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NUMERICAL SIMULATION OF FREE SURFACE SEEPAGE IN SATURATED SOIL USING SMOOTHED PARTICLE HYDRODYNAMICS

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Abstract. In this paper we present numerical simulations of free surface seepage in saturated soil using Smoothed Particle Hydrodynamics. The modeling of the water seepage is based on mixture theory. Soil skeleton and water are depicted by different kinds of particles. The water particles move according to momentum equations while the soil particles are fixed in position. The interactions between these two phases, consisting of drag force and buoyancy force, are added to the governing equations of the water phase. A non-linear drag force model is adopted. Numerical results are compared with that from literature. It is demonstrated that seepage surface and pressure field can be obtained with satisfactory accuracy with the present model.

1 INTRODUCTION

Strong soil water coupling is common in natural environment and engineering practices. For example, landslide and debris flow are natural hazards mainly caused by the motion of soil-water mixture. Other examples are bed erosion, dam break, water jet excavation, etc. In these phenomenons coupling models usually used in soil mechanics, i.e., Biot consolidation theory, are no longer appropriate, because there is high relative velocity between soil and water, and the flow is mainly in turbulent regime. Moreover, numerical simulation of these problems is challenging, because free surface flow of fluid phase and large deformation of soil phase are difficult to model in the FEM method.

Smoothed particle hydrodynamics is a meshless method widely used in astrophysics and computational fluid dynamics. With an update Lagrangian scheme, it models large deformation and material boundary conveniently. Owing to this property, it is a promising tool in the problems involving strong soil water coupling. In this paper we present the simulation of free surface seepage in saturated soil, which is a essential and preliminary step of the complete simulation of coupling problems. First we present our free seepage model based on SPH and then numerical examples are performed to validate it.

2 SEEPAGE MODEL

2.1 Governing equations

Mixture theory [1, 2] is a model category involves all phases presenting in soil mixture, i.e., soil, water and gas. It describes the conservation laws of each constituent while taking the inter-constituent interaction into consideration. In this work we consider soil as undeformable so the governing equations consisting only of mass and momentum conservation for water:

$$\frac{\partial(\tilde{\rho}_f \phi_f)}{\partial t} = -\nabla \cdot (\tilde{\rho}_f \phi_f \boldsymbol{v}_f) \tag{1}$$

$$\frac{\partial(\tilde{\rho}_f \phi_f \boldsymbol{v}_f)}{\partial t} + \nabla \cdot (\tilde{\rho}_f \phi_f \boldsymbol{v}_f \otimes \boldsymbol{v}_f) = \nabla \cdot \boldsymbol{\sigma}_f + \tilde{\rho}_f \phi_f \boldsymbol{g} - \boldsymbol{f}_s \tag{2}$$

where $\tilde{\rho}_f$ is the intrinsic density of water; ϕ_f is the volume fraction of water; $\boldsymbol{v}_f$ is the Darcy velocity, also called unit discharge; $\boldsymbol{g}$ is the gravity and $\boldsymbol{f}_s$ is the force between fluid phase and soil phase. The stress tensor of fluid has the form $\boldsymbol{\sigma} = -p\boldsymbol{I} + \phi_f \boldsymbol{\tau}_f$, where p is the pressure, $\boldsymbol{I}$ an identity tensor and $\boldsymbol{\tau}_f$ the shear stress tensor. The interaction force $\boldsymbol{f}_s$ consists of drag force $\boldsymbol{D}$ and buoyancy force $-\phi_s \nabla p$. For simplicity, time and spatial derivatives of volume fraction are neglected. Substituting interaction force $\boldsymbol{f}_s$ in Eq. (2), omitting the subscript f and rewriting the equations in material derivatives, we have

$$\frac{\mathrm{d}\rho}{\mathrm{d}t} = -\rho \nabla \cdot \boldsymbol{v} \tag{3}$$

$$\frac{\mathrm{d}\boldsymbol{v}}{\mathrm{d}t} = -\frac{\nabla p}{\rho} + \frac{\boldsymbol{\tau}}{\rho} - \frac{\boldsymbol{D}}{\phi \rho} + \boldsymbol{g} \tag{4}$$

where ρ is the intrinsic density of water and the over-bar is omitted. We can see that by neglecting the time and spatial derivatives of ϕ Eq. (1) and (2) are greatly simplified. The form of equation (4) is similar to the momentum equation used by Lasere et al [3] and Shao [4]. But if fully coupled simulation with deformable soil is performed, the Eq. (2) should be used.

