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FORM-FINDING OF EXTENSIVE TENSEGRITY USING TRUSS ELEMENTS AND AXIAL FORCE LINES

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Summary. Tensegrity structure, which consists of cables and struts, are expected to be used as systems for cosmological, foldable and/or inflatable structures. The equilibrium shape of the tensegrity can be determined by iteration of solving the tangent stiffness equation. Here, it is rational to use the truss elements for struts and the axial force line elements for cables. In this study, a way to find the shapes of "extensive tensegrity", which counts their self-weight and permits support conditions of statically indeterminate. As results of numerical examples, even the case where many solutions exist under the same loading conditions like the tower tensegrity, expected one equilibrium solution can be obtained, and its equilibrium path can be drawn.

1 INTRODUCTION

Tensegrity structures have very unique morphology that is formed by continuous tension and discontinuous compression, and so many researchers have been tried to determine their shapes. Force Density Method [1] (FEM) is one of the form-finding method for tensegrity structures used most frequently. The method gives equilibrium solutions by a linear stiffness equation without any iteration and is useful, for example, for form-finding of cable net structures under constant external forces and stable support conditions. However, we have to choose suitable force density ratio between every element to find smooth and proportionate shape with unified element size. Furthermore, in order to get the spatial shapes of the “pure” tensegrities, which is in state of self-equilibrium without self-weight and external forces, FDM needs to determine the feasible sets of force density by non-linear analysis before solving the linear stiffness equation. Since Vassart and Motro[2], some procedures to find the feasible sets of pure tensegrities have been proposed[3]-[5].

On the other hand, the authors have developed the measure potential which produces the elements with virtual stiffness, and have applied to the form-finding of the cable net structures, membrane pneumatic structures and tensegrity structures[6]-[8]. The measure potential can be defined freely as a function of "element area" or "element length", so if we define the potential of a triangular element as is proportionate to its area, the element behaves as soap film and the form-finding of an isotonic surface realizes. Moreover, if we define the potential of a line element as is proportionate to (n+1)-th power of its length, its axial force proportionate to n-th power of its length and we call it "n-th axial force line element". Especially, when n=1, the stiffness equation becomes linear and the process of computation becomes quite equal to FDM. (In that meaning, our idea for this potential may be close to Miki and Kawaguchi's one.) However, n that magnitude is bigger than 2, gives the solutions more regulated element length and brings the performance of the form-finding better. Our recent paper[8] has tried to apply the measure potential to the pure tensegrities, and here we used the axial force line elements with n=2 for cables and the rigid bars for the struts. NR method by iteration of solving the tangent stiffness equation converges surely and perfect equilibrium solutions can be obtained.

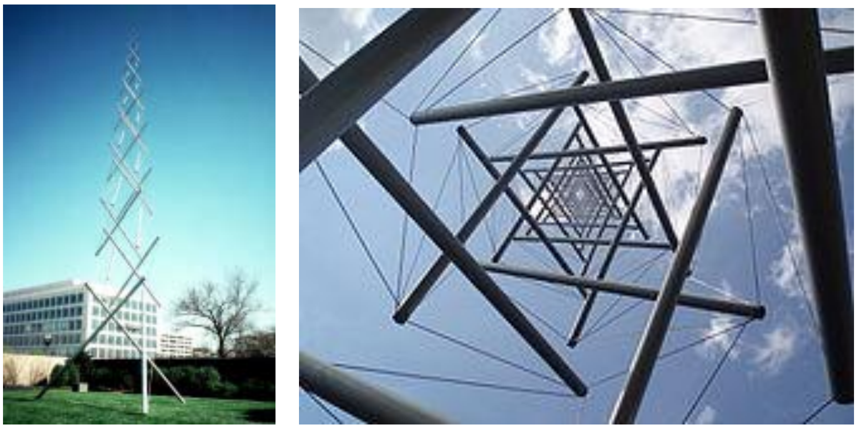


Figure 1 Snelson's needle tower

Also in this study, elements for struts are modified to truss element with real material stiffness in other to get better convergence. Moreover, this modification made it easy to apply the form-finding analysis to extensive tensegrity structures, which allow external forces and connection between struts.

In this study, some numerical examples of form-finding for tower tensegrities just like Snelson's needle tower (Figure 1) are shown. This type of tensegrity has self-weight and requires stable support conditions, therefore spatial forms may be obtained even if FDM uses any value for force densities. However, a tensegrity has many equilibrium shapes corresponding to one condition of connectivity and loading, and then it is difficult to obtain an expected solution such as Figure 1 that the modules with uniform geometry are lined up in vertical direction in good order. This study shows that the combination of the loading control and the displacement control is effective to find the equilibrium of self-reliance with its self-weight. Letting top nodes of control points displace up compulsorily and searching where the control points have no reaction forces gives us a solution with tower geometry.

