Shear fatigue strength of reinforced concrete members without transverse reinforcement according to the Compression Chord Capacity Model

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Abstract

Although the shear fatigue behaviour of elements without shear reinforcement has been studied for a while, there is still a need for a simple mechanical model that evaluates the shear fatigue strength. The Compression Chord Capacity Model (CCCM), previously developed by the authors, is a simplified model for the shear strength prediction of reinforced and prestressed concrete members with and without transverse reinforcement, with I, T or rectangular cross-sections. This model represents a useful tool for structural design and assessment in engineering practice. In this paper, the CCCM has been extended to assess the fatigue shear strength of RC elements. This extension is consistent since the existing observed fatigue failure modes show similarities with the initial assumption of the model, which considers that the member shear strength is controlled by the shear capacity of the flexural compression chord. Three different approaches to take into account the influence of the fatigue have been combined with the CCCM. In general, a good performance of the model combined with the three different approaches was observed when analysing the ratio V_{test}/V_{pred} through a database of 87 tests previously published by other authors, showing a low scatter (less than 15 %) and a significant safety given by the minimum ratio and the 5th percentile.

Keywords: fatigue, reinforced concrete, shear strength, beam, compression chord, mechanical model.

1. Introduction

Shear fatigue failures in reinforced concrete elements without shear reinforcement may govern the design in structures subjected to a high number of load cycles as wind towers, offshore structures, bridge decks, precast slabs for ballastless tracks and so forth. A number of approaches to assess or design the fatigue shear strength are based on the value of the predicted monotonic shear strength. In this sense, as the shear strength of reinforced concrete (RC) elements is still a hot topic, different mechanical approaches are being discussed for future codes [1]. However, most of the current design equations for shear strength of RC elements included in different codes are empirical, they were initially derived on the basis of a number of experimental data with which they showed a good correlation, for example, EC- 2 [2] is based on Zsutty [3], and ACI 318-14 [4] is based on [5].

Since the beginning of the study of the shear fatigue behaviour of RC elements without shear reinforcement, failure modes have been well comprehended and described. In 1958, Chang and Kesler [6,7] classified the different modes of fatigue failure in two groups: the first one is related to the fatigue of the longitudinal reinforcement in tension (Fig. 1a), and the second is described as the failures whereas the compression zone at the top of the diagonal or shear crack, subjected to combined compression and shear, became so small to resist the applied load. In the last group, the authors differentiated between the failures where the compression zone failed as soon as the diagonal shear crack was formed (Fig. 1b), and the failures where the formation of the diagonal shear crack preceded a number of cycles to the failure of the compression zone (Fig. 1c). In any case, the crack patterns of the failures represented by Fig. 1b are similar to the cracks presented in Fig. 1c and they are also similar with those observed in tests under quasi-static loads.

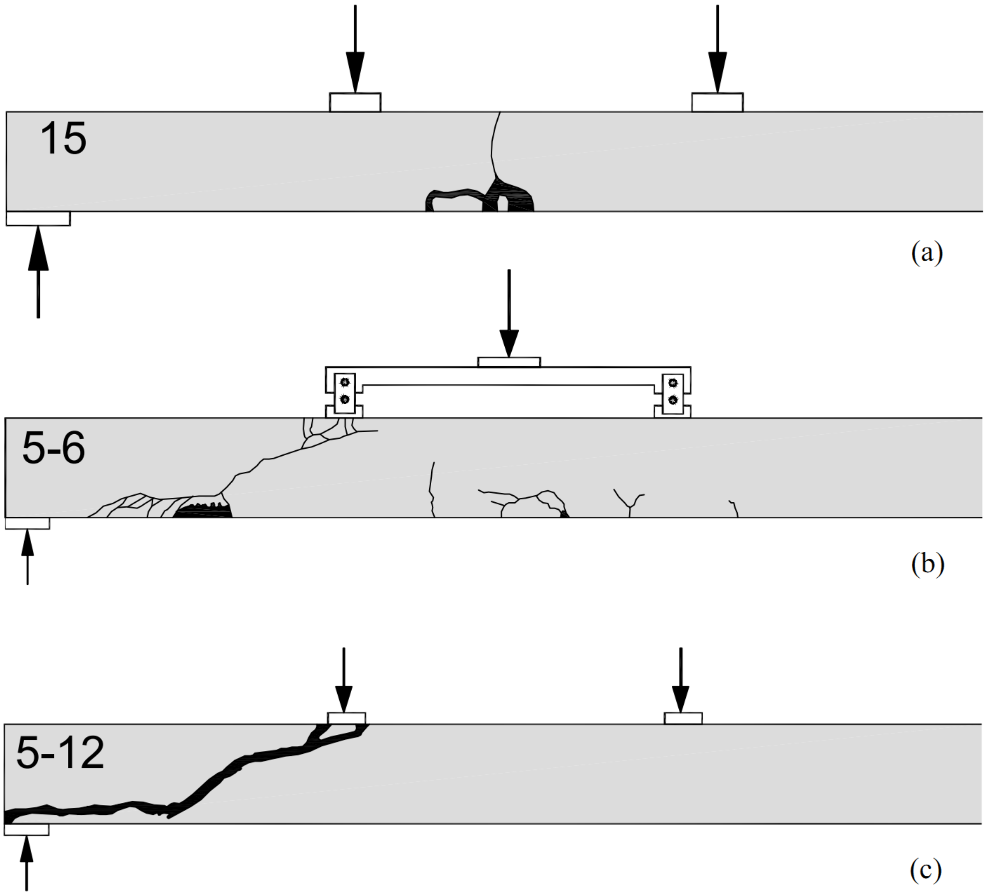

Fig. 1. Types of fatigue failure defined by Chang and Kesler [6,7].

It should be mentioned that during their experimental tests, Chang and Kesler [6,7] observed splitting cracks in some cases at the level of the longitudinal reinforcement (Fig. 1c), also called by other authors delamination cracks [8], and concluded that this splitting behaviour was considered as a secondary failure due to the fact that the crack was developed after the beam had “failed”.

The modes of failure and crack pattern described and observed in Fig. 1b-c were adopted or observed by different researchers, [8–10] among others. Frey and Thurlimann [10] observed a critical shear crack that crossed the bending cracks, and the large width of the diagonal crack did not allow any aggregate interlock, so the beam failed due to fatigue of the compression chord.

Rombach and Kohl performed 20 fatigue tests on RC beams without shear reinforcement [11] and concluded that most beams were able to carry the cyclic loads even after a wide shear crack ($\omega > 1$ mm) had formed. In such cases, load transfer through crack friction was no longer possible. Thus, the compression zone appeared to be the dominating mechanism to carry the shear load at the ultimate limit state. This assumption was confirmed in the same research through finite element simulations which shown that between 76 % and 88 % of the total shear strength was transferred through the compression zone at failure.

Gallego et al. [12] presented a shear fatigue mechanical model based on the detailed study of a fatigue shear test in which the formation of the diagonal crack did not suddenly lead to failure (as the case in beam Fig. 1c) and the test continued with the propagation of the crack in both senses: towards the load application point and towards the support. Fatigue failure was finally caused by the destruction of the compression zone that had been reduced due to the propagation of the upper branch of the shear crack. The proposed model showed to be in good agreement with the experimental data and it is able to estimate the number of cycles before failure. However, the goal of the proposed model was to provide a physical understanding of the shear fatigue process but its application for daily engineering practice is too complex.

