Optimal sizing and location of reactive power compensation in offshore HVAC transmission systems for loss minimization

Jovana Dakic, Marc Cheah-Mane, Eduardo Prieto-Araujo, Oriol Gomis-Bellmunt
CITCEA-UPC, Department of Electrical Engineering, Universitat Politècnica de Catalunya,
Av. Diagonal 647, 08028, Barcelona, Spain. Email: jovana.dakic@upc.edu

***Abstract*—This paper investigates the use of HVAC transmission systems with reactive power compensation as a potential cost effective solution compared to HVDC. A multi-objective optimisation technique is presented to identify the optimal reactor location and value. Two conflicting objective functions are considered: power losses and investment costs of installed compensation devices. Impacts of reactive power compensation on HVAC transmission system is analysed. The results show different optimal values, number and location of reactors depending on which objective function is prioratised. Also, a sensitivity analysis is presented to evaluate the impact of different voltage transmission levels and distances.**

I. Introduction

Year by year, an increase in the installed capacity of offshore wind power plants has been witnessed, therefore the power and distance of the transmission line are also increasing [1]. High Voltage AC (HVAC) and High Voltage DC (HVDC) are the current transmission options in reaching the target of building efficient, reliable and cost-effective systems. Although HVDC does not have charging currents, the cost for short distances is not competitive with HVAC. Besides the cost-effectiveness, HVAC is a more mature technology, so it might be recognized as the preferred option if proper compensation is installed.

Installing adequate reactive power compensation at the appropriate location highly contributes to reducing power losses and regulating voltage at the point of connection of a wind power plant. A number of papers are focused on the optimisation of reactive power compensation to achieve the most cost-effective solution of the transmission system [2], [3]. The variation of costs including reactive power compensation only at the both ends of the cable have been presented for different rated powers, voltages and transmission distances [2]. Further on, the improvement of the active power transfer capability over long distances including mid-cable compensation are shown in [3].

The purpose of this paper is to identify the optimal reactor size and location of an HVAC offshore transmission system for minimizing power losses and costs. Firstly, a power flow analysis of an HVAC transmission system is performed using Kirchhoff's circuit laws. Different configurations are defined according to potential shunt reactor locations and the number of reactors (from one to three) presented in Fig 1.

A multi-objective optimisation is introduced, where power losses and investment costs will be set as two objectives. The chosen method to weight the importance that costs and power losses have on the selection of the optimal solution is the linear weighted method. Power losses are obtained from the power flow analysis and the investment cost of compensation devices are calculated based on the absorbed reactive power. A wide set of optimal solutions will be obtained using a Pareto front (sum of objective functions multiplied by weighting coefficients). System constraints that are taken in account follow grid-codes-based requirements at the point of connection typically used in standard industrial practice.

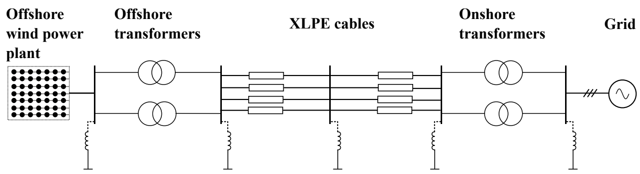


Fig. 1. Offshore wind transmission system diagram

Depending on which objective function is given more importance, different optimal solutions are shown according to the number of reactors and their location. Additionally, different voltage transmission levels and distances have been applied. Finally, total costs are analysed considering a trade-off between the additional cost of installing reactive power compensation and the savings from the reduction of transmission power losses. Also, total costs are compared to HVDC systems.

II. System Description

The diagram of an offshore wind transmission system is shown in Fig 1. The offshore AC collection grid is operating at 33 kV. The voltage from the collector grid to the offshore transmission cables is stepped up with an offshore transformer. Also, an onshore transformer is used to step up the voltage to the onshore grid operating level, which is 400 kV. The preferred transformer topology mainly depends on economic aspects, resulting in two transformers in parallel, rated at 60 % of the nominal power of offshore wind power plant [4]. The offshore wind power plant is connected to the grid using cross-linked polyethylene (XLPE) cables. Today, XPLE cables are widely applied in offshore industry. They can be either single-core or three-core, but three-core cables have the advantage due to reduced power losses and less installation cost [5].

