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THE APPLICATION OF LAGRANGIAN VORTEX METHODS TO THE PREDICTION OF HYDRODYNAMIC DAMPING OF FLOATING BODIES

J. MICHAEL R. GRAHAM*

* Department of Aeronautics, Imperial College,
London SW7 2AZ, UK
e-mail: m.graham@imperial.ac.uk

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Summary. *Lagrangian vortex methods of simulating the vortex shedding which occurs at the bilges and sharp edges of floating bodies under oscillatory flow conditions due to incident waves and motion of the body in response are presented. Local forces are taken from discrete vortex simulations of the flow around an isolated edge representing the bilge section of the hull. Both the classical, meshless, potential flow vortex method and a vortex-in-cell viscous simulation are used. The flow around the bilge of a typical long floating hull in beam waves is treated on a sectional basis and the isolated edge results are matched to the outer three-dimensional wave potential flow provided by a standard surface panel method. The advantage of this matching procedure is that the more computationally expensive vortex flow simulation is limited to the local bilge section for which universal results may be computed whereas the large scale wave-hull interaction which extends out many hull- or wave-lengths from the body is solved by the less computationally intensive panel method. This procedure thus provides an efficient method replacing empirical vortex damping coefficients, as presently used, by a more rational method based on flow physics. Results for regular waves generating sinusoidal flows around right angle edges, edges fitted with flat plate bilge keels and rounded edges are presented and some comparisons made with measured data from laboratory wave tank tests and results of full Navier-Stokes simulations.*

1 INTRODUCTION

Viscous forces usually need only be considered when predicting hydrodynamic forces on a floating body in oscillatory flow without mean velocity when the relevant Keulegan-Carpenter number, U_0T/b, is much greater than one. Here U_0 is the amplitude of relative velocity of the incident waves and/or oscillatory body motion, T the period and b a relevant length, typically the beam. Because of amplitude limits on U_0, due for example to wave breaking, viscous forces are usually much smaller than wave potential forces unless the body is ‘small’ enough not to be in the wave diffraction regime. However it is well established that for certain body motions the viscous component of the damping, although small relative to total wave potential forces, determines the response amplitude of the body because the latter are principally inertia forces with very little contribution to damping. The best known example is the roll damping of ship or barge hulls in beam waves. Other important cases include slow drift motions in sway and surge and damping of the free surface elevation in moonpools.

Viscous damping arises from both direct boundary layer effects (skin friction and displacement) and also from the effects of flow separation on the pressure distribution. The direct boundary layer effects are normally negligible at full scale but may be important in model tests. Flow separation usually occurs in cross-flows about local regions of high curvature on the body surface such as the bilges. For sufficiently sharp edges (right angles, bilge keels) the separated flow is essentially independent of the Reynolds number and an inviscid treatment is possible, but if the edges are rounded a viscous treatment is required. Navier-Stokes computations have been carried out for entire separated flow fields of this type including the free surface [eg. [1, 2]]. A disadvantage of a full Navier-Stokes field computation including the body and the relevant extent of the free surface is the large flow domain which must be simulated. This should cover several wavelengths of the incident waves for minimum effect of domain truncation at the outer boundaries[3]. The present method, described here, takes advantage of Greens function methods to solve the wave potential field which is the major part of the flow field for which the effects of viscosity may be neglected. These methods impose the correct outer radiation conditions for the wave field through the choice of Greens function without the need to consider any finite outer boundary. The viscous part of the calculation can then be limited to a smaller inner flow field. The flow in this region may be computed by a method, suitable for the separated and/or viscous flow, which is matched to the outer potential flow field in a similar manner to classical boundary-layer theory.

2. METHOD OF ANALYSIS

The method embeds a solution of the inner viscous flow field within the outer potential flow following a Helmholtz split of the velocity field. It is convenient to consider the case of non-steep waves:

$$H(\text{waveheight})/L(\text{wavelength}) \equiv \varepsilon << 1,$$

so that the wave potential flow can be treated by linearised analysis.