Fernández-Ruiz et al. [8] combined the principles proposed by Gallego et al. [12] with the Critical Shear Crack Theory (CSCT) [13]. The proposal showed consistent agreement to test data. Note that CSCT was initially based on a data fit as commented by Campana [14]. However, their authors demonstrated, in 2015, that the failure criterion is mainly related with aggregate interlock, dowel effect, residual tensile strength, and the arch action at the concrete chord [15]. In any case, the aggregate interlock component was clearly not observed in the tests performed by Frey and Thurimann [10] or Rombach and Kohl [11]. For this reason, it seems more appropriate to develop a model where the failure criterion fits with the one observed in shear fatigue tests.

In this paper, the combination of the Compression Chord Capacity Model (CCCM) [16] with different approaches for fatigue is proposed as a valid alternative to assess the fatigue shear strength of RC elements. The use of the CCCM seems to be consistent and fit with the usual fatigue shear failures due to the similarity between the description of the failures and the initial assumption done for the derivation of the CCCM, as will be explained. The mechanical background of the CCCM will allow combining it with a fatigue model for the concrete tensile strength, among other possibilities. The use of the CCCM leads to a simple design procedure, which have an excellent agreement with available test results.

2. Shear strength under monotonic loads

The shear strength of RC and PC members is still a hot research topic. In this paper three models will be used, the Compression Chord Capacity Model (CCCM) [16], the formulation included in current Eurocode 2 [2], and the Level of Approach II for members without stirrups included in Model Code 2010 [17].

2.1 Compression chord capacity model (CCCM)

The CCCM is a simplified model derived from a more general mechanical model [18]. The original or background model, referred also as the Multi-Action Shear Model (MASM), takes explicitly into account the different commonly accepted shear transfer actions: (a) the shear transferred by the un-cracked concrete chord, also called arching action; (b) the shear transferred across web cracks (through residual tensile stresses at MASM); (c) the dowel action in the longitudinal reinforcement if shear reinforcement is provided; (d) tension in the vertical, or inclined, steel reinforcement (stirrups) if it exists; and the interaction between them. The derivation of close-form expressions for the four shear transfer actions is presented in [18].

In this model, the critical shear crack is considered to initiate at a point where, at failure, the bending moment equals the flexural cracking moment of the beam, and reaches the neutral axis at a distance of 0.85d from its initiation (Fig. 2). If no other premature failure takes place, the member shear strength will be controlled by the shear capacity of the flexural compression chord, as it is the last element to initiate softening, reducing its capacity as the crack propagates (Fig. 2b). Associating the initiation of the shear failure with the stage represented by point A in Fig. 2b [19] results in significant simplification of the problem without significant loss of accuracy—the association allows the formulation of a failure criterion to be expressed in terms of concrete stresses in the compression chord. Failure is considered to occur when, at any point in the compression chord, the principal stresses (σ_1 and σ_2) reach the Kupfer's biaxial failure envelope [20] in the compression-tension branch, Fig. 2d. Moreover, the failure takes place generally for compressive stresses less than $0.5 \cdot f_{cm}$ [18], and failure is more conditioned by the concrete tensile strength rather than the compressive strength. In any case, this failure criterion depends on f_{cm} and f_{ct}, which have less scatter than other parameters needed in kinematical criteria, such as crack openings, relative sliding between their faces, fracture energy, and so forth. Although it had been clearly documented that resisting actions that are primary in some cases, may be secondary in other cases—depending on the load conditions, beam geometry, and concrete type, among other factors—it was necessary to simplify the MASM model to obtain more compact equations for practising engineers, avoiding the use of four different equations to compute the contribution to the shear strength of the concrete and the stirrups. In this framework, the MASM was transparently simplified taken into account both the Eurocode 2 and the ACI perspectives [16,21]. The main premise of the MASM simplification is that, at failure, both the residual tensile stresses and the dowel action are small compared to the shear resisted by the un-cracked concrete chord and/or the shear reinforcement contribution. For simplification purposes, average safety values of the residual tensile stresses and the dowel action transfer actions were considered.

The resulting expression, particularized for RC members without transverse reinforcement, is presented in Eq. (1). However, in some members, e.g. one-way slabs with low levels of longitudinal reinforcement, the shear contribution due to residual stresses along the crack may be comparable to the contribution of the uncracked zone, since *x/d* is small. For this situation, a minimum shear strength was proposed, see Eq. (2). $V_{cu,min}$ takes explicitly into account the residual tensile stresses action in the case of a reduced un-cracked compression chord transfer action. The complete derivation of Eq. (1) and Eq. (2) may be found in [16]. Note that the CCCM was derived in a general way, and the model is valid for RC and PC members, with or without transverse reinforcement, with rectangular or T- or I-cross sections, and it can consider the influence of internal tensile stresses, or it can be applied for steel fibre reinforced concrete members or beams reinforced with FRP bars [22].

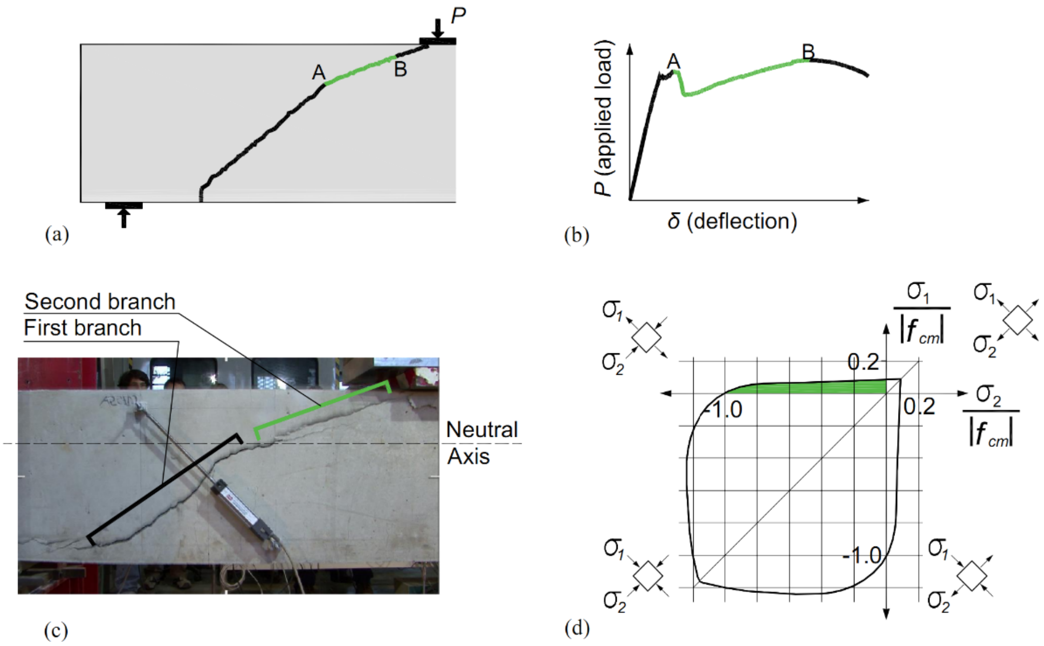

Fig. 2. Critical shear crack propagation: a) qualitative scheme of crack trajectory; b) schematic load-displacement curve; c) critical crack in a tested beam; d) adopted failure envelope for concrete under biaxial stress state.

Table 1. Summary of the CCCM equations for rectangular beams without shear reinforcement.

