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NONLINEAR DYNAMIC RESPONSES OF SHELL STRUCTURES USING VECTOR FORM INTRINSIC FINITE ELEMENT METHOD

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Abstract. In this paper, in order to compute nonlinear dynamic responses of shell structures, formulations of the internal forces of the shell element in vector form intrinsic finite element (VFIFE) method are developed. This novel shell element is named by VFIFE-DKT element. These elements are to compute internal forces from the deformations and the motion of the shell structures. The VFIFE method is a particle-based method. They have three key VFIFE processes such as the point value description, path element and convected material frame. Thus, the shell structure is represented by finite particles. Each particle is subjected to the external forces and internal forces. The particle satisfies the Newton's Law. A fictitious reversed rigid body motion is used to remove the rigid body motion from the deformations of the element. The internal forces of the element in deformation coordinates satify the equilibrium equations. Through the numerical examples of the benchmark structures undergo extermly-large displacements, rotation and motion, the proposed procedures using the novel element demonstrates its accuracy and efficiency.

1 INTRODUCTION

In past years, large displacement of the shell structures using finite element method have been proposed, but it is still difficulties in analyzing general shell structures. Clearly, the numerical unreliability and the insufficient knowledge of the actual physical sensitivity of the shell structures, the bad analysis results will be created. For example, the isoparametric element formulations in FEM of the shell structure subjected to pure bending, the membrane strains should equal zero. Unfortunately, these element formulations cannot represent pure bending state, since the membrane stiffness includes the coupling terms between the membrane and bending stiffness. When the thickness ratio to the radius of curvature decreases in a shell element, underestimated results occur which is called membrane locking. In order to alleviating the shear locking as well as the membrane locking in shell element, numerous approaches such as the use of reduced and selective integration [1,2] were usually adopted. All of those studied thus far possess at least one spurious zero-energy mode. Huang [2] has been used assumed membrane strains in the orthogonal curvilinear coordinate system applied to avoid membrane locking behavior. Three-node triangular element using assumed natural strain (ANS) formulation to suppress the locking effect was proposed by references [3,4].

Recently, an approach named the vector form intrinsic finite element (VFIFE) method for the large geometrical analysis in continuous media was proposed by reference [5]. This method has been successfully applied to the dynamic analysis of the 2D elastic frame, space truss structure, and the elastic-plastic space truss structure obtained by references [6-8]. Motion analysis of 3D membrane structure using VFIFE method was developed by reference [9]. The extremely large displacement analysis of elastic-plastic 2D structures using the VFIFE method has been introduced [10]. Differences and similarities between the VFIFE and explicit finite element method has been discussed [10]. Our objective in this paper is to develop a computing method of the internal forces from deformed shell element using VFIFE method. An efficient shell analysis procedure to avoid the membrane locking in thin shell structures is considered.

2 INTERNAL FORCES OF THE VFIFE-DKT ELEMENT

2.1 Internal forces of the membrane element

In this section, the internal forces from computing deformations of the membrane element in deformation coordinates are introduced. The corresponding internal forces satisfy the equilibrium. The deformation displacements using fictitious reversed rigid body motion of the particles *i,j* and k must be transferred from global coordinates to the deformation coordinates. The deformation displacements and relative position vectors can be computed in deformation coordinates from:

$$\hat{\boldsymbol{\eta}}_n^d = \mathbf{Q}\boldsymbol{\eta}_n^d \text{ , } n=j, k \tag{1}$$

$$\hat{\mathbf{x}}_n^a = \mathbf{Q}\mathbf{x}_n^a \text{ , } n=j, k \tag{2}$$

in which

$$\hat{\boldsymbol{\eta}}_j^d = \{\hat{\eta}_{jx}^d \quad 0 \quad 0\},\ \hat{\boldsymbol{\eta}}_k^d = \{\hat{\eta}_{kx}^d \quad \hat{\eta}_{ky}^d \quad 0\} \tag{3}$$

The deformation displacements and strains of the membrane element can be represented as:

$$\hat{\mathbf{u}}_m^d = \sum_{n=j}^{k} N_n \hat{\boldsymbol{\eta}}_n^d \text{ , } n=j, k \tag{3}$$

where N_n is a shape function of the particle n. Similar to FEM process, the strains of the element can be computed from:

$$\hat{\boldsymbol{\varepsilon}}_m = \mathbf{B}_m \hat{\mathbf{d}}_m^* \text{ , } n=j, k \tag{4}$$

where

$$\hat{\mathbf{d}}_m^* = \left\{ \hat{\eta}_{jx}^d \quad \hat{\eta}_{kx}^d \quad \hat{\eta}_{ky}^d \right\} \tag{5}$$