Furthermore, in this study, another numerical example is shown. As mentioned above, tower tensegrity have so many solutions for a condition, so the searching equilibrium paths attracts us and that gives us a lot of information to make clear the character of tensegrity. As a result of computation, five main paths, in which the shape deforms keeping symmetry, have found and they are independent each other.

2 FORM-FINDING BY THE TANGENT STIFFNESS METHOD

2.1 Tangent stiffness equation

Let the vector of the element edge forces independent of each other be indicated by **S**, and let the matrix of equilibrium which relates **S** to the general coordinate system by **J**. Then the nodal forces **U** expressed in the general coordinate follow the equation:

$$\mathbf{U} = \mathbf{J}\mathbf{S} \tag{1}$$

The tangent stiffness equation is expressed as the deferential calculus of Eq. (1),

$$\boldsymbol{\delta}\mathbf{U} = \mathbf{J}\boldsymbol{\delta}\mathbf{S} + \boldsymbol{\delta}\mathbf{J}\mathbf{S} = (\mathbf{K}_0 + \mathbf{K}_G)\boldsymbol{\delta}\mathbf{u} \tag{2}$$

In which, $\mathbf{K_0}$ is the element stiffness which provide the element behaviour in element (local) coordinate, and $\mathbf{K_G}$ is the tangent geometrical stiffness. $\boldsymbol{\delta}\mathbf{u}$ is nodal displacement vector in general coordinate.

2.2 Element potential function

In order to regulate the element behavior in element (local) coordinate, we define the element measure potential, which is expressed as the function of measurement such as element length or element area. Defining element measure potential is equal to assuming the "virtual" elemental stiffness. Moreover, it has no relationship with material's stiffness.

Let element measure potential is P, and let the vector of elements' measurements whose component is independent of each other is **s**

$$\mathbf{S} = \frac{\partial P}{\partial \mathbf{s}} \tag{3}$$

Then we can get the element edge force **S**.

2.3 Axial force line element

The line element is connected with nodal point 1 and nodal point 2. Supposing that the element measure potential is proportional to the power of length of line element, the element measure potential can be expressed as:

$$P = Cl^{n+1} \tag{4}$$

The axial line element force can be obtained by differential calculus of element measure potential:

$$N = nCl^{n} \tag{5}$$

where, C is the coefficient to be able to set freely.
Let $\boldsymbol{\alpha}$ are the components of cosine vector of an axial force line element, which connects node 1 and node 2, and we can rewrite the Eq. (1) as:

$$\begin{bmatrix} \mathbf{U_1} \\ \mathbf{U_2} \end{bmatrix} = \begin{bmatrix} \mathbf{-\alpha} \\ \boldsymbol{\alpha} \end{bmatrix} N \tag{6}$$

Substituting the Eq. (6) to the above Eq. (2), and make it matrix.

$$\delta \begin{bmatrix} \mathbf{U_1} \\ \mathbf{U_2} \end{bmatrix} = \mathbf{K_T^L} \delta \begin{bmatrix} \mathbf{u_1} \\ \mathbf{u_2} \end{bmatrix} \tag{7}$$

$$\mathbf{K_T^L} = nCl^{n-2} \begin{bmatrix} \mathbf{e} + (n-2)\boldsymbol{\alpha\alpha}^\mathbf{T} & -\mathbf{e} - (n-2)\boldsymbol{\alpha\alpha}^\mathbf{T} \\ -\mathbf{e} - (n-2)\boldsymbol{\alpha\alpha}^\mathbf{T} & \mathbf{e} + (n-2)\boldsymbol{\alpha\alpha}^\mathbf{T} \end{bmatrix} \tag{8}$$

For the Eq. (5), in the case of n=2, the element forces become constant, and for the Eq. (7), the tangent geometrical stiffness of line element becomes the same form as truss element's. Therefore, the axial forces can be designated as a constant value.
In addition, in the case of n=2, axial force is proportional to the length of line element, and Eq. (7) is linear. However, in the case of n>2, iterative steps are required because of nonlinearity. The magnitude of n become larger, the length of all line elements on the solution surface tend to be more uniform [6],[7].