Equation	Expressions	
Concrete contribution	$V_{cu} = 0.3\zeta \frac{x}{d} f_{cm}^{2/3} bd \not< V_{cu,min}$	(1)
Minimum shear strength	$V_{cu,min} = 0.25\left(\zeta K_c + \frac{20}{d_0}\right) f_{cm}^{2/3} bd$	(2)
Factors	**Expressions**	
Size and slenderness effect	$\zeta = \frac{2}{\sqrt{1+\frac{d_0}{200}}}\left(\frac{d}{a}\right)^{0.2} \not< 0.45$	(3)
Relative neutral axis depth	$\frac{x}{d} = \alpha_e \rho_l \left(1+\sqrt{1+\frac{2}{\alpha_e \rho_l}}\right) \approx 0.75\left(\alpha_e \rho_l\right)^{1/3}$	(4)

Taking into account that $0.30 \cdot f_{cm}^{2/3}$ is equal to the concrete tensile strength, f_{ct}, Eqs. (1) and (2) may be rewritten as Eqs. (5) and (6).

$$V_{cu} = \zeta \frac{x}{d} f_{ct} bd \not< V_{cu,\min} \quad (5)$$

$$V_{cu,\min} = 0.833\left(\zeta K_c + \frac{20}{d_0}\right) f_{ct} bd \quad (6)$$

2.2 Eurocode 2

Eurocode 2 [2] adopts, for RC members without shear reinforcement, the empirical formulation given by Eqs. (7) and (8) for the particular case of beams without axial forces and removing the material safety factor. In the following, f_{ck} will be considered equal to f_{cm}.

$$V_{cu} = 0.18k\left(100\rho_l f_{ck}\right)^{1/3} bd \not< V_{cu,\min} \quad (7)$$

$$V_{cu,\min} = 0.035k^{3/2} f_{ck}^{1/2} bd \quad (8)$$

2.3 Model Code 2010 – Level II of approximation

The shear strength prediction according to Model Code 2010 depends on the level of approximation. For members without shear reinforcement, the second level of approximation offers the best results, being level I a simplification. The equations needed to obtain the shear strength according to the level II of approximation are presented in Eqs. (9)-(12):

$$V_{R,c} = k_v\sqrt{f_{ck}}zb_w \quad (9)$$

$$k_v = \frac{0.4}{1+1500\varepsilon_x}\frac{1300}{1000+k_{dg}z} \quad (10)$$

$$\varepsilon_x = \frac{\frac{M}{z}+V}{2E_sA_s} \quad (11)$$

$$k_{dg} = \frac{32}{16+d_g} \nless 0.75 \quad (12)$$

Level II of approximation of the Model Code 2010 requires an iterative procedure when applied to experimental data because the term ε_x depends on the bending moment (*M*) and shear force (*V*) at the ULS. In any case, this procedure does not require iterating for designing. In the following, f_{ck} will be considered equal to f_{cm}.

2.4 Comparison for quasi-static tests

The comparison between the predictions by Eurocode 2, Model Code 2010 (level of approximation II) and the CCCM and the experimental results of reinforced and prestressed concrete beams failing in shear is presented in [16,23]. For RC members without stirrups, a database developed by ACI-DafStb [24], and previously published, was used to perform the comparisons. The main results are presented in Fig. 3. For the 784 beams included in the database, the average value of the ratio V_{test}/V_{pred} is equal to 1.17 for the CCCM predictions, 1.22 for MC2010 and 1.10 for EC-2. The Coefficients of Variation (CoV) equal 18.5 %, 22.8 % and 27.9 % respectively. Despite the lower scatter of the CCCM predictions, the most important difference in the correlations is that EC-2 tends to be more unsafe for beams with high effective depths compared to CCCM or MC2010 (Fig. 3).

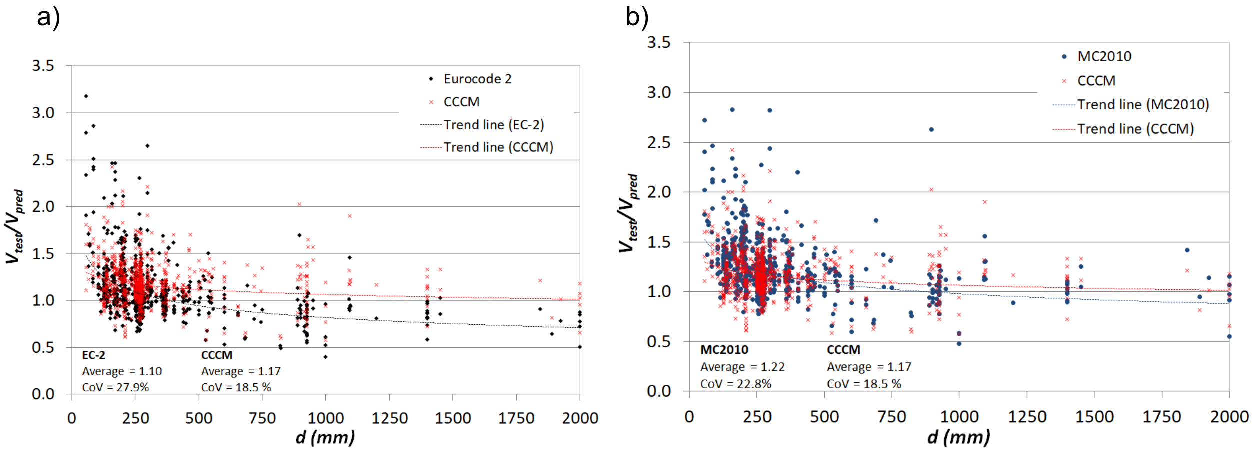

Fig. 3. Correlation between the predictions and the experimental results as a function of the effective depth, d: a) Eurocode 2 vs. CCCM; b) MC2010 vs. CCCM.

3. Shear fatigue strength approaches for RC members

The Eurocode 2 [2] approach for the verification of the fatigue due to shear effects is based on a fatigue model initially derived for compression stresses. For concrete under compression the following condition must be verified (Section 6.8.7 of current EC-2):

$$\frac{\sigma_{c,\max}}{f_{cd,fat}} \leq 0.5 + 0.45\frac{\sigma_{c,\min}}{f_{cd,fat}} \leq \begin{Bmatrix} 0.9\, for\ f_{ck} \leq 50\, N/mm^2 \\ 0.8\, for\ f_{ck} > 50\, N/mm^2 \end{Bmatrix} \tag{13}$$

Eurocode 2 also proposes the use of Eq. (13) to compute the fatigue strength of members without shear reinforcement (when the maximum and the minimum shear force have the same sign). For that purpose, Eq. (13) is rewritten as presented in Eq. (14) and Figure 4, where V_{ref} is the shear strength of the beam subjected to quasi-static load and equal to V_c given by Eq. (7).

$$\frac{V_{\max}}{V_{ref}} \leq 0.5 + 0.45\frac{V_{\min}}{V_{ref}} \leq \begin{Bmatrix} 0.9\ up\ to\ C50/60 \\ 0.8\ greater\ than\ C55/67 \end{Bmatrix} \tag{14}$$

Equations (13) and (14) are the particularization of an *S-N* curve for a fixed number of load cycles. As discussed in [12], the general form of Eq. (14) for any number of cycles is presented in Eq. (15).