Base on stress-strain relationships, the stresses of the element can be written as

$$\hat{\boldsymbol{\sigma}}_m = \mathbf{D}\hat{\boldsymbol{\varepsilon}}_m = \mathbf{D}\mathbf{B}_m \hat{\mathbf{d}}_m^* \tag{6}$$

where D is material matrix. In this study, the linear elastic material and an isotropic material are considered. For membrane element, due to the internal virtual work δW_m equals the virtual strain energy δU_m, equivalent internal forces can be obtained.

$$\hat{\mathbf{f}}^* = \int_{V_e} \mathbf{B}_m^T \hat{\boldsymbol{\sigma}}_m dV = \left\{ \hat{f}_{jx} \quad \hat{f}_{kx} \quad \hat{f}_{ky} \right\} \tag{7}$$

The remaining three internal forces in $\hat{x}-\hat{y}$ plane can be computed from the equilibrium equations.

$$\hat{f}_{ix} = -(\hat{f}_{jx} + \hat{f}_{kx}) \tag{8}$$

$$\hat{f}_{jy} = -(\hat{f}_{ky}\hat{x}_k^a - \hat{f}_{jx}\hat{y}_j^a - \hat{f}_{kx}\hat{y}_k^a) / \hat{x}_j^a \tag{9}$$

$$\hat{f}_{iy} = -(\hat{f}_{jy} + \hat{f}_{ky}) \tag{10}$$

2.2 Internal forces of the VFIFE-DKT element

Due to the internal virtual work equals the virtual strain energy, equivalent internal forces can be obtained.

$$\hat{\mathbf{m}} = (2A_a \int_0^1 \int_0^{1-\hat{\lambda}} \mathbf{B}_b^T \mathbf{D}_b \mathbf{B}_b dA_a)\mathbf{d}_b^T = \left\{ \hat{m}_{ix} \quad \hat{m}_{iy} \quad \hat{m}_{jx} \quad \hat{m}_{jy} \quad \hat{m}_{kx} \quad \hat{m}_{ky} \right\} \tag{11}$$

Where

$$\mathbf{D}_b = \int_{-\frac{h}{2}}^{\frac{h}{2}} \mathbf{D}_{b1} z^2 dz \tag{12}$$

$$\mathbf{D}_{b1} = \frac{E}{1-\upsilon^2} \begin{bmatrix} 1 & \upsilon & 0 \\ \upsilon & 1 & 0 \\ 0 & 0 & \frac{1+\upsilon}{2} \end{bmatrix} \tag{13}$$

$$\mathbf{B}_b(\hat{\xi}, \hat{\lambda}) = \frac{1}{2A} \begin{bmatrix} \hat{y}_{ki}\mathbf{H}_{x,\xi}^T + \hat{y}_{ij}\mathbf{H}_{x,\lambda}^T \\ -\hat{x}_{ki}\mathbf{H}_{y,\xi}^T - \hat{x}_{ij}\mathbf{H}_{y,\lambda}^T \\ -\hat{x}_{ki}\mathbf{H}_{x,\xi}^T - \hat{x}_{ij}\mathbf{H}_{x,\lambda}^T + \hat{y}_{ki}\mathbf{H}_{y,\xi}^T + \hat{y}_{ij}\mathbf{H}_{y,\lambda}^T \end{bmatrix} \tag{14}$$

where **B** is the strain-displacement transformation matrix in deformation coordinates. Base on the equilibrium equations, three shear forces of the element can be computed from:

$$\sum \hat{F}_z = 0\,,\ \hat{f}_{iz} + \hat{f}_{jz} + \hat{f}_{kz} = 0 \quad (15)$$

$$\sum \hat{M}_x = 0\,,\ \hat{m}_{ix} + \hat{m}_{ix} + \hat{m}_{kx} = -(\hat{f}_{jz}\hat{y}_j + \hat{f}_{kz}\hat{y}_z) \quad (16)$$

$$\sum \hat{M}_y = 0\,,\ \hat{m}_{iy} + \hat{m}_{jy} + \hat{m}_{ky} = \hat{f}_{jz}\hat{x}_j + f_{kz}\hat{x}_k \quad (17)$$

Six internal forces from computing the membrane deformations, three shear forces and six moments from computing the VFIFE-DKT element are obtained, respectively. Due to all the force directions at each particle are defined in the global coordinates. Since the all forces in deformation coordinates must be rotated from fictitious configuration V_r, then, the force directions in configuration V need be transformed into global coordinates.