2.2 Constitutive relations

There are two ways to calculate pressure in SPH: one is to treat water as a weakly-compressible fluid (WCSPH), the other is incompressible SPH (ISPH) [4]. It is reported that the ISPH results in better pressure results. However, in the ISPH, the Poisson equation has to be solved, so the computation is more complex and time consuming. Recent researches show that, if density diffusion term, like δ-SPH, is applied in the WCSPH, good results of pressure can be obtained [5]. Therefore, we use weakly-compressible formulation. The pressure of water is calculated from an equation of state

$$p = B\left[\left(\frac{\rho}{\rho_0}\right)^\gamma - 1\right] \tag{5}$$

where ρ_0 denotes the reference density of water, i.e., $\rho_0 = 1000\text{kg/m}^3$; B is a pressure coefficient related to the configuration of the problem; γ is taken as $\gamma = 7$ for water.

The shear stress $\boldsymbol{\tau}$ is calculated from the velocity gradient

$$\boldsymbol{\tau} = \mu(\nabla\boldsymbol{v} + (\nabla\boldsymbol{v})^{\mathrm{T}}) \tag{6}$$

where μ is the dynamic viscosity of water and $\nabla\boldsymbol{v}$ is the velocity gradient.

2.3 Drag force

The most used drag force model is the Darcy's law which is linear, valid in laminar flow. Based on experiment data, non-linear models have been proposed and proven to be more realistic. A quadratic form of non-linear drag force [3] is adopted in this work, which has the following form

$$\boldsymbol{D} = \frac{\mu\phi}{k}\boldsymbol{v} + \frac{1.75\phi\rho}{\sqrt{150k}n^{3/2}} \parallel \boldsymbol{v} \parallel \boldsymbol{v} \tag{7}$$

where k is the intrinsic permeability related only to the property of the soil material

$$k = \frac{\phi^3 D_{50}^2}{150(1-\phi)^2} \tag{8}$$

where D_{50} is used as an equivalent diameter of soil. One advantage of the adopted form of drag force is that in free flow area, where $\phi = 1$, the drag force drops to zero automatically. Therefore, the Eq. (4) can be applied to whole computational domain. There is no need to divide flow domain into free flow area and seepage area.

3 SPH FORMULATIONS

Conventional numerical methods based on mesh, such as FEM and FVM, have difficulty in solving Eq. (3) and (4) in seepage problems due to the presence of free surface. Additional numerical techniques, like level set method, have to be applied [3]. SPH is widely used to model free surface flow. Therefore it is applied here to solve the system of equations. In SPH the material is represented by particles carrying physical variables moving with material velocity. By using an update Lagrangian scheme, we track the position change of the particle system and the carried variables, hence the problem is numerically solved.

3.1 Fundamentals

In SPH, a field function can be approximated by the following integral interpolation

$$f(\boldsymbol{x}) = \int_{\Omega} f(\boldsymbol{x}')W(r,h)\mathrm{d}\Omega \tag{9}$$

where Ω is the integral domain; W is the weighting function, usually called kernel function in SPH literature; r denotes the Euclidean distance between $\boldsymbol{x}$ and $\boldsymbol{x}'$, i.e., $r = \parallel \boldsymbol{x} - \boldsymbol{x}' \parallel$. Finally, h is called smoothing length determining the size of the kernel. The Wendland C^6 function is used as kernel function in this work

$$W(r,h) = \alpha_d(1-\frac{1}{2}q)^8(1+4q+\frac{25}{4}q^2+4q^3) \tag{10}$$

where $q = r/h$. α_d is a coefficient chosen to fulfill the normalization condition. The chosen kernel function is compactly supported. It is obvious that $W = 0$ when $q \geq 2$.