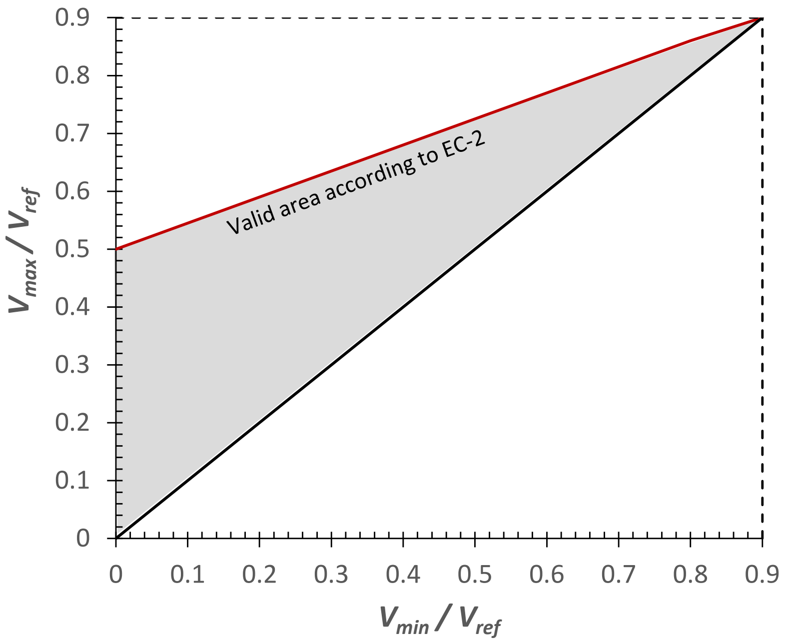

Fig. 4. Goodman diagram for cases up to C50/60 (Eq. 14). In grey, valid range of shear forces according to EC-2.

$$\frac{V_{\max}}{V_{ref}} \leq C + \frac{\log N}{m}\left(\frac{V_{\min}}{V_{ref}} - C\right) \not< 0.5 \tag{15}$$

where C and m are equal to 0.9 and 15 respectively, and N is equal to 5 million load cycles to derive Eq. (14).

Eurocode 2 does not include any formulation for the fatigue of concrete under tension stresses. However, for pure tension and tension-compression with relatively high tensile stresses, Model Code 2010 [17] proposes the following *S/N* curve:

$$\log N = 12\left(1 - \frac{\sigma_{ct,\max}}{f_{ct}}\right) \tag{16}$$

Consequently, for MC2010, the tensile strength diminishes for increasing values of N, according to Eq. (17):

$$\sigma_{ct,\max} = f_{ct}\left(1 - \frac{\log N}{12}\right) \tag{17}$$

Model Code 2010 presents also a specific *S/N* curve for the shear design of members without shear reinforcement, presented in Eq. (18). In this case, the fatigue requirement will be met, if under cyclic loading, the number of cycles corresponding to the required service life is smaller than or equal to the number of cycles to failure, N, given by Eq. (18).

$$\log N = 10\left(1 - \frac{V_{\max}}{V_{ref}}\right) \tag{18}$$

where V_{max} is the maximum shear force under the relevant representative values of permanent loads including prestress and maximum cyclic loading, and V_{ref} is the design shear resistance under monotonic loads, given by Eq. (9) for Model Code 2010.

Note that the shear fatigue verification equation presented in EC-2, Eq. (14), is only valid for $N = 5 \cdot 10^6$ cycles, meanwhile that shear fatigue verification given in MC-2010 does not consider the amplitude of the cycles, i.e. the ratio between V_{min} and V_{max} or V_{ref}. To overcome these limitations, Fernández-Ruiz et al. [8] derived a design approach by using the principles of Fracture Mechanics applied to quasi-brittle materials, see Eq. (19).

$$\frac{V_{max}}{V_{ref}} = \eta \frac{1}{R + N^{1/m}(1-R)} \nless 0.5 \tag{19}$$

where R is equal to the ratio V_{min}/V_{max}, m is an empirically derived coefficient equal to 17, and the threshold of 0.5 also refer to the average response of the test results, and the authors recognize that these values could be adapted, if necessary, to respect a target safety level. The term η is considered equal to 1.10 [8], to take into account that the quasi-static reference strength, V_{ref}, is aimed at quasi-static failures in cases when loading duration is about one hour time (typical testing time). However, tests failing in fatigue loading are typically performed at much higher loading rates, e.g. 1 Hz. This implies that for $N \rightarrow 1$, the observed strength at higher loading rates should be higher than the corresponding one for a reference (quasi-static) test. The value $\eta = 1.10$ is equivalent to the MC-2010 increase on the concrete compression strength for tests performed at a loading rate of 1 Hz with respect to 1 hour-time failure. In any case, the authors recognized in the original paper that a more refined investigation of this parameter will be necessary. In fact, considering $\eta = 1.10$ implies that for the particular cases where $R = V_{min}/V_{max} \rightarrow 1$, V_{max} tends to $1.10V_{ref}$

for N cycles, which would mean that, in those cases, maintaining a constant high value of the applied load is a beneficial effect for shear strength. According to EC-2 (Eq. (15) and Fig. 4), if $R = V_{min}/V_{max} \rightarrow 1$, V_{max} tends to $0.9V_{ref}$, which fits better with the long term effects on the compressive strength. As a compromise solution, in this paper, $\eta = 1.0$ will be considered when combining this fatigue model with the CCCM or Eurocode 2. Equation (19), expressed in terms of R, may be expressed also in terms of V_{min}/V_{ref} (see Eq. 20), to facilitate the comparison with the other fatigue models considered (Fig. 5).

$$\frac{V_{max}}{V_{ref}} = \eta N^{-1/m} + \frac{V_{min}}{V_{ref}}\left(1 - N^{-1/m}\right) \nless 0.5 \qquad (20)$$

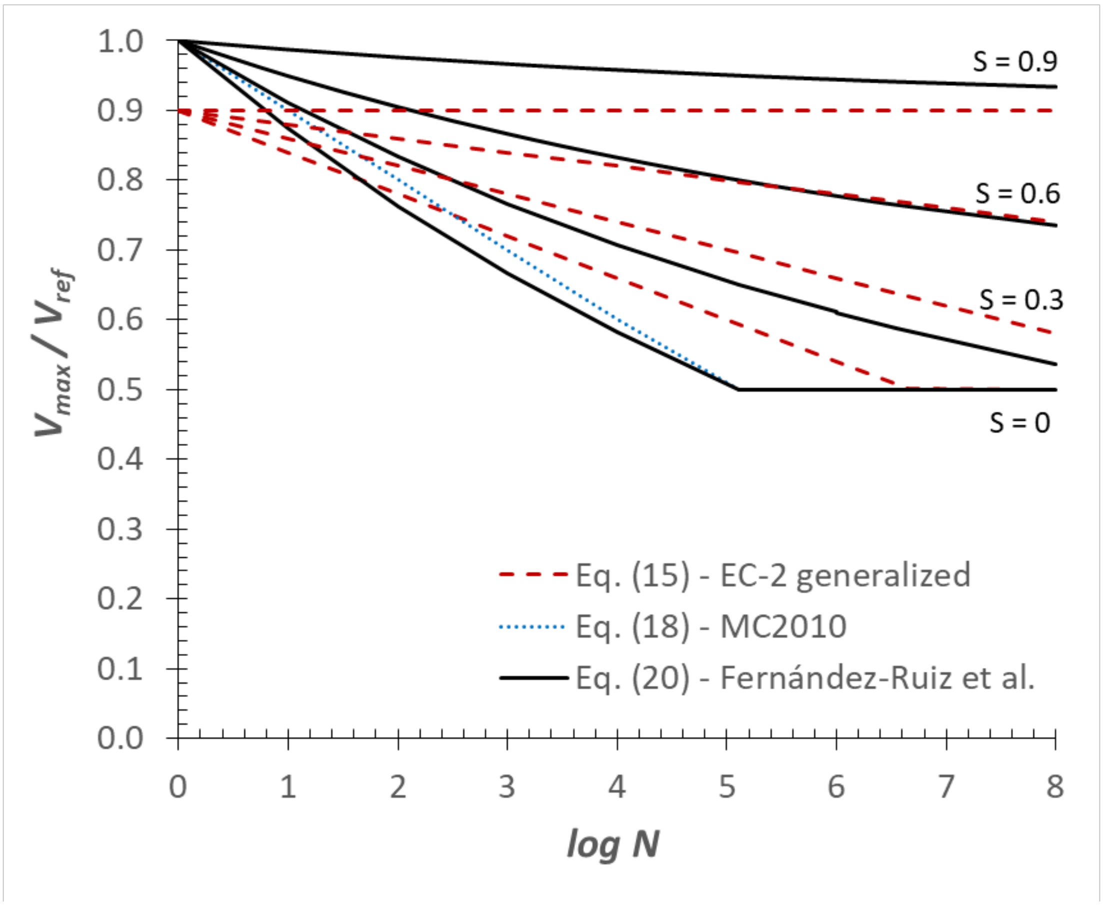

Fig. 5. Comparison of S/N curves studied for different values of $S = V_{min}/V_{ref}$.