2.4 Truss element with real stiffness for struts

In the Ref.[8], rigid bares are used for struts, but it seems that they causes the convergence worse. However, it became evident that the rigid-bars bring the aggravation of convergence, because of the non-linearity of the degeneration matrix.
Therefore, the ordinary truss members are applied to struts in this study, and the convergence property was improved dramatically instead of sacrificing just one degree of freedom for an element. Namely, when a huge value was applied to Young's modules, the member behaves like a rigid-bar. Referentially, the element force equation and the tangent stiffness equation of a truss member are shown in Eq.(9) and (10), respectively.

$$N = \frac{EA}{l_0} \Delta l \tag{9}$$

$$\delta \begin{bmatrix} \mathbf{U_1} \\ \mathbf{U_2} \end{bmatrix} = \left\{ \frac{EA}{l_0} \begin{bmatrix} \boldsymbol{\alpha\alpha}^\mathbf{T} & -\boldsymbol{\alpha\alpha}^\mathbf{T} \\ -\boldsymbol{\alpha\alpha}^\mathbf{T} & \boldsymbol{\alpha\alpha}^\mathbf{T} \end{bmatrix} + \frac{N}{l} \begin{bmatrix} \mathbf{e} - \boldsymbol{\alpha\alpha}^\mathbf{T} & -\mathbf{e} + \boldsymbol{\alpha\alpha}^\mathbf{T} \\ -\mathbf{e} + \boldsymbol{\alpha\alpha}^\mathbf{T} & \mathbf{e} - \boldsymbol{\alpha\alpha}^\mathbf{T} \end{bmatrix} \right\} \delta \begin{bmatrix} \mathbf{u_1} \\ \mathbf{u_2} \end{bmatrix} \tag{10}$$

where, EA is elongation rigidity, and l_0 is non stressed length of the member.

3 COMPUTATIONAL EXAMPLES

3.1 Comparison of element application for struts

The example compares two models of element application for struts,one is to use rigid bodies and the other is to use truss. Both models have no external force, so the solutions should be in self-equilibrium state. Figure 2 is the primary shape consists of 64 axial force line elements and 9 rigid bodies with four-nodes. Here, the coefficient and the power in Eq.(5) are designated as c=2 and n=2, respectively. As a result, the equilibrium shown in Figure 3 can be obtained. Figure 4 is the primary shape where the tetrahedral truss-units are placed instead of the rigid bodies of previous model. The equilibrium shown in Figure 5 can be obtained. Comparing these two models, all the nodes are located at almost same position and equilibrium shapes are evaluated as equal.

Figure 6 is comparison of the convergent process of maximum unbalanced force. When the rigid body is applied to struts, the convergence process is gradual as the unbalanced force reduces to half in an incremental step. On the other hand, the truss units make the convergence process accelerate, and the uniqueness of the tangent stiffness method that the unbalanced force converges suddenly and rapidly can be recognized.