4 EXAMPLES

4.1 Large displacement of pull-out of an open cylinder

In this case, the large displacement of shell structure, pull-out of an open cylinder structure subjected to a pulling force with the same amplitude F at particles A and D (Fig. 1) is considered. The cylinder radius and thickness are R=4.953 and t=0.094. Due to the symmetric conditions, one-eighth of the cylinder model as 2×8×16 VFFIE shell elements.

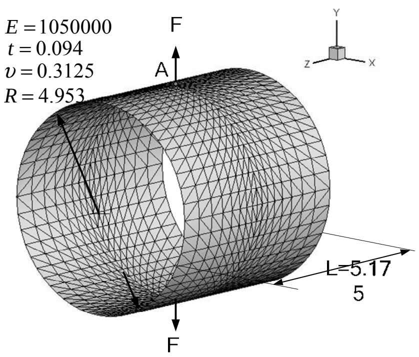

Figure 1. An open-cyclinder subjected to pulling force F at particles A and D

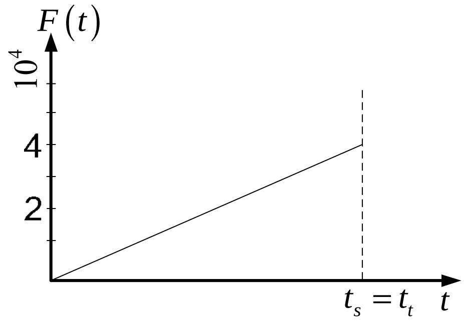

Figure 2. Time function of the Pulling force F(t).

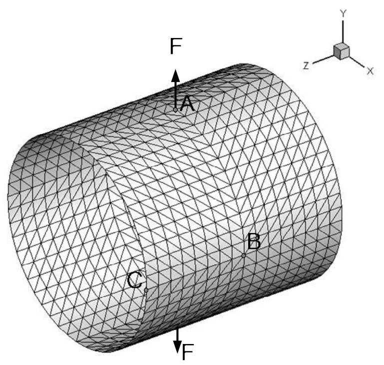

Figure 3. The open-cyclinder subjected to pulling force F at particles A and D

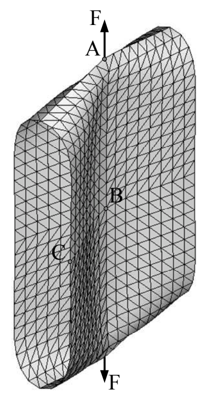

Figure 4. Deformed configuration of the cyclinder subjected to F_{max}=10×10^4

The material properties are: the Young's modulus E= 10.5×10^6, Poisson ration ν=0.3125 and mass density ρ=10, respectively. Time step size is 5×10^{-5} . In this study, the pulling force function F(t) and maximum force F_{max}= 10×10^4 (Fig. 2) are given. In order to verify the capability of use of the co-rotational 8-node degenerated thin-walled element to compute the large displacements of the cylinder, this structure is subjected to force F_{max}=10×10^4(Fig. 3). Figure 4 shows deformed configuration of the cyclinder. In Figs 5, it is confirmed that the VFIFE results well agree with the results obtained by reference [11]. This example illustrates the accuracy and capability of the proposed procedures in computing large displacement of the open cylinder structure.

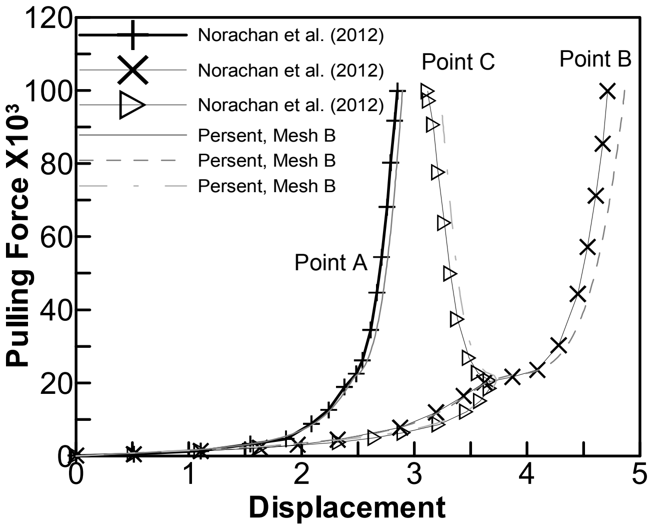

Figure 5. Displacements of the particle A and B versus pulling forces (F_{max}=10×10^4)