Figure 5 compares the *S/N* curves given by MC-2010 (Eq. 18), the generalized expression of EC-2 (Eq. 15) and by Eq. (20). Note that the MC-2010 expression does not depend on the amplitude of the load cycles and that an asymptote for $V_{max}/V_{ref} = 0.5$ has been considered in Fig. 5, although the original formulation does not present this threshold value. This asymptote will be considered along this paper for the combination of the fatigue models with EC-2 to the CCCM shear models, as most research performed last years by different authors related to fatigue shear strength tends to confirm it [8,11].

In all cases, the fatigue strength depends on the reference shear strength considered for quasi-static tests, V_{ref}. In Section 4, the shear fatigue strength predictions of reinforced concrete members without shear reinforcement will be compared with empirical results, considering three possibilities for V_{ref}: the shear strength predicted by the CCCM (Eq. 5), by EC-2 (Eq. 7) and by MC2010 (Eq. 9). It is important to highlight that the CCCM predictions depend on the tensile strength, f_{ct} in Eq. (5), so it is possible to directly consider the reduction of the tensile strength for increasing values of N according to MC-2010 (Eq. 17).

4. Verification of the considered shear models with experimental results

4.1 Database

The database of fatigue tests on shear critical beams published in [8] by Fernández-Ruiz et al., based on the database initially developed by Gallego et al. [12], will be used. The database includes 87 tests reported in [6,7,10,25–31]. The distribution of the different relevant parameters of the beams included in the database is presented in Fig. 6. Table 2 presents the range of variables in the database of fatigue tests on shear critical beams and compares it with the range of variables for the quasi-static tests included in the database by ACI-DafStb [24]. It is important to emphasize that the distribution of parameters for the database of fatigue tests on shear critical beams is not homogeneous and, in fact, it does not represent the distribution of structural members built in real construction. As an example, 85 % of the beams present an effective depth, d, lower than 250 mm, and only 6.9 % of the tested beams present an amount of longitudinal reinforcement lower than 1 %.

Note also that 48 % of the included tests presents a value of $R = V_{min}/V_{max}$ lower than 0.05 and R is higher than 0.5 only for 4.6 % of the tests. For this reason, in the authors' opinion, it is important to avoid fitting the model parameters to this database, as it can drive to unsafe predictions when dealing with elements of characteristics out of the tested range, as beams of larger size or with lower longitudinal reinforcement. Coefficient m in Eqs. (19) and (20) was, however, fitted to this database, but it could be revised in case of further testing in the future.

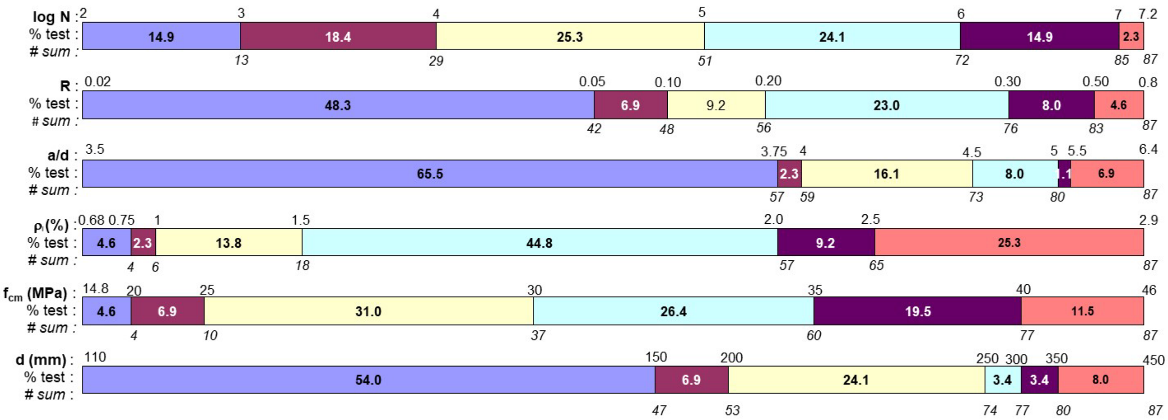

Fig. 6. Distribution of the key parameters in the analysed database for fatigue tests.

Table 2. Range of variables in the databases.

	Fatigue tests [8]		Quasi-static tests [24]	
	Min	Max	Min	Max
# tests	87		784	
b_w (mm)	100	400	50	3005
d (mm)	110	450	57	3000
f_{cm} (MPa)	14.8	46	12.9	139
ρ_l (%)	0.68	2.9	0.1	6.6
a/d	3.5	6.4	2.4	8.1

4.2 Fatigue consideration in CCCM, Eurocode 2 and Model Code 2010

The main advantage of using a mechanical model is that it can be extended to cover different circumstances without the need for empirical adjustments, but taking into account the basic behaviour of the materials. In the CCCM, the shear strength depends of the tensile strength of the concrete, f_{ct} in Eq. (5) and (6). Therefore, it is possible to use the CCCM to predict the fatigue resistance to shear by incorporating the fatigue behaviour of the concrete tensile strength. Model Code 2010 predicts (Eq. 17) that the tensile strength decreases as a function of the number of load cycles, *N*. Combining Eq. (17) with Eqs. (5) and (6) will allow to predict the fatigue shear strength, as it is presented in Eqs. (21-22):

$$V_{cu} = \zeta \frac{x}{d} f_{ct} \left(1 - \frac{\log N}{12}\right) bd \nless V_{cu,min} \quad (21)$$

$$V_{cu,min} = 0.833 \left(\zeta K_c + \frac{20}{d_0}\right) f_{ct} \left(1 - \frac{\log N}{12}\right) bd \quad (22)$$

Note that the previous equations do not take into account the amplitude of the load cycles nor the frequency, as the MC-2010 pure tension and tension-compression fatigue model given by Eq. (17) do not consider them. In any case, any improvement to this tensile fatigue model at material level could naturally be incorporated into the CCCM. As previously commented, an asymptote for $V_{max}/V_{ref} = 0.5$ has been considered for the shear fatigue strength and the term 1-log*N*/12 has to be considered not lower than 0.5. in Eqs. (21)-(22).

On the other hand, the Eurocode 2 model for the prediction of the shear strength of RC members without shear reinforcement is an empirical equation, see Eqs. (7) and (8). In this case, a concrete member resists fatigue due to shear effects when Eq. (14) is verified. As previously commented, Eq. (14) is the particular case of a *S-N* curve for *N* equal to 5 million load cycles (Goodman diagram represented in Fig. 4). In order to be able to compute EC-2 predictions for a different number of load cycles, the general Eq. (15) will be used in this paper, combined with EC-2 and CCCM quasi-static shear strengths. The equation proposed by Fernández Ruiz et al. [8], Eq. (19), will be also taken into account combined with EC-2 and CCCM. In addition, the shear fatigue formulation included in Model Code 2010 will be also used, that is, the combination of the shear strength given by Eqs. (9)-(12) and the S/N curve of Eq. (18). In all considered cases, the critical section is considered to be the same under quasi-static loading and fatigue loading.

