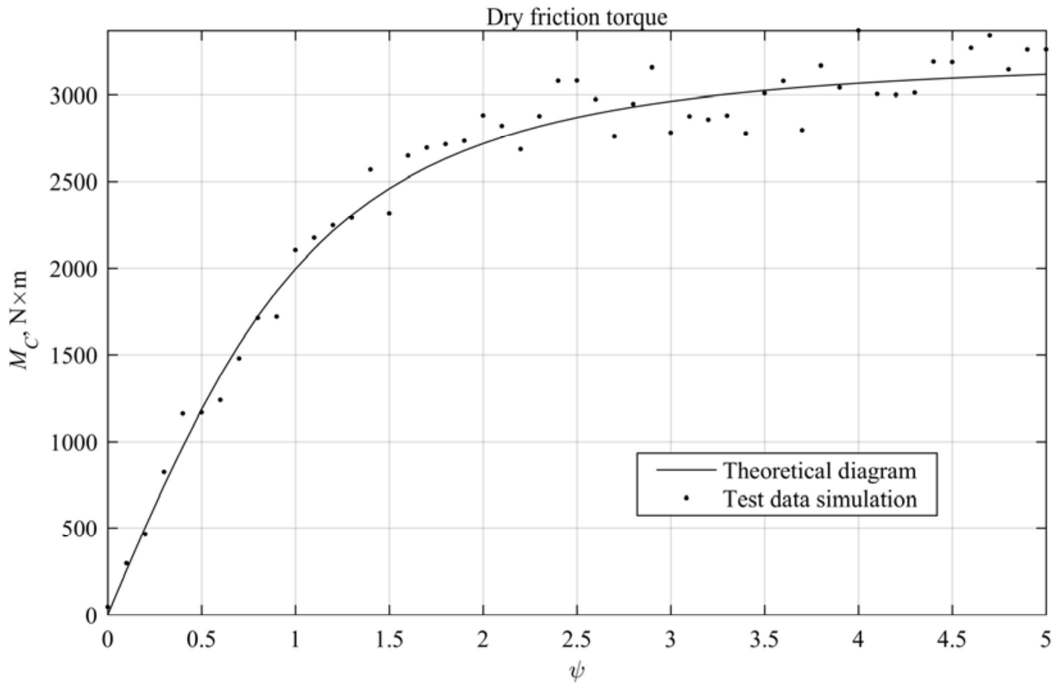

Figure 4. Dependence of the friction torque on the ratio between the spinning angular velocity and sliding velocity, ψ

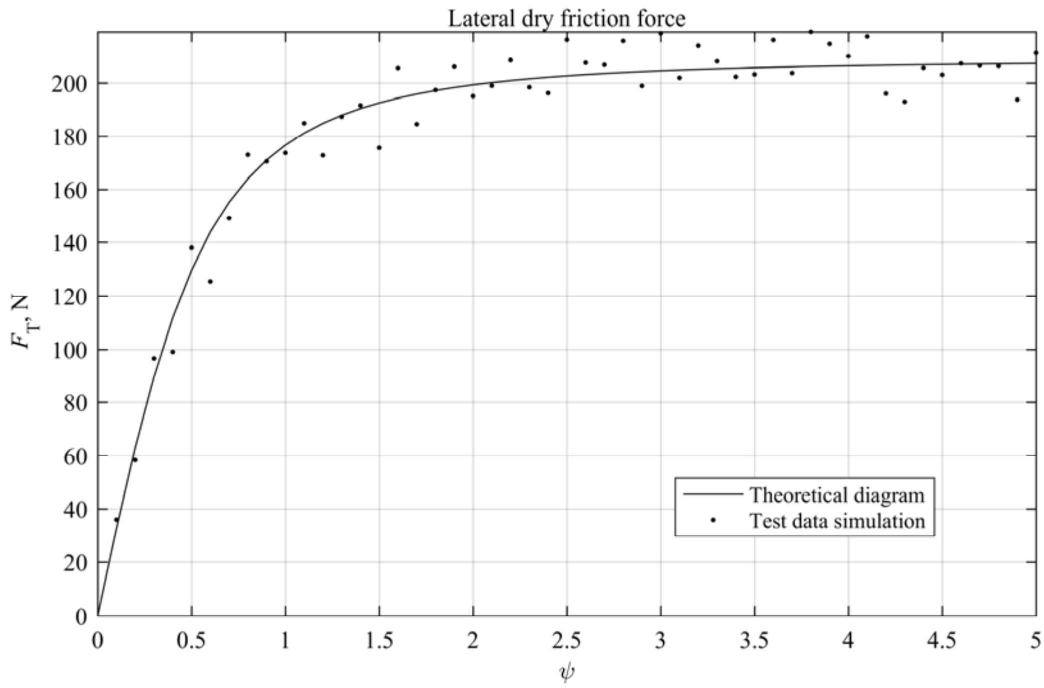

Figure 5. Dependence of the lateral friction force on the ratio between the spinning angular velocity and sliding velocity, ψ