For the present case a slender floating body is assumed with length much greater than beam b, assumed to be O(L) so that the body is not small with respect to the wave field and therefore in the diffraction regime. The response amplitude due to the waves is O(H) and it is assumed that the body has one or more edge regions of small radius of curvature, R << b (eg. bilges). A (non-unique) Helmholtz split of the flow field is made by writing:

$$\underline{U}(x,t) = \nabla\phi + \underline{Ur} \tag{1}$$

The potential flow field φ is obtained from any Greens function method for the incident, diffracted and radiated fields satisfying the normal velocity boundary condition zero relative to the body surface for the incident and diffracted waves and all the degrees of freedom in which the body responds. This outer flow drives an inner rotational flow field $\underline{U}_r$ satisfying modified equations:

$$\partial \underline{Ur} / \partial t + \underline{U}.\nabla \underline{U} = -\, 1/\rho \, \nabla p + \nu \nabla^2 \underline{Ur} \tag{2}$$

with boundary conditions on the body surface ($\underline{n}$ normal , $\underline{s}$ tangential):

$\underline{U}_r.\underline{n} = 0$ to satisfy zero normal velocity and

$\underline{U}_r.\underline{s} = -\partial\varphi/\partial s$ to remove the outer flow slip velocity.

The inner flow field for a long body in beam waves, typically the worst roll/sway/heave case for a ship or barge hull, varies slowly (O(beam/length) in the lengthwise direction compared with the cross-sectional variation and may be approximated, as conventionally, by a series of locally two-dimensional sections (Figure 1).

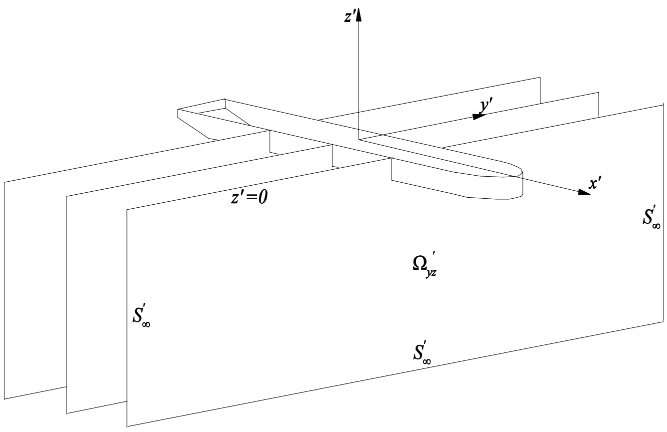

Figure 1. Strip theory showing 2-D hull sections.

Therefore in each inner sectional flow:

$$\partial \omega / \partial t + \underline{U}.\nabla \underline{\omega} = \nu \nabla^2 \omega \tag{3}$$

where $\omega \equiv (\nabla \times U_r)_1$, the dominant component of vorticity which is in the axial direction and normal to the section.

For a sharp-edge which fixes the separation and a high Reynolds number, the viscous term in equation 3 may be neglected since the effects of skin friction and diffusion of shed vórtices are small. Therefore:

$$D\omega/Dt \equiv \partial \omega / \partial t + \underline{U}.\nabla \underline{\omega} = 0 \tag{4}$$

This equation used together with the Biot-Savart law to compute inviscid oscillatory flow around a sharp right-angle edge is the classical two-dimensional discrete vortex method

(DVM) in which Lagrangian point vortex particles with unchanging circulation convect the vorticity field in the cross-section as the flow develops in time. Discrete vortices shed from the separation point are made to satisfy a Kutta-Joukowski condition at the edge to represent the separating vortex sheet convecting with the local velocity field. The evolution equations are time-stepped using a second order Runge-Kutta method. An instance of the resulting vorticity distribution (figure 2) shows how the vortex structures tend to form one counter-rotating pair per flow cycle, which convects away from the edge under its own self-induced velocity, when the driving (outer) flow field is oscillatory. The case shown is from a random sequence of waves incident on a right-angled bilge fitted with a small bilge-keel.

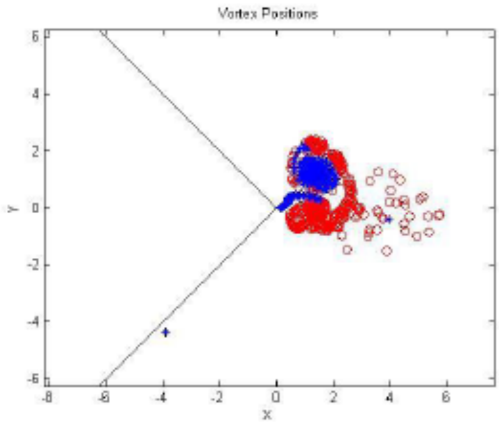

Figure 2.Vortex shedding generated by oscillatory flow around an isolated 90º edge representing a bilge with small bilge-keel (DVM computation).