Table 3. Comparison between predicted shear strengths and experimental results of beams included in the fatigue database.

Shear model (V_{ref})	Fatigue model	V_{test}/V_{pred} Mean	CoV(%)	Min.	5%
CCCM Eqs. (5) and (6)	Generalized S/N EC-2 curve Eq. (15)	1.10	14.6	0.69	0.84
	Fernández-Ruiz et al. [8] Eq. (19) with η = 1.0	1.21	13.4	0.80	0.92
	Tension-compression MC-2010 Eq. (21)	1.17	14.7	0.82	0.89
Eurocode 2 Eqs. (7) and (8)	Generalized S/N EC-2 curve Eq. (15)	1.05	15.8	0.61	0.77
	Fernández-Ruiz et al. [8] Eq. (19) with η = 1.0	1.15	15.7	0.71	0.84
Model Code 2010 Eqs. (9)-(12)	S/N MC2010 curve Eq. (18)	1.52	21.9	0.97	1.08
CSCT as presented in [8]	Fernández-Ruiz et al. [8] Eq. (19) with η = 1.1	1.00	15.1	0.65	0.75

4.3 General comparison

Table 3 presents the correlation between predicted shear strength and the experimental results of the beams included in the analysed database. It may be observed that considering the quasi-static shear strength, V_{ref}, given by CCCM, offers always a lower scatter (lower Coefficient of Variation) than using any other approach. Moreover, the very simple combination (Eq. 21) of the tensile fatigue model presented in MC-2010 with the CCCM offer also very good results, in spite of not considering the amplitude of the load cycles. Note that Eurocode 2 based correlations present the lowest safety for the beams included in the database meanwhile the Model Code 2010 predictions present the highest coefficient of variation and the highest safety. Just for comparison, Table 3 also includes the statistics included in Fernández-Ruiz et al. [8] for the combination of the Critical Shear Crack Theory (CSCT) and the fatigue approach by Fernández-Ruiz et al. [8]. As commented previously, coefficient *m* was fitted with this database and the CSCT, for that reason the mean value of V_{test}/V_{pred} equals 1.00. In any case, note that considering the quasi-static shear strength, V_{ref}, given by CCCM, as proposed in this paper, offers always a lower scatter (lower Coefficient of Variation) and higher safety (higher minimum value and 5th percentile) than using the CSCT [8].

The following procedure has been used to obtain the V_{test} values used to derive Tables 3, 4 and 5. The number of cycles producing failure for each test, *N*, which is the main result of the experimental tests, has been considered as an input for the predictions. The relationship V_{min}/V_{max}, applied during the tests, has also been considered as an input for the different predictions. Consequently, the value of V_{max} is the predicted value for each used procedure, taking into account V_{ref} from CCCM, Eurocode-2, or Model Code 2010 for quasi-static tests.

4.4 Detailed analysis

From the general comparison, it results that the fatigue strength prediction using CCCM obtain better results, with lower scatter, compared to the predictions using Eurocode-2 or Model Code 2010. In this section, the predictions will be examined in more detail.

The experimental correlations for different subsets of beams (according to *d* and ρ_l) are presented in Table 4. As commented previously, all members in the fatigue database present depths lower than 450 mm and, consequently, the application of EC-2 model to big beams, with the depth higher than 500 mm, could probably lead to unconservative results (see Fig. 3a for quasi-static tests). This trend, observed for quasi-static tests, is also seen for the EC-2 prediction of fatigue tests: the average value of the ratio V_{test}/V_{pred} clearly decreases when *d* increases, obtaining average unsafe results for beams with $d \geq 300$ mm. The same trend is observed for the results according to the CSCT or MC2010. For example, for the predictions according to MC2010, V_{test}/V_{pred} changes from 1.58 for beams under 200 mm of effective depth to 1.31 for the beams over 350 mm of effective depth. This reduction is much lower for the predictions based on the CCCM.

Table 4. Average values of the ratio V_{test}/V_{pred} for different subsets of beams (d and ρ_l).

Shear model (V_{ref})	Fatigue model	d (mm) < 200	d (mm) 200 - 300	d (mm) ≥ 300	ρ_l (%) < 1.5	ρ_l (%) 1.5- 2.5	ρ_l (%) ≥ 2.5
	# Tests	51	24	12	18	47	22
CCCM Eqs. (5) and (6)	Generalized S/N EC-2 curve Eq. (15)	1.11	1.11	1.04	1.07	1.11	1.12
	Fernández-Ruiz et al. [8] Eq. (19) with η = 1.0	1.22	1.22	1.14	1.20	1.20	1.23
	Tension-compression MC-2010 Eq. (17)	1.15	1.24	1.12	1.16	1.18	1.16
Eurocode 2 Eqs. (7) and (8)	Generalized S/N EC-2 curve Eq. (15)	1.12	**0.96**	**0.90**	**0.96**	1.01	1.20
	Fernández-Ruiz et al. [8] Eq. (19) with η = 1.0	1.23	1.06	**0.99**	1.08	1.10	1.31
Model Code 2010 Eqs. (9)-(12)	S/N MC2010 curve Eq. (18)	1.58	1.50	1.31	1.42	1.51	1.62
CSCT as presented in [8]	Fernández-Ruiz et al. [8] Eq. (19) with η = 1.1	1.04	**0.95**	**0.92**	**0.91**	**0.99**	1.10

In bold: un-safe results (<1)

Table 5. Average values of V_{test}/V_{pred} for different subsets of beams in the database (S).

Shear model (V_{ref})	Fatigue model	$S = V_{min}/V_{ref}$ < 0.1	0.1 – 0.2	0.2 – 0.3	0.3 – 0.4	0.4 - 0.5	> 0.5
CCCM Eqs. (5) and (6)	Generalized S/N EC-2 curve Eq. (15)	1.12 (#53)	1.05 (#16)	1.13 (#11)	1.06 (#3)	1.09 (#2)	1.03 (#2)
	Fernández-Ruiz et al. [8] Eq. (19) with η = 1.0	1.23	1.17	1.24	1.15	1.13	1.01
	Tension-compression MC-2010 Eq. (17)	1.14	1.15	1.24	1.25	1.34	1.33
Eurocode 2 Eqs. (7) and (8)	Generalized S/N EC-2 curve Eq. (15)	1.11 (#51)	**0.96** (#27)	**0.98** (#5)	**0.90** (#2)	**0.99** (#1)	**0.94** (#1)
	Fernández-Ruiz et al. [8] Eq. (19) with η = 1.0	1.22	1.07	1.06	**0.94**	**0.97**	**0.93**
Model Code 2010 Eqs. (9)-(12)	S/N MC2010 curve Eq. (18)	1.52 (#50)	1.44 (#17)	1.57 (#13)	1.37 (#4)	2.03 (#1)	1.68 (#2)
CSCT as presented in [8]	Fernández-Ruiz et al. [8] Eq. (19) with η = 1.1	1.03 (#57)	**0.98** (#23)	**0.87** (#3)	**0.82** (#2)	**0.82** (#1)	**0.79** (#1)

In bold: un-safe results (<1). In parenthesis: number of beams in each subset for the different shear models considered for computing the ratio V_{test}/V_{pred}. The total number of beams is coincident in all cases, but the distribution of the beams in each subset depends on V_{ref}.