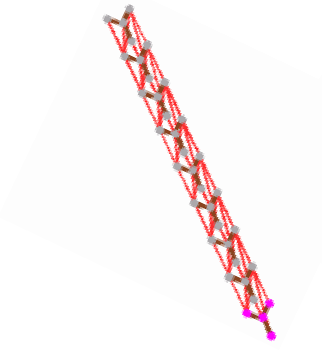


Figure 2 Primary shape

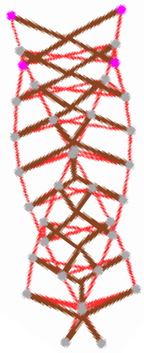


Figure 3 equilibrium shape

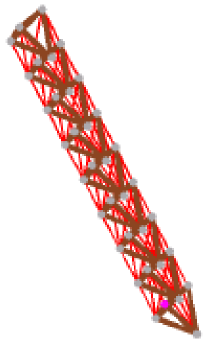


Figure 4 Primary shape

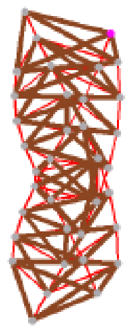


Figure 5 equilibrium shape

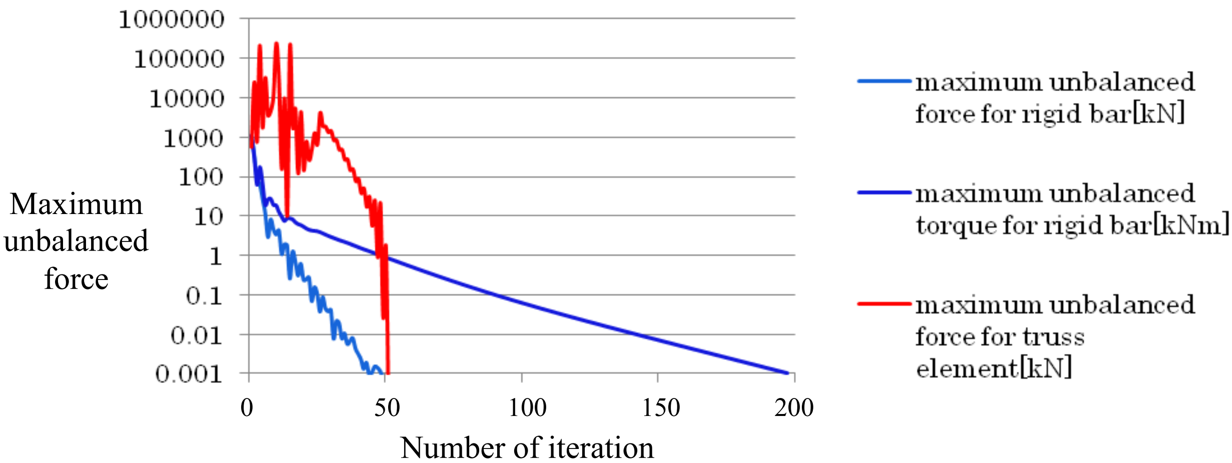
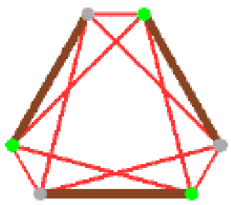
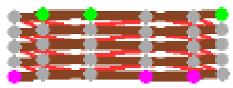


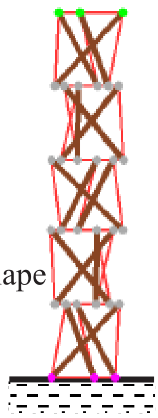
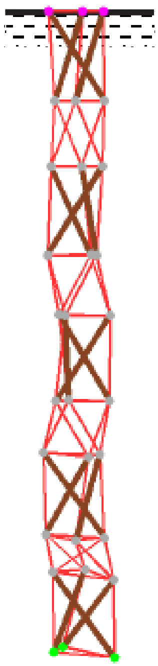
Figure 6 Comparison of convergent process

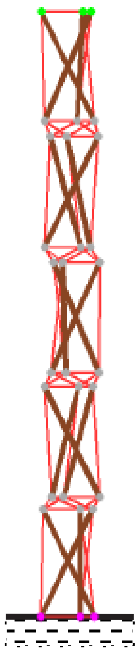
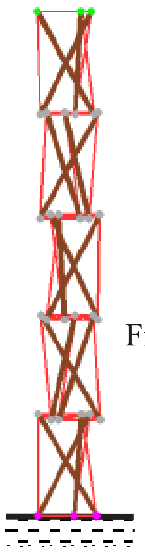
3.2 Tower shape with gravity

The model with five units layered has the same connectivity as the needle tower by Snelson, as shown in Figure 1. Figure 7 is the primary shape in which (a) is top view and (b) is side view. Where, non-stressed length of the struts is 1m respectively. The initial idea was to obtain the objective equilibrium solution with self-weight via a solution of self-equilibrium without any nodal force (Figure 8 (a)). Figure 8 (b) is a solutions with 0.1kN of self-weight for each node under the condition of *n*=5, *C*=1.5 in eq.(5), but in almost cases of *n* and *C*, similar solution will be obtained. They are different from the expected shape of tower.

The second idea is to once displace the top nodes of the tower compulsorily up to a height corresponding to the expected shape. Then displacing them gradually down as to the reaction forces of the control points become zero, we can get the solution of self-reliance with its self-weight. Figure 9 (a) is the shape with the displacement of 5m which is 5 times for the length of a strut. Here, eq. (5) is set by *n*=5, *C*=1.5. After that, 0.2m of compulsory displacement is added to the equilibrium of Figure 9 (a) step by step, and the reaction forces have changed from negative to positive at 5th step. Then releasing the restriction of the control points gives the perfect equilibrium shape of self-reliance with its self-weight as shown in Figure 9 (b).
However, depending on the values of *n* and *C*, the expected solution may not be obtained. Figure 10 shows an example in case of *n*=5, *C*=1.5, but this is also a perfect equilibrium solution. This fact suggests us the existence of so many solutions for a connectivity condition.