By particle summation, Eq. (9) can be written in the following form

$$f(\boldsymbol{x}_i) = \sum_{j=1}^{n} f(\boldsymbol{x}_j)W_{ij}m_j/\rho_j \tag{11}$$

where n is the number of particles in the support domain of kernel W_{ij} centered at particle $\boldsymbol{x}_i$. Following a similar way, the gradient of $f(\boldsymbol{x})$ can be calculated as

$$\nabla f(\boldsymbol{x}) = \sum_{j=1}^{n} f(\boldsymbol{x}_j)\nabla_i W_{ij}m_j/\rho_j \tag{12}$$

3.2 Discretization of governing equations

We use subscript a and b to denote fluid particles and i and j for soil particles in the subsequent content. The mass conservation equation is discretized as

$$\frac{\mathrm{d}\rho_a}{\mathrm{d}t} = \sum_b m_b(\boldsymbol{v}_a - \boldsymbol{v}_b)\cdot\nabla_a W_{ab} + \delta h c_0 \sum_b \frac{m_b}{\rho_b}\boldsymbol{\psi}_{ab}\cdot\nabla_a W_{ab} \tag{13}$$

In the right side of Eq. (13), the second term is the density diffusion named δ term. It is the key point to obtain a smooth pressure results in our simulation. In Eq. (13), c_0 is the reference speed of sound, δ is a constant determining the diffusion effect, and $\boldsymbol{\psi}_{ab}$ is

$$\boldsymbol{\psi}_{ab} = 2(\rho_a - \rho_b)\frac{\boldsymbol{r}_{ab}}{\parallel \boldsymbol{r}_{ab} \parallel^2} \tag{14}$$

where $\boldsymbol{r}_{ab} = \boldsymbol{x}_a - \boldsymbol{x}_b$ is the vector from $\boldsymbol{x}_b$ to $\boldsymbol{x}_a$.

The discretized momentum equation is

$$\frac{\mathrm{d}\boldsymbol{v}_a}{\mathrm{d}t} = -\sum_b m_b \left(\frac{p_a}{\rho_a^2} + \frac{p_b}{\rho_b^2}\right) \nabla_a W_{ab} + \sum_b \frac{2 m_b \mu \boldsymbol{r}_{ab} \cdot \nabla_a W_{ab}}{\rho_a \rho_b (\| \boldsymbol{r}_{ab} \|^2 + 0.01 h^2)} \boldsymbol{v}_{ab} - \sum_j \frac{m_j}{\phi_a \rho_a \rho_j} (K_{aj}^A \boldsymbol{v}_{aj} + K_{aj}^B \| \boldsymbol{v}_{aj} \| \boldsymbol{v}_{aj}) W_{aj} \tag{15}$$

where $\boldsymbol{v}_{aj}$ is the relative velocity between fluid particle a and soil particle j; ρ_j is the partial density of soil particle j. the coefficients K_{aj}^A and K_{aj}^B are

$$K_{aj}^A = \frac{\mu \phi_a}{k_a}, \quad K_{aj}^B = \frac{1.75 \phi_a \rho_a}{\sqrt{k_a} \phi_a^{3/2}} \tag{16}$$

representing respectively linear and non-linear effect. In Eq. (15) the discretization of the gradient of shear stresses follows the treatment in [4]. The volume fraction of fluid particle is interpolated over the neighboring soil particles

$$\phi_a = \sum_j \frac{m_j}{\rho_j} (1 - \phi_j) W_{aj} / \sum_j \frac{m_j}{\rho_j} W_{aj} \tag{17}$$

where ϕ_j is the volume fraction of soil at particle j.