Table 5 presents the average value of the ratio V_{test}/V_{pred} for different subsets of beams, grouped according to the value of $S = V_{min}/V_{ref}$. The values considered for V_{ref} depend on the shear model used and, consequently, the number of beams in each group varies for the different shear models and it is presented in parenthesis in Table 5. Figure 7 compares the *S-N* used diagrams with the experimental results in terms of V_{max}/V_{ref} versus *log N,* considering the reference quasi-static strengths, V_{ref}, by CCCM (red dots) and EC-2 (blue x) for different values of $S = V_{min}/V_{ref}$. Note that, for simplicity, Figure 7 does not represent the results given by the MC-2010 or CSCT shear models. This figure reveals that for higher values of S, the fatigue strength increases, but in most tests, S were kept lower than 0.1 or 0.2, and only one test presented a value higher than 0.6. From Figure 7, it can be seen that the tension-compression fatigue model in MC-2010 (Eq. 17), the blue dotted line in Fig. 7, does not consider the amplitude of the cycles, and it is probably too conservative for $S \geq 0.5$ and low level of loads. This behaviour is in agreement with the difficulty of incorporating in a so simple fatigue model the cycles previous to the propagation of the shear crack for low level of loads. The fatigue model by Fernández-Ruiz et al. [8] seems to present the best agreement according to Fig. 7 for the different ranges of S. However, looking to the detailed results already presented in Table 5, the average safety of the fatigue model by Fernández-Ruiz et al. [8] decreases as S increases (amplitude of the cycles decreases), independently of the shear model used. In fact, for tests with $S > 0.1$ the fatigue model by Fernández-Ruiz et al. [8] combined with the shear models by CSCT or EC-2 give un-safe results. Conversely, the MC-2010 considered fatigue models given by the tensile fatigue model, Eq. (17), and the shear fatigue model, Eq. (18), do not consider the influence of the loading amplitude, and the safety of the results increases when S increases, but obtaining safe results for all subsets studied in Table 5. Based on these results, it seems that the influence of the amplitude of the cycles, S, should be considered, but this influence is lower than predicted by the fatigue model of Fernández-Ruiz et al. [8] or the generalized S/N curve in EC-2 (Eq. (15)).

Finally, Figure 8 compares the three considered fatigue models and experimental results in terms of V_{max}/V_{ref} versus V_{min}/V_{ref} (Goodman diagrams), considering the reference quasi-static strengths, V_{ref}, by CCCM (red dots) and EC-2 (blue x), for different log N values. As observed, the fatigue model by Fernández-Ruiz et al. [8] shows a good agreement, being more conservative for the highest values of N. However, a few number of tests are available for log N = 6 and 7. Therefore, more experimental data is required to verify the performance of the model in these cases.

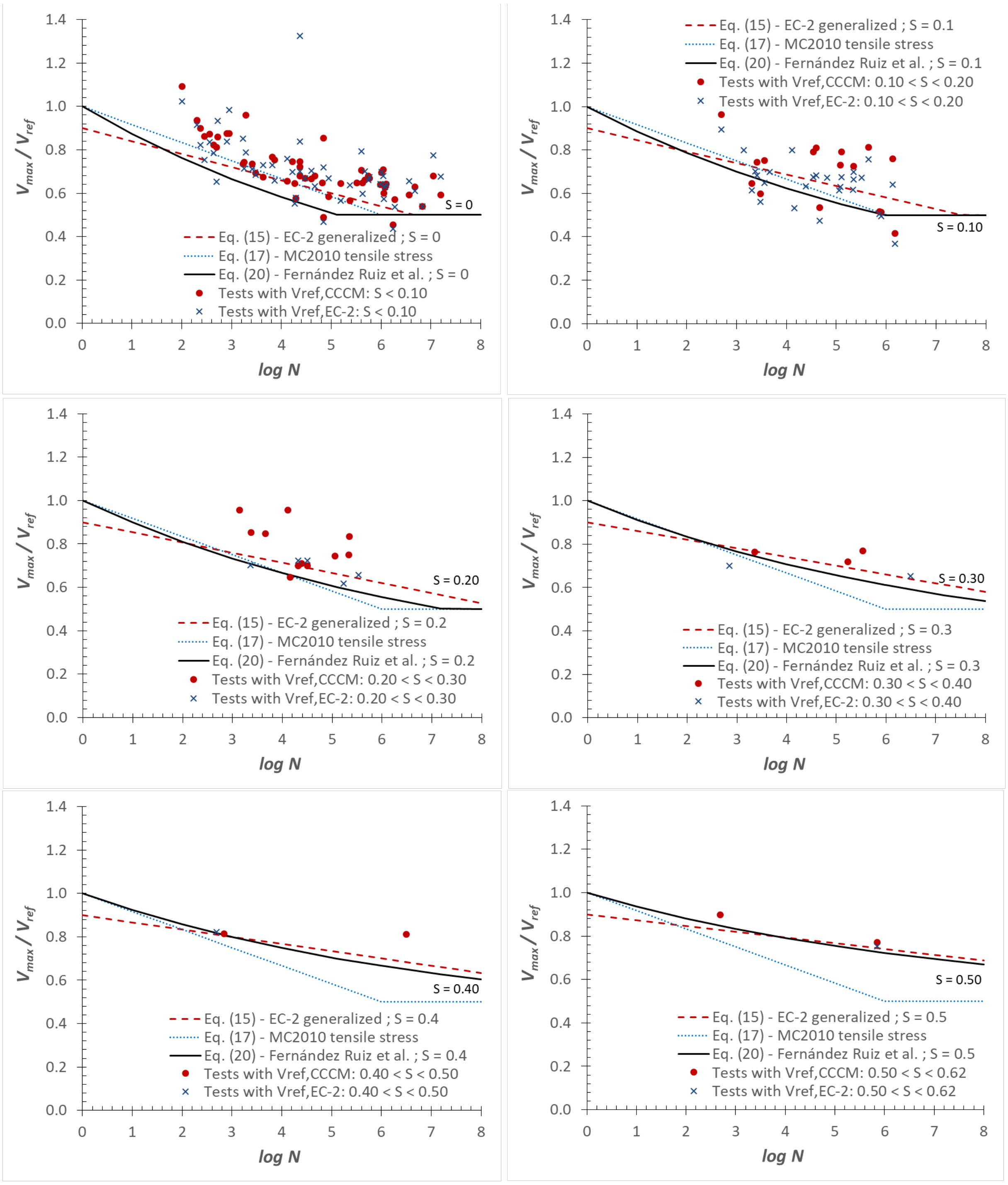

Fig. 7. Comparison of S/N curves of the three considered fatigue models and experimental results in terms of V_{max}/V_{ref} vs. log N.