The model factors could be obtained by the nonlinear least squares fitting [13]:

$$\min \sum_{k=1}^{N} \left\| f\left(x_k\right) - y_k^2 \right\| \tag{1.6}$$

where x_k are the values of ψ and υ and y_k are measured values of the longitudinal force $F_{\parallel}$, lateral force $F_{\perp}$ and friction couple M_{ν}. The dry friction factor could be obtained as

$$f = \frac{F_0}{N} \tag{1.6}$$

The fitting of the test data results in the following coefficients of the investigated model obtained with 95% confidence bounds:

Table 1: Coefficients of the model

Coefficient	Theoretical value	Experimental value	Bounds
f	0.3	0.301	0.296, 0.305
F_0	72219	72370	71200, 73530
M_0	3215	3252	3182, 3322
a	0.690	0.714	0.603, 0.825
m	1.595	1.607	1.348, 1.867
k_x	0.130	0.129	0.128, 0.131

3 CONCLUSIONS

- The model of the dry friction with combined kinematics is considered accounting for the contact pressure distribution obtained from the finite element simulation of the quasi-static deformed state of the pneumatic tire.
- The factor of the rolling friction as well as other coefficients of the model based on analytical approximations are obtained on the background of the numerical simulation of the steady rolling of the tire.
- The dimensionless dependencies of the longitudinal dry friction force, lateral dry friction force, and dry friction torque are obtained.
- The experimental data are simulated by adding of random noise to the theoretical curves, then the model factors are obtained from noised curves by nonlinear least squares procedure.
- The good correlation between the exact model coefficients and the ones obtained from the simulated test curves is shown, therefore the possibility of identification of the model parameters after typical tests is proven.

4 ACKNOWLEDGEMENTS

These investigations were carried out within the framework of the state program of the scientific research works on the section No.: AAAA-A17-117021310382-5.

REFERENCES

[1] Ivanov, A. On Singular Points of Equations of Mechanics. *Doklady Mathematics* (2018) **97(2)**:167-169.

[2] Ivanov, A.P. On the Equilibrium of Magic Stones. *Mechanics of Solids* (2018) **53(2)**: 26-31.

[3] Kireenkov, A.A. Improved theory of the combined dry friction in problems of aviation pneumatics dynamics. *Proceedings of the 7th International Conference on Coupled Problems in Science and Engineering, COUPLED PROBLEMS 2017* (2017):1293-1298.

[4] Kireenkov, A.A. Improved friction model of the aviation tyre contact with the landing strip. *IFAC-PapersOnLine* (2018) **51(2)**:890-894. (DOI 10.1016/j.ifacol.2018.04.027)

[5] Kireenkov, A.A., Nushtaev, D.V., Zhavoronok, S.I. A new approximate model of tyre accounting for both deformed state and dry friction forces in the contact spot on the background of the coupled model. *MATEC Web of Conferences* (2018) **211(08003)**. (DOI 10.1051/matecconf/201821108003)

[6] Kireenkov, A.A., Zhavoronok, S.I. Coupled dry friction models in problems of aviation pneumatics dynamics. *International Journal of Mechanical Sciences* (2017) **127**:198-203. (DOI 10.1016/j.ijmecsci.2017.02.004)

[7] Kireenkov, A.A., Zhavoronok, S.I. Implementation of analytical models of the anisotropic combined dry friction in problems of pneumatics' dynamics. *MATEC Web of Conferences* (2018) **211(08004)**. (DOI 10.1051/matecconf/201821108004)

[8] Kireenkov, A.A. Further development of the theory of multicomponent dry friction. *Proceedings of the 6th International Conference on Coupled Problems in Science and Engineering*, *COUPLED PROBLEMS 2015* (2015):203-209.

[9] Kireenkov, A.A. Three-dimensional model of combined dry friction and its application in non-holonomic mechanics. *Proceedings of ENOC-2005, Eindhoven, Netherlands, August 7-12* (2005):571-577.

[10] Ramodanov, S.M., Kireenkov, A.A. Controllability of a rigid body in a perfect fluid in the presence of friction. *Proceedings of the 7th International Conference on Coupled Problems in Science and Engineering, COUPLED PROBLEMS 2017* (2017):179 – 184.

[11] Semendyaev, S.V. Solid system with two massive eccentrics on a rough plane: rotational case. *IFAC-PapersOnLine* (2018) **51**(2):884-889.

[12] Zhavoronok, S.I., Kireenkov, A.A. On the effect of the anisotropic dry friction and the deformed state of tires on the shimmy initiation. *Proceedings of the 7th International Conference on Coupled Problems in Science and Engineering, COUPLED PROBLEMS 2017* (2017):216-226.

[13] Strutz, T. Data Fitting and Uncertainty (*A practical introduction to weighted least squares and beyond*). 2nd edition, Springer Vieweg, 2016, ISBN 978-3-658-11455-8.