Three voltages levels are considered V_i: 110, 150 and 220 kV. The number of cables (n) connected in parallel is determined by the rated power of wind power plant and transmission voltage. Due to the capacitance of the cable and generated reactive power, compensation is required to increase transferred active power to the grid and reduce power losses [2]. All possible reactor locations are specified in Fig. 2.

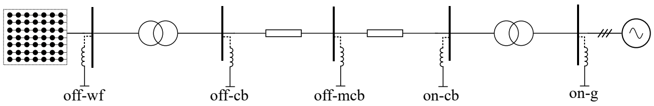


Fig. 2. Reactor locations: offshore wind power plant (off-wf), offshore cable (off-cb), offshore middle of the cable (off-mcb), onshore cable (on-cb), onshore grid (on-g)

III. System Modelling

A. Modelling of components

The elements modelled within the offshore wind transmission system are presented in the following subsections:

1) Cable model: For the steady state analysis cable it is more adequate to represent the cable with its equivalent π circuit. Taking in account the characteristic impedance $\underline{Z}_c$ and characteristic angle $\underline{\theta}$ [6]:

$$\underline{Z}_c = \sqrt{\frac{\underline{Z}}{\underline{Y}}} \quad \underline{\theta} = l\sqrt{\underline{Z}\,\underline{Y}} \tag{1}$$

$$\begin{aligned} \underline{Z} &= R + j\omega L \\ \underline{Y} &= j\omega\frac{C}{2} \end{aligned} \tag{2}$$

where R is the resistance, L is the inductance, C is the capacitance of the cable and l is the transmission distance.

In 3 and Fig. 3(a) $\underline{Z}_\pi$ and $\underline{Y}_\pi$ are defined [6]:

$$\begin{aligned} \underline{Z}_\pi &= \underline{Z}_c \sinh(\underline{\theta}) = \underline{Z}l\frac{\sinh(\underline{\theta})}{\underline{\theta}} \\ \underline{Y}_\pi &= \frac{\tanh(\frac{\underline{\theta}}{2})}{\underline{Z}_c} = \frac{\underline{Y}l}{2}\frac{\tanh(\frac{\underline{\theta}}{2})}{\frac{\underline{\theta}}{2}} \end{aligned} \tag{3}$$

2) Transformer model: The transformer is presented with impedance $\underline{Z}_T$ and admittance $\underline{Y}_T$ referred to primary or secondary side as in Fig. 3(b):

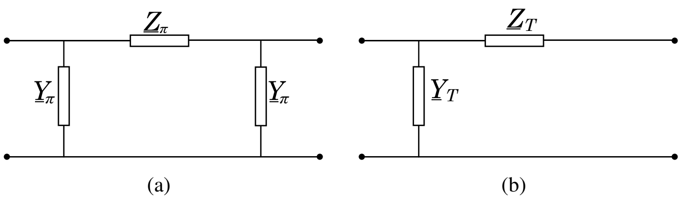


Fig. 3. Single-line models for elements: a) cable b) transformer

$$\begin{aligned} \underline{Z}_T &= R_T + jX_T \\ \underline{Y}_T &= G_T - jB_T \end{aligned} \tag{4}$$

$$R_T = P_{Cu}^{loss}(\frac{U_{nT}}{S_{nT}})^2 \quad X_T = \sqrt{(\frac{u_k(\%)}{100}\frac{U_{nT}^2}{S_{nT}})^2 - R_T^2} \tag{5}$$

$$G_T = \frac{P_{Fe}^{loss}}{U_{nT}^2} \quad B_T = \frac{i_o(\%)}{100}\frac{S_{nT}}{U_{nT}^2} \tag{6}$$

where P_{Cu}^{loss} are the copper losses, P_{Fe}^{loss} are the iron losses, u_k is the short circuit voltage and i_o is the open circuit current.

3) Shunt reactor model: Shunt reactor is presented with an admittance $\underline{Y}_l = 1/j\omega L_r$ where L_r is the inductance of the reactor.