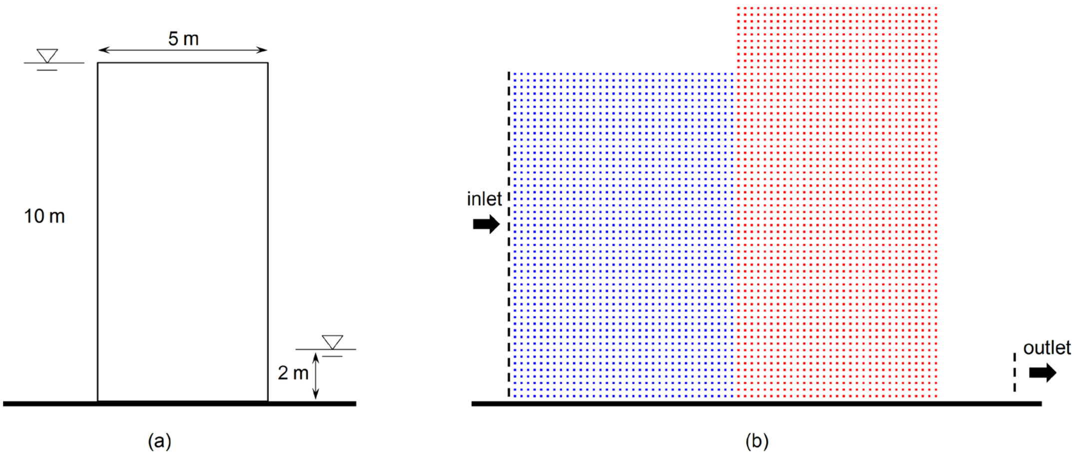

Figure 1: Simulation setup: (a) Geometry of the rectangular dam; (b) Boundary conditions and initial particle configuration used in the simulation. The blue particles denote water and red particles denote soil.

4 NUMERICAL EXAMPLES

The free surface seepage in a rectangular dam with tail water is investigated. The geometry and boundary condition is shown in Figure 1(a). The dam is 10 m in height and 5 m in width. The upstream water level is 10 m and downstream tail water is 2 m deep. Figure 1(b) shows the configuration of simulation. The dam and water are discretized by soil particles and water particles respectively. The inlet boundary condition is set 7 m away from the dam, while outlet boundary condition is 3 m away. The impermeable bottom boundary is modeled by dummy particles fixed in position. Initially there is no particle downstream. The soil particles are fixed in position, thus they are only used in the calculation of fluid volume fraction and drag force.

The following material parameters are used in the simulation: soil partial density 1350 kg/m^3; soil intrinsic density $\tilde{\rho}_s$ = 2700 kg/m^3; D_{50} = 10 mm; dynamic viscosity $\mu = 1 \times 10^{-3}$ Pa·s. The chosen material constants results in relatively large permeability. However, the permeability only affects the seepage velocity, not the shape and position of the seepage surface.

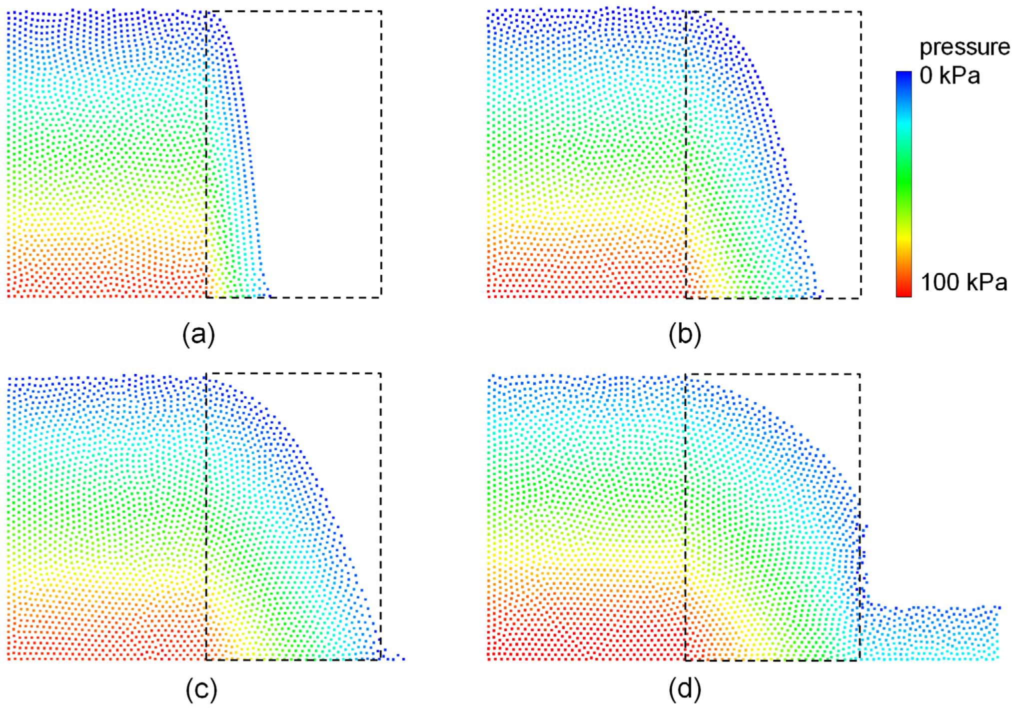

Figure 2: Seepage in the rectangular dam. The colored legend applies to all four figures.