(a)

(b)
Figure 7 Primary shape and connectivity


(a)

(b)
Figure 8 Shapes obtained by initial idea


(a)

(b)
Figure 9 Shapes obtained by second idea

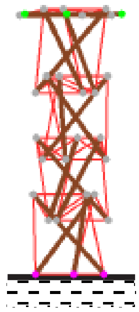


Figure 10 Another solution by *n*=5,*C*=1.5

3.3 Path finding of tower tensegriy

It was suggested in Chapter 3.2 that tower shape tensegrity have a variety of equilibrium corresponding to one connectivity. Therefore, it is attractive for us to search the equilibrium path under the self-weight of struts. The model is a tower with two units layered, in order to observe the transition of the equilibrium shape with partial buckling. Figure.11 is the primary shape in which (a) is top view and (b) is side view. Five nodes of lower unit is fixed. The coefficient and the power in Eq.(5) are designated as c=2 and n=2, respectively. Moreover, nodal self-weight is 0.1[kN] and elongation rigidity is 2.0×10^9[kN], and five node
The analysis proceeds as follows;

1) Initial load is triggerd on five nodes of upper unit that are control points. Obtained solution be the primary solution of path finding. Depending on the magnitude of initial load deffernt pathes can be found. (Only for the path in Figure 16, the primary solution is obtained by 'snap through' during searching the path expressed by Figure 15.)
2) Path is drawn by incremental analysis with combination of the load control and the displacment control. The control is swiched by the current tangent of the path.

As the results of computation, five paths independent each other (Figure 12-16) have found and the all the solutions on these paths have symmetric shape. On Figure 17, all the paths found by the analysis are gathered on a coordinate, the point where the path crosses the horizontal axis is the solution of self-reliance with its self-weight. Therefore, thirteen of self-reliance solutions have been found by the analysis.

However, all the paths and all the solutions may not provided by this analysis. They are "some of all". Eigenvalue analysis of tangent stiffness matrix may be necessary to discover this equilibrium system completely.

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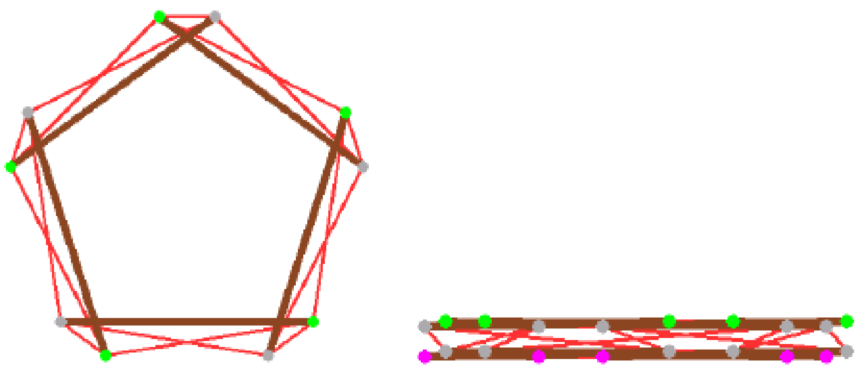

(a) (b)