4) Offshore wind power plant: It is assumed that the power plant controller of the wind power plant ensures that the total reactive power injection of the wind turbines at the transformer is 0 [7]. If the wind turbines were expected to absorb reactive power for compensation purposes, the transformer would need to be oversized accordingly.

5) Grid: The grid is modeled with a Thevenin's equivalent ($\underline{U}_{grid}$) and impedance ($\underline{Z}_{grid} = R_{grid} + j\omega L_{grid}$) which is calculated from the short circuit ratio (SCR) and X/R ratio.

B. Power flow

In Fig. 4, the model for power flow calculation is presented. Depending on the number of cables and transformers, the final impedances and admittances are presented as the impedances of the single elements parameters connected in parallel in per units.

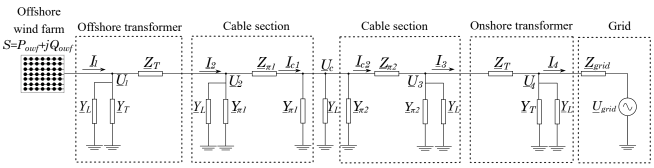


Fig. 4. Single-line model for power flow calculation

The power flow equations are:

$$\begin{aligned} \underline{U}_1\underline{I}_1^* &= P_{owf} + jQ_{owf} \\ \underline{U}_1 &= \underline{U}_2 + \underline{Z}_{T1}\underline{I}_2 \\ \underline{I}_1 &= (\underline{Y}_{T1} + \underline{Y}_l)\underline{U}_1 + \underline{I}_2 \\ \underline{U}_2 &= \underline{U}_c + \underline{Z}_{\pi1}\underline{I}_{c1} \\ \underline{I}_2 &= (\underline{Y}_{\pi1} + \underline{Y}_l)\underline{U}_2 + \underline{I}_{c1} \\ \underline{I}_{c1} &= (\underline{Y}_{\pi1} + \underline{Y}_l)\underline{U}_c + \underline{I}_{c2} \\ \underline{U}_c &= \underline{U}_3 + \underline{Z}_{\pi2}\underline{I}_{c2} \\ \underline{I}_{c2} &= (\underline{Y}_{\pi2} + \underline{Y}_l)\underline{U}_c + \underline{I}_3 \\ \underline{U}_3 &= \underline{U}_4 + \underline{Z}_{T2}\underline{I}_3 \\ \underline{I}_3 &= (\underline{Y}_{T2} + \underline{Y}_l)\underline{U}_4 + \underline{I}_4 \\ \underline{U}_4 &= \underline{U}_{grid} + \underline{Z}_{grid}\underline{I}_4 \end{aligned} \tag{7}$$

The power losses in the system are expressed as ohmic losses, i.e. $\sum R_i I_i^2$ where R_i is the equivalent resistance of element i and I_i is the current through that element. Losses from all elements are considered as follows:

$$P_{loss} = P_{loss}^{onTR} + P_{loss}^{cb} + P_{loss}^{offTR} \quad [MW] \tag{8}$$

where is P_{loss}^{onTR} are losses of onshore transformers, P_{loss}^{cb} are losses of the cable and P_{loss}^{offTR} are losses of offshore transformers.

C. Cost modelling

The expressions of the costs are presented in the following subsections:

1) HVAC components: Cost of cable C_{cb} is presented [8]:

$$C_{cb} = \frac{(A + Be^{CS_{rated}} + D) \cdot (9n + 1)}{10E} \cdot l \tag{9}$$

where the constant values (A, B, C, D, E) are defined in Table I, which are dependent on the cable voltage [9], [10] and S_{rated} is the rated apparent power of the cable in [MW]. Cost of switchgears is calculated [11]:

$$C_{gis} = 0.0117 \cdot U_{rms} + 0.0231 \tag{10}$$

where U_{rms} is the transmission voltage [kV].