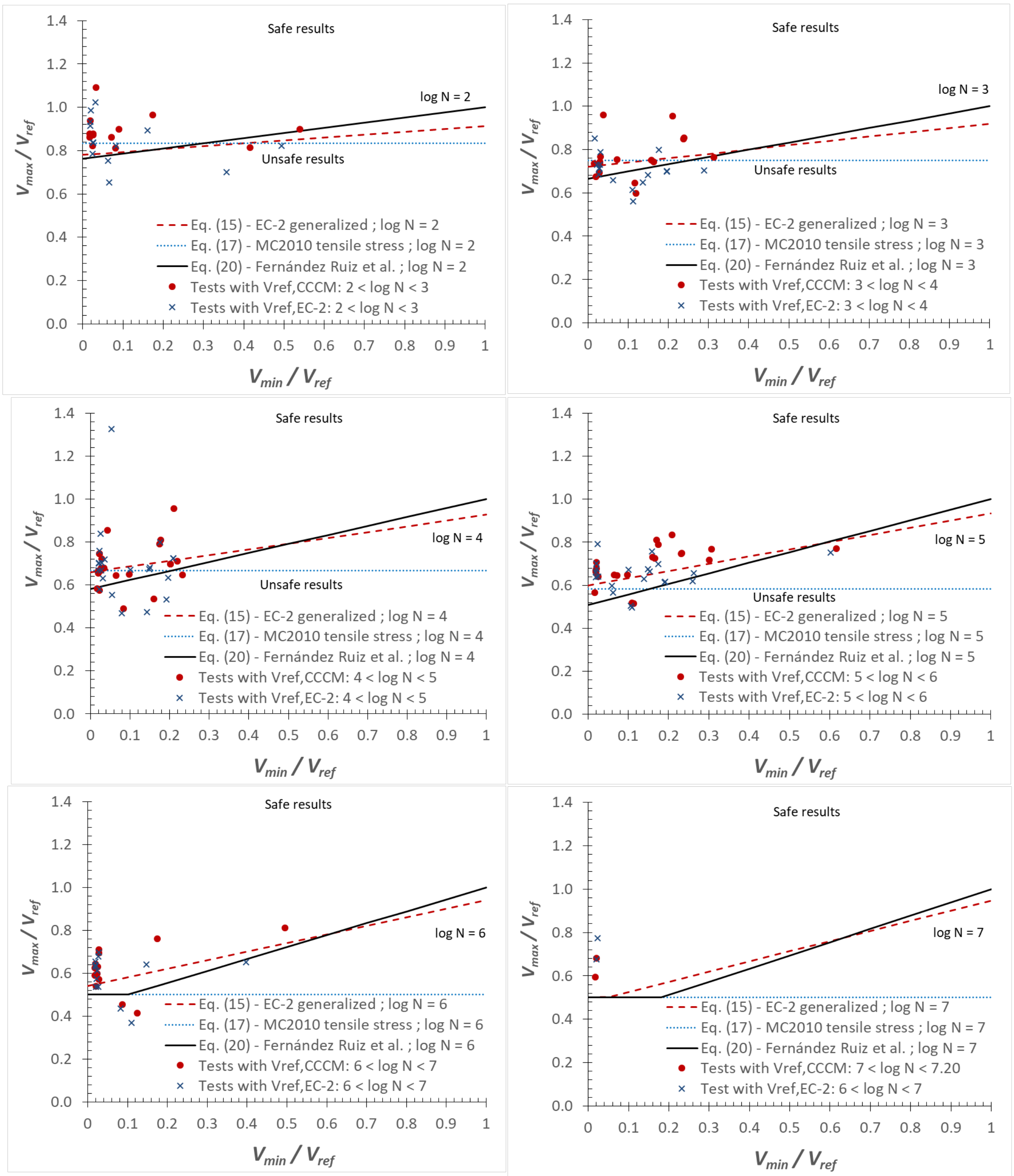

Fig. 8. Comparison of the three considered fatigue models and experimental results in terms of V_{max}/V_{ref} vs. V_{min}/V_{ref} (Goodman diagrams).

5. Conclusions

This paper presents the extension of the Compression Chord Capacity Model (CCCM), previously developed by the authors, to assess the fatigue shear strength of RC elements. For this purpose, different approaches for fatigue stresses have been combined with the model. Since the CCCM is a mechanical model, it has been possible to incorporate the fatigue resistance to shear through the fatigue behaviour of the concrete tensile strength (see Eqs. 21-22). The three considered fatigue approaches are: a) the generalized S/N curve from Eurocode 2; b) the Fernández-Ruiz et al. approach; and c) the tensile fatigue model of MC-2010. The performance of the CCCM combined with three different approaches for the fatigue strength were studied comparing the average ratio V_{test}/V_{pred} for a database of 87 tests. The results were also compared with the predictions by Eurocode-2, Model Code 2010 and the Critical Shear Crack Theory (CSCT). The following conclusions have been drawn:

- When comparing the CCCM model with the Eurocode 2 predictions, the first one gives better results with less scatter and higher safety (higher 5th percentile). Model Code 2010 predictions for beams failing on fatigue shear are too conservative and present the highest scatter.
- From the different fatigue models considered, the one of Fernández-Ruiz et al. combined with the CCCM gives the best approach also in terms of scatter and safety. However, it is necessary to take into account that the average safety of the model decreases as the amplitude of the cycles decreases ($S = V_{min}/V_{ref}$ increases). However, the vast majority of tests showed values of S around 0.1 or 0.2 and new tests with $S \geq 0.3$ would be needed. It must be taken into account that the fatigue approach by Fernández-Ruiz et al. include an empirical coefficient fitted with the same database used, and that this database is not representative of real construction.

- The combination of CCCM with the simple model of MC-2010 for fatigue tensile strength, which does not consider the amplitude of the number of cycles, gives also good results and can be used as a simplification. In this case, the average safety increases when *S* increases for low level of loads (V_{max}/V_{ref}), but the average predictions are always safe for the different subsets analysed.
- The application of the model for beams or slabs with depths higher than 500 mm should be studied in the future since all RC members included in the fatigue database showed a lower value and some considered models even offers average unsafe results for beams with *d* higher than 200 or 300 mm.
- A few number of tests are available for log *N* higher than 6. More experimental data is required to verify the performance of the shear fatigue models in these cases.

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Notations

A shear span. See reference [16] for complete definition regarding shear in beams

b width of the cross-section of a beam. For T or I-shaped is equal to the flexural effective compression flange width

b_w width of the web on T, I or L beams. For rectangular beams $b_w = b$

d effective depth of the cross-section

d_0 effective depth of the cross-section, d, but not less than 100 mm

d_g maximum size of the aggregate

f_{cd} design compressive strength of concrete

$f_{cd.fat}$ design fatigue strength of concrete $f_{cd,fat} = k_1 \beta_{cc}(t_0) f_{cd}\left(1-\frac{f_{ck}}{250}\right)$

f_{ck} characteristic compressive strength of concrete

f_{cm} mean compressive strength of concrete

f_{ct} tensile strength of concrete, in MPa, not greater than 4.60 MPa

k size effect factor according to Eurocode 2

k_1 coefficient of the definition of $f_{cd,fat}$ to be found in the National Annex of Eurocode 2. The recommended value of N=10^6 cycles is 0.85

m constant depending on the material properties

t_0 time of the start of the cyclic loading on concrete in days

x neutral axis depth of the cracked section, obtained assuming zero concrete tensile strength

z internal level arm, considered equal to 0.9d

C constant equal to 0.9

K_c relative neutral axis depth, x/d, but not greater than 0.20

N number of cycles to failure (fatigue loading)

R ratio V_{min}/V_{max}

S ratio V_{min}/V_{ref}

V_{cu} concrete contribution to the shear strength

$V_{cu,min}$ minimum concrete contribution to the shear strength

V_{max} design value of the maximum applied shear force under frequent load combination. According to MC2010, is the maximum shear force under the relevant representative values of permanent loads including prestress and maximum cyclic loading

V_{min} design value of the minimum applied shear force under frequent load combination in the cross-section where V_{max} occurs

V_{ref} quasi-static reference strength

α_e modular ratio, $\alpha_e = E_s/E_{cm}$

$\beta_{cc}(t_0)$ coefficient for concrete strength at first load application

ρ_l longitudinal tensile reinforcement ratio referred to the effective depth *d* and the width *b*.

ζ size and slenderness effect coefficient, given by Eq. (3)

$\sigma_{c,max}$ maximum compressive stress at the fibre under the frequent load combination

$\sigma_{c,min}$ minimum compressive stress at the same fibre where $\sigma_{c,max}$ occurs. If $\sigma_{c,min}$ is a tensile stress, then $\sigma_{c,min}$ should be taken as 0

$\sigma_{ct,max}$ maximum tensile stress in the concrete

Conflict of interest

None.

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