For a given geometry, right angled edge or right-angled edge plus bilge keel a single computation for a sinusoidal outer flow or a random oscillatory velocity sequence with specified characteristic period T can be used through matching[4] to provide the force on each ship hull section. To calculate the response of the floating body the procedure first computes the potential flow and the body response amplitudes to the incident and diffracted wave potential. This is then applied as an outer boundary condition to the inner rotational flow field in the time domain to provide the viscous force contribution to be added to the wave potential forces. In the case of a sharp-edged bilge this inner flow field need only be computed once since the flow field is solely a function of the non-dimensional time (t/T) and the edge angle (usually 90º). A universal force may therefore be applied through a geometric scaling relationship for any hull cross-section having the same bilge edge angle. A rounded bilge or one fitted with a bilge keel has an additional length scale (radius of curvature or length of bilge keel) which formed into a Keulegan-Carpenter number provides another dimensionless parameter. The combined force on the body is then input into the time domain matrix equation for the body dynamics (for the three transverse degrees of freedom: heave, sway and roll, in the present case). The response amplitudes are recomputed and the whole procedure iterated to convergence which is rapid (less than ten iterations at roll resonance).

3. RESULTS

Figure 3 shows the roll response amplitude operator (RAO) for a rectangular section transport barge in regular beam waves, where the vortex damping has been computed using the above isolated edge technique matched to a linearised outer wave potential panel method. This compares well with laboratory wave-flume tests[5] on a 1/124 scale model. But the peak 'potential' RAO which neglects vortex damping is about double the measured value.

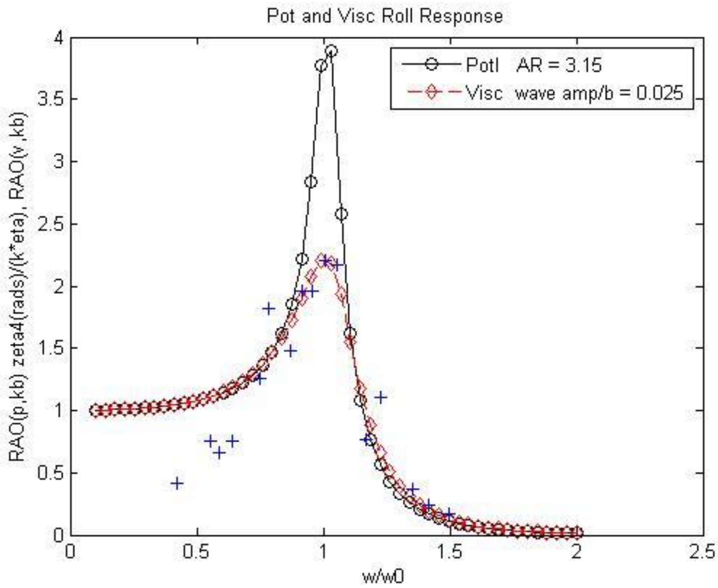

Figure 3. Roll RAO (free roll) for a rectangular section barge hull with sharp bilges in regular waves, computations: o, measured : +[5].

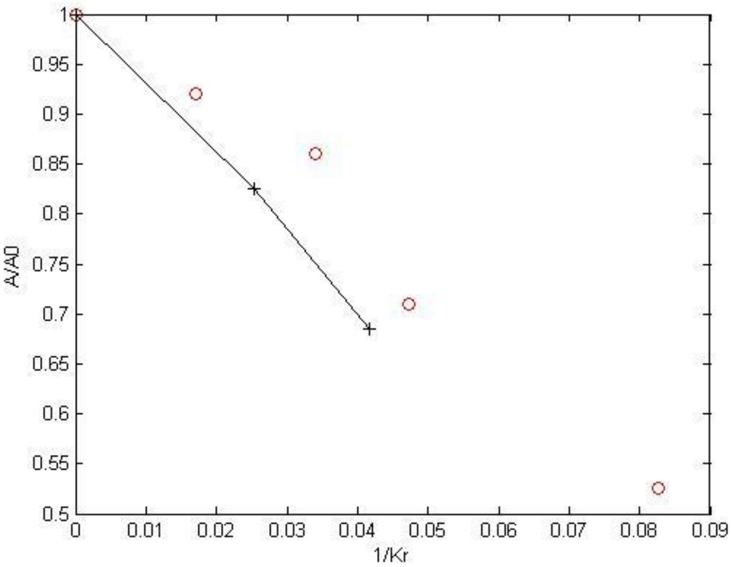

Figure 4. Ratio of rounded edge damping force coefficient to sharp edge value. Isolated edge, present DVM computations: **o** , previous [6] : +.