Cost of transformer is dependent of its rated power S_{TR} [MVA] and assumed based on [12]

$$C_{TR} = 0.0427 \cdot S_{TR}^{0.7513} \tag{11}$$

The substation cost is defined [10], [13]:

$$C_{ss} = 2.534 + 0.0887 \cdot P_{owf} \tag{12}$$

where P_{owf} is the rated power of the offshore wind power plant [MW]. Investment costs of compensation equipment is derived from [2], [11], [14], [15]:

$$C_{react} = K \cdot Q_l + P \tag{13}$$

The constant values (K, P) are defined in Table II, which are dependent on the location of reactor, $Q_l = Y_l \cdot U_{rms,nom}^2$ is reactive power compensated by the reactors, Y_l is mentioned in Section III-A3 and $U_{rms,nom}$ is the nominal transmission voltage. Power losses C_{loss} are presented in form of cost:

$$C_{loss} = 8760 \cdot c_{owf} \cdot t_{owf} \cdot C_{energy} \cdot P_{loss} \tag{14}$$

where c_{owf} is the capacity factor of the wind power plant, t_{owf} is the life time of the wind power plant in years, C_{energy} is the cost of energy in €/MWh and P_{loss} is defined in Equation 8.

TABLE I
COEFFICIENTS FOR XLPE SUBMARINE AC CABLES [9], [10]

	30 kV	70 kV	150 kV	220 kV	400 kV
A	0.411	0.688	1.971	3.181	5.8038
B	0.596	0.625	0.209	0.11	0.044525
C	0.041	0.0166	0.0166	0.0116	0.0072
D			$17 \cdot 10^4$		
E			8.98		

TABLE II
COEFFICIENTS FOR SHUNT REACTORS

Location	K	P
Onshore	4.2	0.8283
Offshore	6.096	1.279
Middle	18.096	1.543

2) HVDC components: Cost of cable C_{cb} is presented in [8] including investment and installation cost:

$$C_{cb} = \frac{(A + BP_{rated} + D) \cdot (9n + 1)}{10E} \cdot l \tag{15}$$

where the constant values (A, B, D, E) are defined in Table III which are dependent on the cable voltage and l is the transmission distance [km]. The VSC converter offshore and onshore are defined in the following equations [8]:

$$C_{dc,off} = 42 + 27 \cdot \frac{P_{N,conv}}{300} \tag{16}$$

$$C_{dc,on} = 18 + 27 \cdot \frac{P_{N,conv}}{300} \tag{17}$$

where $P_{N,conv}$ is the rated power of the converter. The cost of VSC converter losses is evaluated as:

$$C_{loss,VSC} = P_{loss,VSC} \cdot 8760 \cdot c_{owf} \cdot t_{owf} \cdot C_{energy} \tag{18}$$

where it is assumed that the VSC losses are 1% of the converted power, obtaining the following equation:

$$P_{loss,VSC} = 0.1 \cdot c_{owf} \cdot P_{owf} \tag{19}$$

From data in [11], it is evaluated that an HVDC substation costs are from 57,9% to 115,4% higher than an HVAC one for the same rated power due to needed additional elements. An average number of 85% is taken in account and the cost function is obtained:

$$C_{ss} = 1.85 \cdot (2.534 + 0.0887 \cdot P_{owf}) \tag{20}$$

The equations for the cost of the transformers and power losses remain the same as in Section III-C1.

TABLE III
COEFFICIENTS FOR DC CABLES [9], [10]

Voltage levels	±80 kV	±150 kV	±220 kV
A	$-0.25179 \cdot 10^6$	$-0.1 \cdot 10^6$	$0.286 \cdot 10^6$
B	0.03198	0.0164	0.00969
D		$22 \cdot 10^4$	
E		8.98	

IV. METHODOLOGY

A. Methodology diagram

The flow diagram for the optimisation problem is presented in Fig. 5.

B. Multi-objective optimisation

A general multi-objective optimisation problem (also called multi-criteria optimisation, multi-performance or vector optimisation problem) is defined as the minimization (or maximization) of the objective function set $F(x) = (f_1(x), ..., f_k(x))$ subject to inequality constraints $g_i(x) \leq 0, i = 1, ..., m$, and equality constraints $h_j(x) = 0, j = 1, ..., p$. where x is an n-dimensional decision variable vector $x = (x_1, ..., x_n)$ [16].