4.2 Nonlinear dynamic response of the hemispherical shell

This example assesses the ability of element to handle finite rotations and large rigid body motions. The geometrical parameters of the hemispherical shell are radius R =10 and thickness t = 0.04. The material properties are taken as Young's modulus E = 6.825×10^7, Poisson's radio ν= 0.3 and density ρ = 2.5 × 10−4. The damping is not considered. Due to the symmetry of the geometry and loading, only a quarter of the hemispherical shell is modeled in Fig. 6. Constant loads are applied on two points as shown in Fig. 7. At x = 0 and y = 0, symmetry boundary conditions are imposed. Only elastic material is considered. The VFIFE results are shown in Fig. 8. It can be seen that the the VFIFE-DKT element provide good results in this case.

5 CONCLUSION

In this paper, a method of computing the internal forces of the sell structure is developed. It can be used to compute nonlinear responses of shell structures subjected to large geometrical changes and complicated excitations. Through two examples, the proposed shell element in VFIFE method demonstrates its accuracy on the nonlinear responses of shell structures. In addition, the shear locking problem usually admitted in the conventional shell element can be removed from this proposed vector from intrinsic shell element.

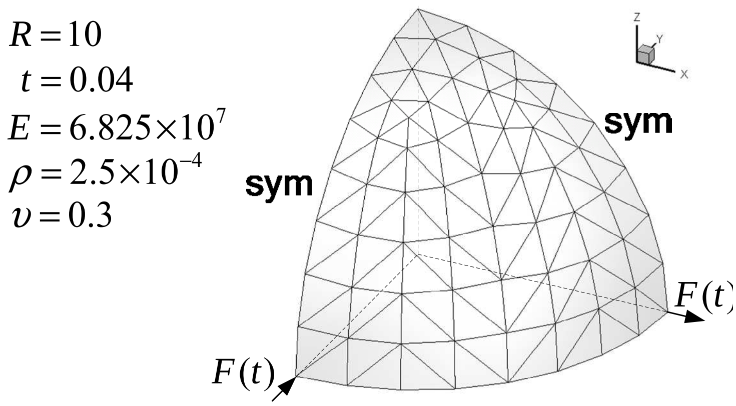

Figure 6. Problem definition for the hemispherical shell

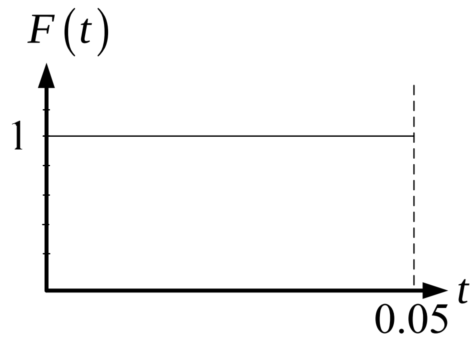

Figure 7. External force function for the ring plate

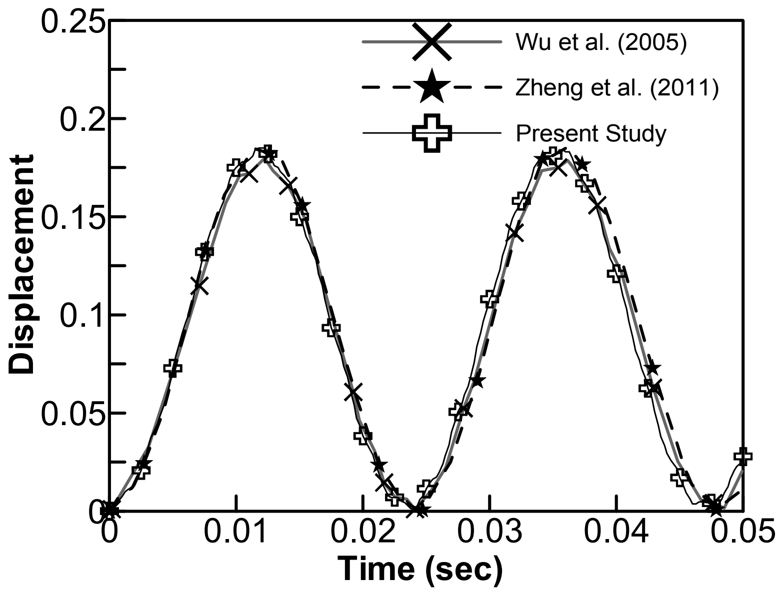

Figure 8. Displacements versus time of the ring plate

6 ACKNOWLEDGEMENTS

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