Figure 11 Primary shape and connectivity

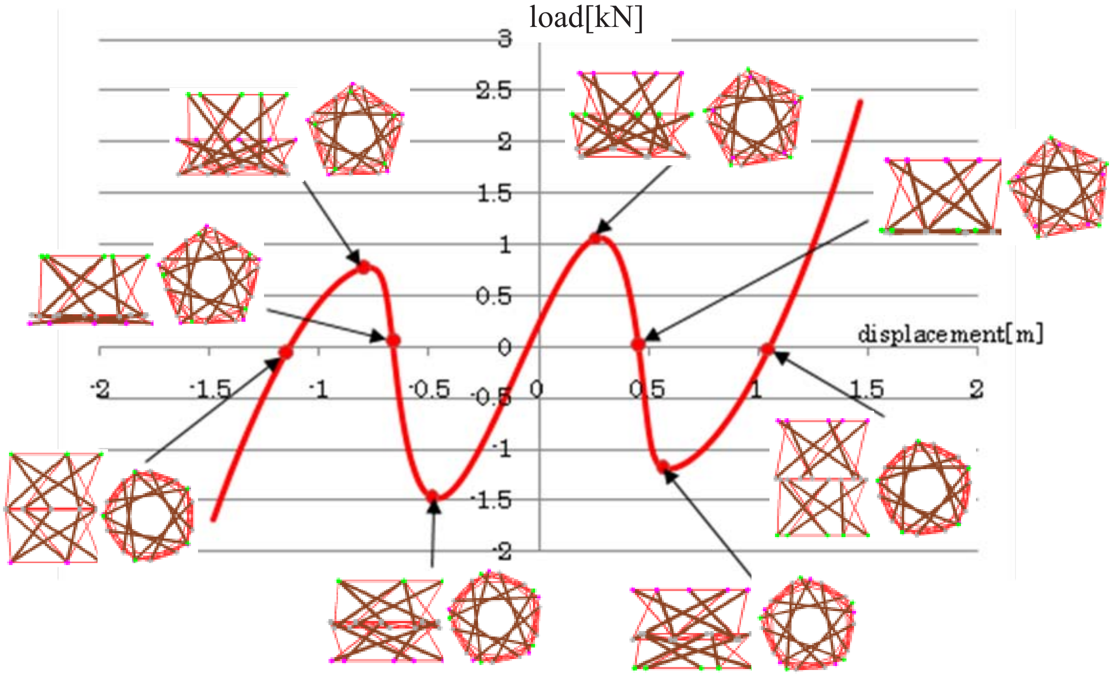

Figure 12 Path from origin of the solution by 2.4[kN] of initial load

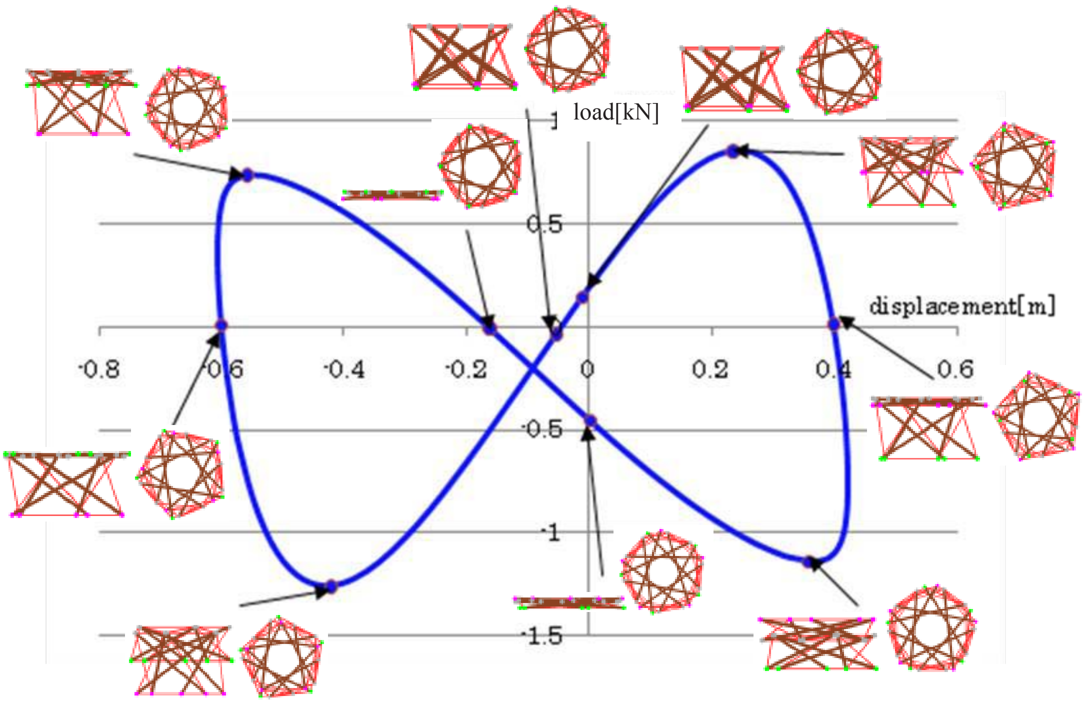

Figure 13 Path from origin of the solution by 0.0[kN] of initial load

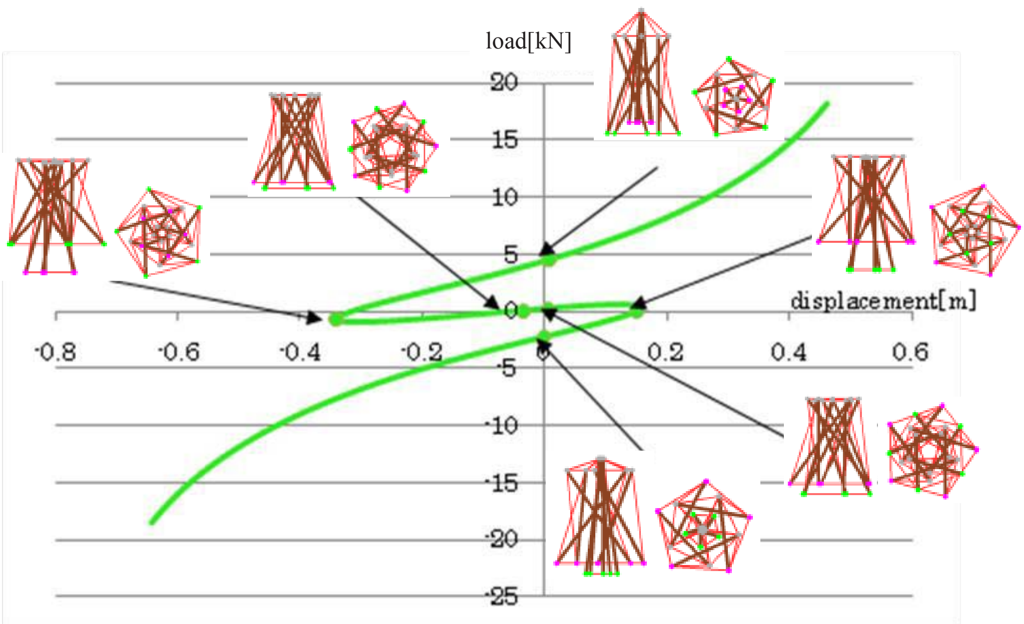

Figure 14 Path from origin of the solution by 1.45[kN] of initial load

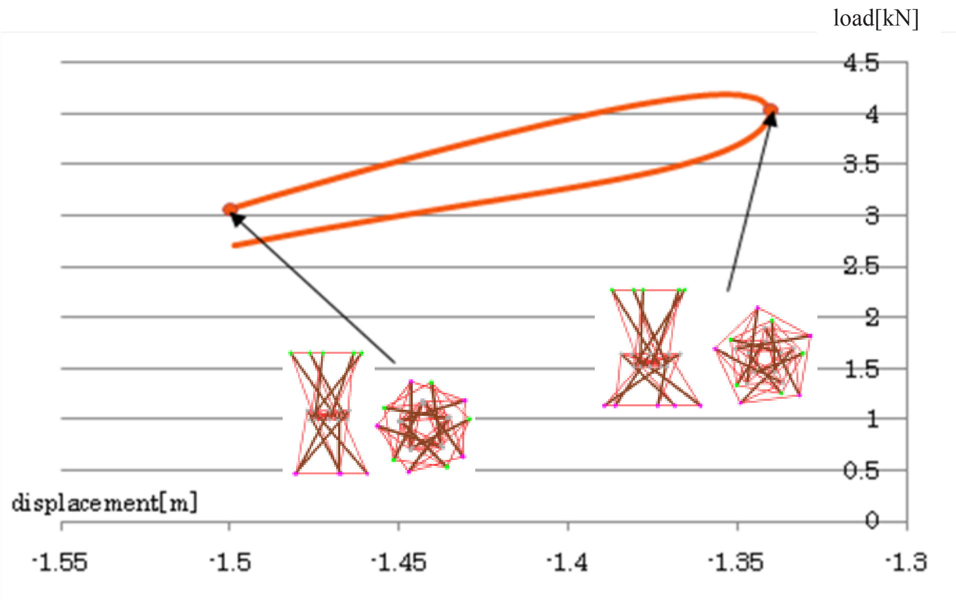

Figure 15 Path from origin of the solution by 2.7[kN] of initial load

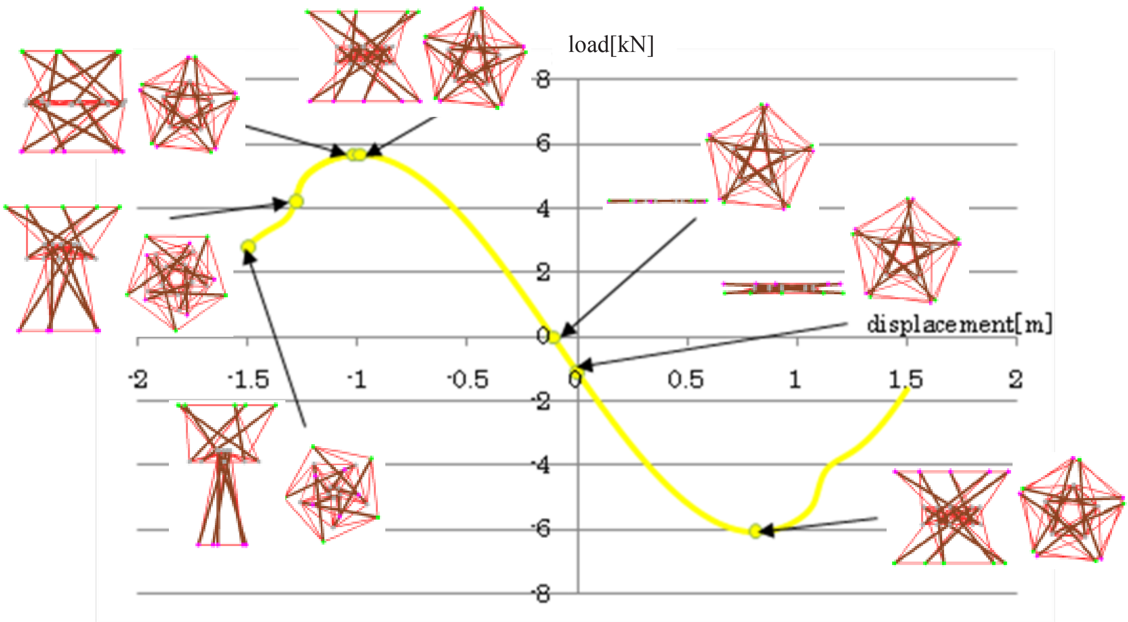