C. Linear weighted method

Linear weighted method is one of the most widespread methods to solve multi-objective programming problems,

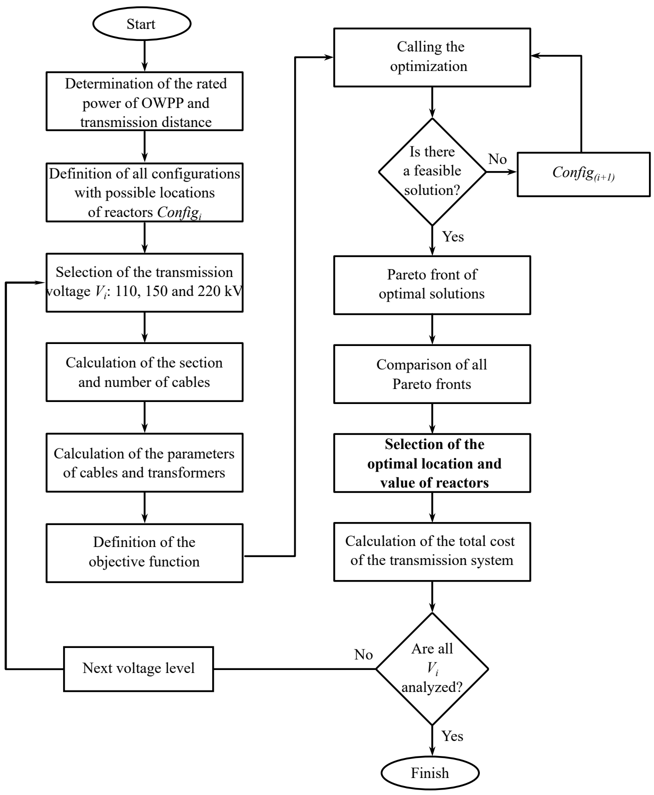


Fig. 5. Model for power flow calculation

and the original problem could be converted to the following form:

$$\min[w_1 f_1(x) + w_2 f_2(x) + ... + w_k f_k(x)] \tag{21}$$

where w_i represents the weight of corresponding objective function for decision makers. If $\sum_{i=1}^{k} w_i = 1, w_i \geq 0$, the weighted sum is said to be a convex combination of objectives. Each of w_i defines a particular optimal solution point in a Pareto front. Thus, w_i values choice is very important to achieve the most convenient optimal result. However the decision maker, in order to choose the coefficients, must have a clear perception of how this choice influence optimal points [17].

D. Optimisation problem

The multi-objective optimisation is defined as [16]:

$$\begin{aligned} F_{obj}(x) &= w_1 f_1(x) + w_2 f_2(x) \\ \text{subject to} \quad g_i(x) &\leq 0, i = 1, ..., k \\ h_j(x) &= 0, j = 1, ..., p \end{aligned} \tag{22}$$

E. Objective functions

The objective functions are defined below:

$$\begin{aligned} f_1 &= \sum_{i=1}^{k} P_{loss_i} [MW] \\ f_2 &= \sum_{i=1}^{m} C_{react_i} [M€] \end{aligned} \tag{23}$$

where P_{loss_i} are the power losses of element i, k is the number of devices, C_{react_i} is the shunt reactor investment cost and m is the number of reactors.

F. Variable vector

The variable vector contains voltages and currents in all nodes and admittances of added reactors:

$$\begin{aligned} x = [&\underline{U}_1, \underline{U}_2, \underline{U}_c, \underline{U}_3, \underline{U}_4, \underline{I}_1, \underline{I}_2, ... \\ &...\underline{I}_{c1}, \underline{I}_{c2}, \underline{I}_3, \underline{I}_4, \underline{Y}_{l1}, \underline{Y}_{l2}, \underline{Y}_{l3}] \end{aligned} \tag{24}$$

G. Equality constraints

The equality constraints $h_j(x)$ are presented by power flow equations in (7) in the following form $Ax = 0$, where A is a matrix containing impedances and admittances of elements.