In the case of a rounded bilge viscous effects reduce the strength of the vortex shedding and hence the vortex induced damping force. It is only then possible to use the inviscid DVM approach as above if the separation point on the rounded edge is specified empirically, usually at the middle of the edge. The behaviour of the averaged vortex damping coefficient A computed by this type of inviscid vortex computation is shown in Figure 4 compared with an earlier DVM computation[6]. The effect of the radius of curvature r of the bilge, expressed as the Keulegan-Carpenter number K_r ($\equiv$ UoT/r), on the coefficient A is shown. The roll RAO for the same case as in Figure 3 but with small bilge rounding radius equal to 1% of the beam, figure 5, shows a 10% increase in the peak RAO due to the reduced damping.

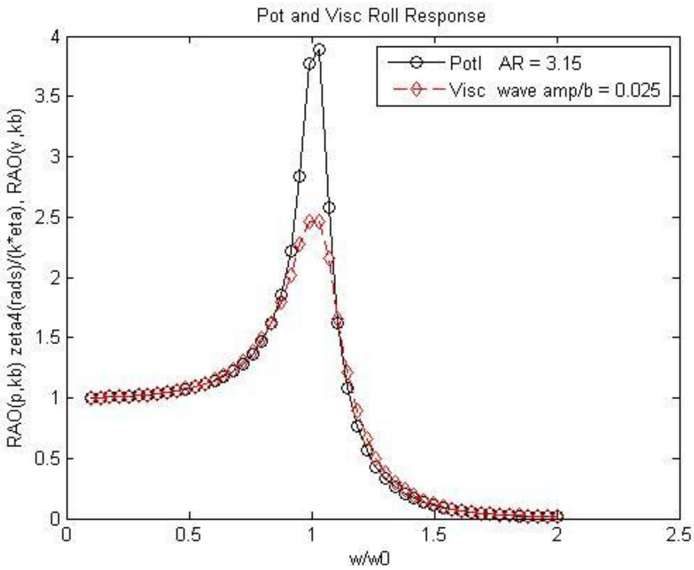

Figure 5. Roll RAO (free roll) as for figure 3 but with 1% bilge rounding, shedding point specified.

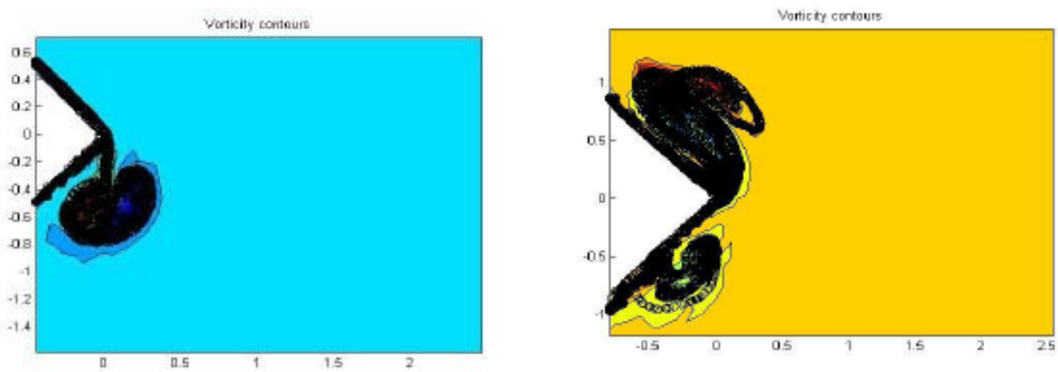

Figure 6. (a) t/T = 0.72 (b) t/T = 1.44
Vortex shedding due to sinusoidal flow around a bilge with small rounding. (VIC, major vortices visualised: $K_r = 20$, $R_r = 100$).

A better but computationally more expensive alternative for rounded edges is to solve the viscous vorticity equation 3 including no-slip on the body surface and separation of the boundary layers, rather than using inviscid results with specified separation. For this a vortex-in-cell (VIC) method[7] on a mesh of non-uniform quadrilateral cells has been used to simulate vortex shedding driven by sinusoidal flow about a slightly rounded edge as a function of two edge parameters, Keulegan-Carpenter number K_r, and Reynolds number R_r ($\equiv U_0r/\nu$). Figure 6 shows the vorticity field shedding from the edge with vortex pairing developing at two time instances.