Figure 2 shows the seepage process in the rectangular dam. The free surface is captured clearly and reasonably. No additional surface detection is needed. The obtained pressure field is smooth and accurate. The results are comparable to results obtained using more complicated and time consuming Imcompressible SPH [4]. The accuracy of pressure is significant, because when solving water-soil coupling problem, the pressure field affects the motion of the mixture a lot.

In Figure 3 we compare the free surface obtained using the present model to that

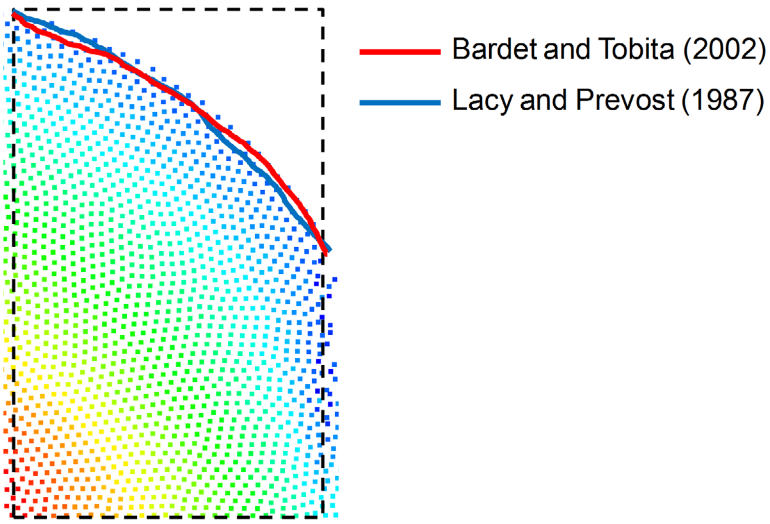

Figure 3: Comparasion of the free surface.

in literature [6, 7]. Good agreement is observed. Iteration or level set are required in obtaining the seepage surface in grid based methods. However, here no special treatment is needed, SPH computes the surface in a natural way. It is demonstrate that with the proposed SPH model, free surface seepage as well as pressure field, are modeled conveniently and with good quality.

5 CONCLUSIONS

In this paper we present a model to simulate free surface seepage in saturated soil using Smoothed Particle Hydrodynamics. The mixture theory is used to derive the governing equation of the seepage. Density diffusion, i.e., δ-SPH is applied to assure the accuracy of the pressure. Numerical examples are performed. It is shown that the present model is able to model free surface seepage conveniently and accurately.

In soils with very low permeability, the present method suffers long computational time because SPH is an explicit method. It is not suitable for seepage in clay. However, based on the mixture theory, the present model has the potential to be extended to solve problems involving strong water-soil coupling, if the mass and momentum conservation of soil phase is considered and soil constitutive models are provided.

REFERENCES

[1] Drew, D.A. Mathematical modeling of two-phase flow. *Ann. Rev. Fluid Mech.* (1983) **15**:261–291.

[2] Pitman, E.B. and Le, L. A two-fluid model for avalanche and debris flows. *Phil.*

Trans. R. Soc. A (2005) **363**:1573-1601.

[3] Larese, A., Rossi, R. and Idelsohn S.R. A coupled PFEM-Eulerian approach for the solution of porous FSI problems. *Computational Mechanics* (2012) **50**:805-819.

[4] Shao, S.D. Incompressible SPH flow model for wave interactions with porous media. *Coastal Engineering* (2010) **57**:304-316.

[5] Molteni, D. and Colagrossi, A. A simple procedure to improve the pressure evaluation in hydrodynamics context using the SPH. *Computer Physics Communications* (2009) **180**:861-872.

[6] Bardet, J.P. and Tobita, T. A practical method for solving free-surface seepage problems. *Computers and Geotechnics* (2002) **29**:451-475.

[7] Lacy, S.J. and Prevost, J.H. Flow through porous media: a procedure for locating the free surface. *International Journal for Numerical and Analytical Methods in Geomechanics* (1987) **11**:585-601.