H. Inequality constraints

The inequality constraints $g_i(x)$ are defined based on voltage, current and reactive power limits. The voltages and currents are restricted by their lower and upper limits in every node as follows:

$$\begin{aligned} U_{i,min} \leq U_i &\leq U_{i,max} \\ I_i &\leq I_{rated} \end{aligned} \tag{25}$$

As well the reactive power delivered to the grid is restricted:

$$Q_{grid,min} \leq Q_{grid} \leq Q_{grid,max} \tag{26}$$

where U_i is the voltage of node i, $U_{i,min}$ and $U_{i,max}$ are the minimum and maximum limits of voltage of node i and are set on $\pm 10\%$. I_i is the current of node i, I_{rated} the rated current of the cable, Q_{grid} the delivered reactive power to the grid and Q_{min} and Q_{max} are the minimum and maximum limits of reactive power delivered to the grid ($\pm 0.1\%$).

V. Optimisation results

The Pareto front considering all combinations of possible reactors locations indicated in Fig. 2 for a voltage of 110 kV and 100 km distance is presented below in Fig. 6. The analysis is done with one (black square), two (red circle) and three (blue diamond) reactors. The cases of interest is the one that has the lowest cost and as well lowest power losses which indicate on the cases C6 and C13. It is evident that giving more priority to power losses it is better to have three reactors (C13).

As C6 is already included in case C13, the voltages and currents for case C13 in every node are shown in Fig. 7, as well the reactive power produced by the reactors. It could be seen that for some weights values, the current reaches the upper limited value, which means that it is in saturation. In these cases, corresponding weight values would not be in consideration in choosing the optimal solution.

The point where the configuration with 3 reactors are more valued could be recognized by increasing the transmission voltage and the distance. It is presented in Fig. 8. Using 100 €/MWh [18] as the average levelised cost of energy in Germany, the cost of power losses and reactive power compensation is shown for distance of 150 km and for voltages 110, 150 and 220 kV for particular locations C6 and C13. For the voltage 220 kV, the solution for configuration C6 is not feasible.

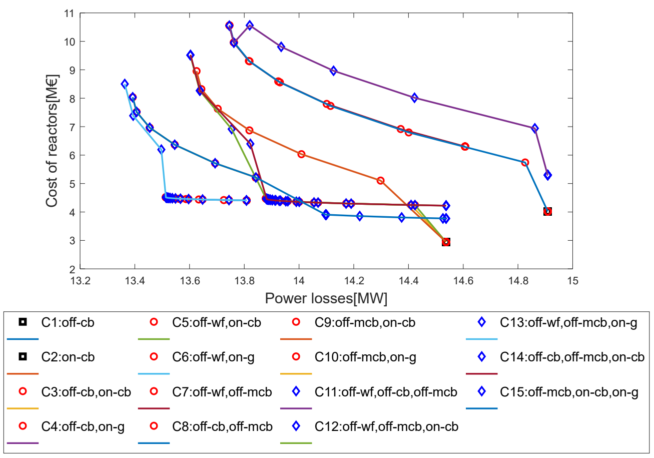
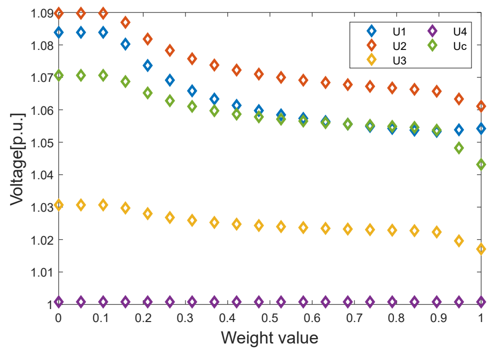
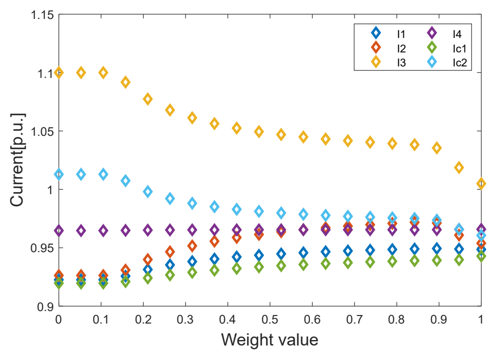