Going beyond the local edge matching method the inner region may be extended completely round the hull section and up to the mean free-surface. As the region is now bigger and lacks the universality of the infinite edge it is much more expensive to compute but a more accurate option if the hull section geometry is complex. The flow field may be simulated by the same methods as before ranging from the DVM up to use of a full Navier-Stokes field solver. An example of the latter is the spectral-element method (*Nektar*[8]) providing a very accurate computation of the inner flow field. Because the waves are linearised for the outer flow a simple rigid lid normal velocity boundary condition for $\underline{U}_r$ has been applied on the mean free surface. This therefore excludes free-surface interactions such as the $O(\varepsilon)$ far field waves radiated by the viscous flow field.

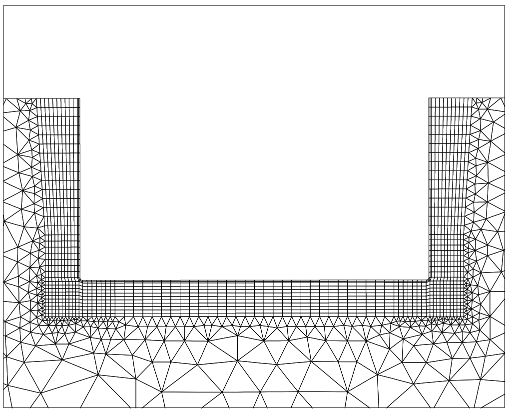

Figure 7. Inner region mesh for Spectral Element Navier-Stokes computation.

Figure 7 shows the mesh used to compute the inner flow field. Examples are shown of two flow fields computed[9] using this mesh and a variation. In the first (Figure 8) for a hull section with bilges which have very small rounding (r/b = 0.01) the structures of numerous shed vortex pairs are seen to be clearly visible in the inner region after several flow cycles. It should be noted that a counter-rotating vortex pair of nearly equal strength which has been shed and stopped growing exerts very little force on the body. In the second case (Figure 9) a hull section with a comparatively large radius of curvature bilge rounding (r/b = 0.125) shows much weaker vortex separation and shedding.

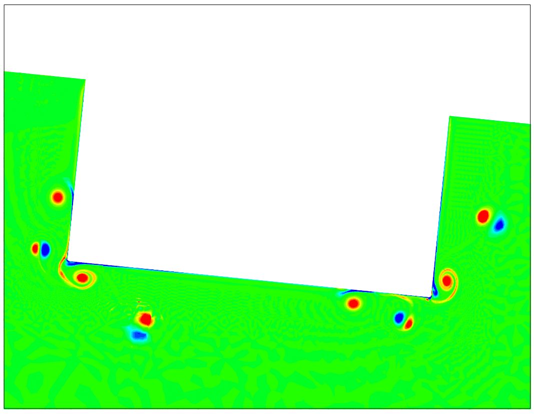

Figure 8. Vortex shedding from hull section with 1% rounded bilges.

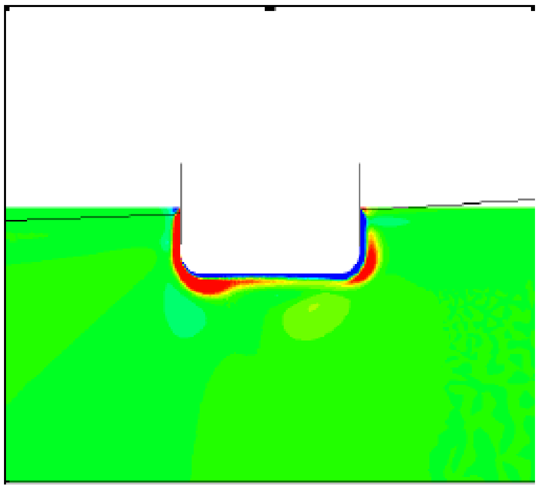

Figure 9. Vortex shedding from hull section with 12.5% rounded bilges.

4. CLOSING REMARKS

A range of methods have been presented for predicting roll motion damping and response of a floating hull in waves based on simulating the separation and vortex shedding which occurs at the bilges of the hull and often dominates the damping forces. All of the methods used except the final one which uses a high order finite element analysis of the Navier- Stokes equations are based on Lagrangian vortex representations of the separated flow. Two-dimensional strip theory analysis, conventional for long ship hulls, has been used throughout for the inner region. The same two-dimensional analysis of the inner separating flow is also appropriate for more general bodies provided the shedding regions can be considered relatively straight compared with the cross-flow dimensions of these regions around the edge. This would be true, for example, of a spar buoy with a truncated vertical, circular, cylindrical geometry but not of a region near the bow or stern of a ship hull.

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