Figure 16 Path found by snap through from another path in Figure 15

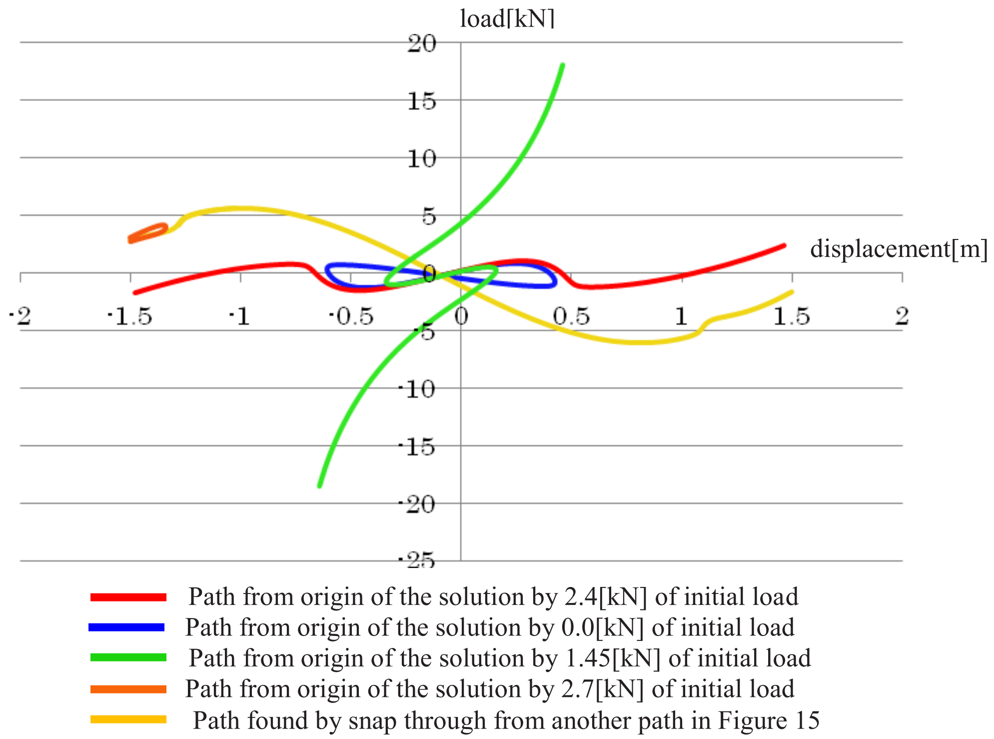

Figure 17 All the paths found in this study

4 CONCLUSIONS

Tensegrity structure has so many morphologies corresponding to one formation of connectivity; therefore, it may be difficult to find an expected shape even if using FDM. On the proposed procedure, the form-finding process is to get the equilibrium solutions of the virtual structure, which consists of the combination of axial force line elements and truss members. Using this virtual structure, the displacement and the length of the struts can be designated freely and it becomes easier to control the equilibrium shapes.

Furthermore, the path finding analysis gives us so many self-reliance solutions of tower tensegrity, and obtained paths bring us interesting information about the complicate equilibrium behavior.

Consequently, the proposed procedure is expected to be a reasonable form-finding process for various types of extensive tensegrity structures including cosmological, foldable and /or inflatable structures.

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