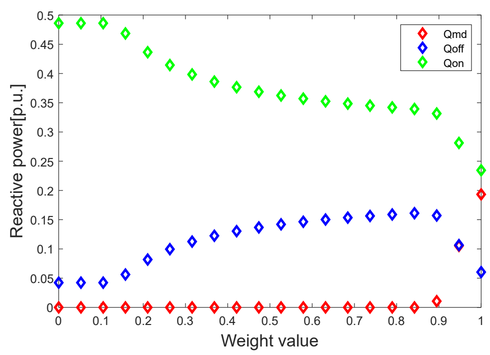
Fig. 6. Pareto front for power losses and cost of reactors



(a)



(b)



(c)

Fig. 7. Configuration 13: off-wf,off-mcb,on-g: a) Voltages, b) Currents, c) Reactive power of reactors

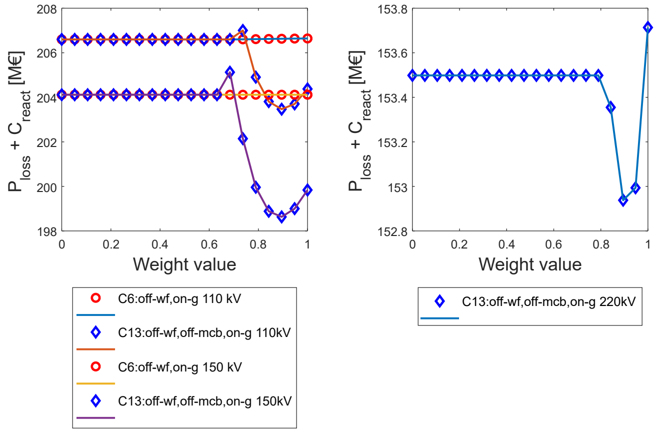


Fig. 8. Cost of power losses and reactive power compensation for different voltages

In Fig. 8, it can be seen that from weight value $w_1 = 0.7845$ to value $w_1 = 1$ there is a cost difference between having two and three reactors. To show this difference, sensitivity cost analysis is done for all 3 voltages for distance of 150 km in Fig. 9 for $w_1 = 0.8947$. Finally, the most optimal solution off-wf, off-mcb, on-g with the transmission voltage 220 kV is determined. The comparison with HVDC system is shown in Fig. 10. It is seen that for a wind power plant of 400 MW with transmission distance of 150 km, HVAC system with appropriate power compensation is still cheaper than HVDC system.

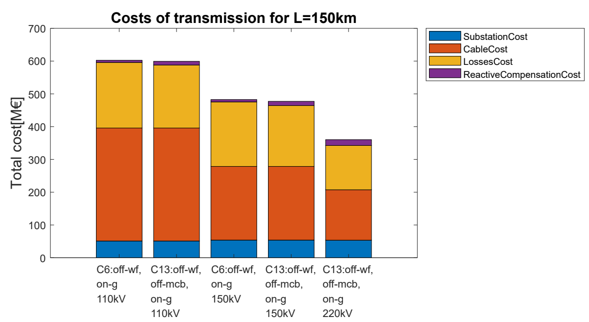


Fig. 9. Comparison of total transmission system cost for two configurations

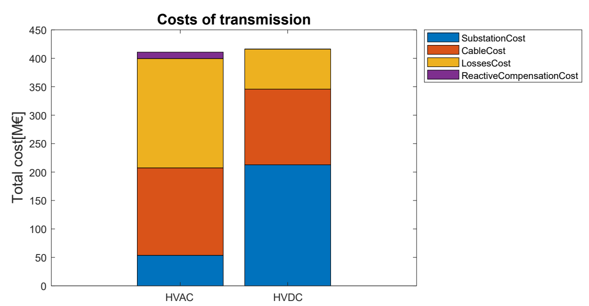


Fig. 10. Comparison of HVAC and HVDC system

VI. Conclusion

The application of multi-objective optimisation for determining the optimal location and value of shunt reactors for reactive power compensation has been presented. The reduction in the power system loss, minimized cost of reactive power compensation as well the improvement of voltage profile were analysed. The results show it is better to have two reactors for low voltages and short distances, while by increasing both of these quantities, the configuration with three reactors provides lower power losses and total cost.

ACKNOWLEDGMENTS

This project has received funding from the European Union's Horizon 2020 research and innovation programme under the Marie Sklodowska-Curie grant agreement no. 765585. This presentation reflects only the authors view. The Research Executive Agency and European Commission are not responsible for any use that may be made of the information it contains. Eduardo Prieto-Araujo is a Lecturer of the Serra Húnter programme.

REFERENCES

[1] Renewable_UK, "Offshore Wind Project Timelines Offshore Wind Projects," Tech. Rep., 2016.

[2] Z. Chen, D. Xiaowei, and Z. Guiping, "Optimisation of reactive power compensation of HVAC cable in off-shore wind power plant," *IET Renewable Power Generation*, vol. 9, no. 7, pp. 857–863, 2015.

[3] K. B. D. Elliott, S. Finney, R. Adapa, C. Brozio, J. Yu, and K. Hussain, "A comparison of AC and HVDC options for the connection of offshore wind generation in Great Britain," *IEEE Transactions on Power Delivery*, pp. 1–12, 2016.

[4] A. R. Henderson, L. Greedy, F. Spinato, and C. A. Morgan, "Optimising Redundancy of Offshore Electrical Infrastructure Assets by Assessment of Overall Economic Cost," *European Offshore Wind Energy Conference*, no. September 2009, pp. 14–16, 2009.

[5] Van Hertem, Dirk, Gomis-Bellmunt, Oriol, and J. Liang, "HVDC grids: for offshore and supergrid of the future," *Wiley*, p. 528, 2016.

[6] J. Machowski, J. W.Bialek, and J. R.Bumby, *Power System Dynamics, Stability and Control*, 2012.

[7] National Grid Electricity System Operator Limited, "The Grid code — National Grid," Tech. Rep. 5, 2019.

[8] H. Ergun, S. Member, D. V. Hertem, S. Member, and R. Belmans, "Transmission System Topology Optimization for Large-Scale Offshore Wind Integration HE European Union has set ambitious goals regarding," *IEEE Transactions on Sustainable Energy*, vol. 3, no. 4, pp. 908–917, 2012.

[9] B. Van Eeckhout, D. Van Hertem, M. Reza, K. Srivastava, and R. Belmans, "Economic comparison of VSC HVDC and HVAC as transmission system for a 300MW offshore wind farm," *European Transactions on Electrical Power*, vol. 20, no. 5, pp. 661–671, 2010.

[10] S. Lundberg, "Performance comparison of wind park configurations," *Power Engineering*, 2003.

[11] ENTSO-E AISBL, "Offshore Transmission Technology," *Entso-e*, pp. 1–44, 2011.

[12] L. P. Lazaridis, "Economic Comparison of HVAC and HVDC Solutions for Large Offshore Wind Farms under Special Consideration of Reliability," *Electrical Engineering*, vol. 12, no. 4, pp. 203–208, 2005.

[13] M. Dicorato, G. Forte, M. Pisani, and M. Trovato, "Guidelines for assessment of investment cost for offshore wind generation," *Renewable Energy*, vol. 36, no. 8, pp. 2043–2051, 2011.

[14] X. Xiang, M. M. C. Merlin, and T. C. Green, "Cost Analysis and Comparison of HVAC , LFAC and HVDC for Offshore Wind Power Connection," *IET 12th International Conference on AC andDC Transmission*, pp. 14–19, 2016.

[15] Y. Deng, "1 . Reactive Power Compensation of Transmission Lines," *Power*, pp. 1–14, 2005.

[16] C.A. Coello Coello, "Theoretical and numerical constraint-handling techniques used with evolutionary algorithms: a survey of the state of the art," *Computer Methods in Applied Mechanics and Engineering*, vol. 191, no. 11-12, pp. 1245–1287, 2002.

[17] L. Zadeh, "Optimality and Non-Scalar-Valued Performance Criteria*," *IEEE Transcations on Automatic Control*, no. 4, 1963.

[18] C. Kost, T. Schlegl, J. Thomsen, S. Nold, J. Mayer, N. Hartmann, C. Senkpiel, S. Philipps, S. Lude, and N. Saad, "Fraunhofer ISE: Levelized Cost of Electricity - Renewable Energy Technologies, March 2018," *Fraunhofer ISE: Levelized Cost of Electricity - Renewable Energy Technologies,*, no. March, 2018.