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GEOMETRY AND STIFFNESS IN THE CASE OF ARCH SUPPORTED TENSILE ROOFS WITH BLOCK AND TACKLE SUSPENSION

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Summary. The paper presents the comparative analysis of cable net roofs supported by truss arches with block and tackle suspension system. The effect of the friction (between the pulley and its shaft) on the internal forces of the supporting arches and on the displacements of the cable net roofs has been analysed. Structures with different number of supporting arches have been compared.

1 INTRODUCTION

The block and tackle suspension system has been invented by Kolozsváry[1] to minimise the bending moment of the supporting arches of tensile roofs by converting the random meteorological roof loads into nearly uniform, symmetric arch loads, based on the well-known principle of block and tackle. The main idea is to suspend the tensile roof by continuous suspension cables, which pass through series of upper and lower pulleys. Pairs of upper pulleys are secured to the truss arch; the lower pulleys are secured to the ridge cable of the roof (Fig. 1). Since the force in the continuous suspension cable is nearly uniform along the arch, the suspension forces acting on the arch are also nearly uniform. This means that the supporting arch can be designed to correspond to the pressure line of uniform arch loads; and the bending moments of the arch can be decreased radically.

The block and tackle suspension system and the first results of the static analysis based on idealised (frictionless) pulleys have been presented in Hincz[2]. Later the author has developed a Dynamic Relaxation[3,4] based procedure for the exact analysis of structures with block and tackle suspension, taking into account the friction of the pulleys. The details of the developed numerical method, the main steps of the analysis and the results of the analysis of a single arch supported tensile roof have been presented in Hincz[5,6].

The current paper presents the comparative analysis of tensile roofs with different number of supporting arches. Since the displacements of the roof are larger in the case of block and tackle suspension system than in the case of conventional suspension with individual suspension cables, it has been interesting to analyse the effect of the geometry of the roofs on the stiffness. The effect of the friction between the pulley and its shaft has also been analysed on the internal forces of the supporting arches and on the displacements of the roof.

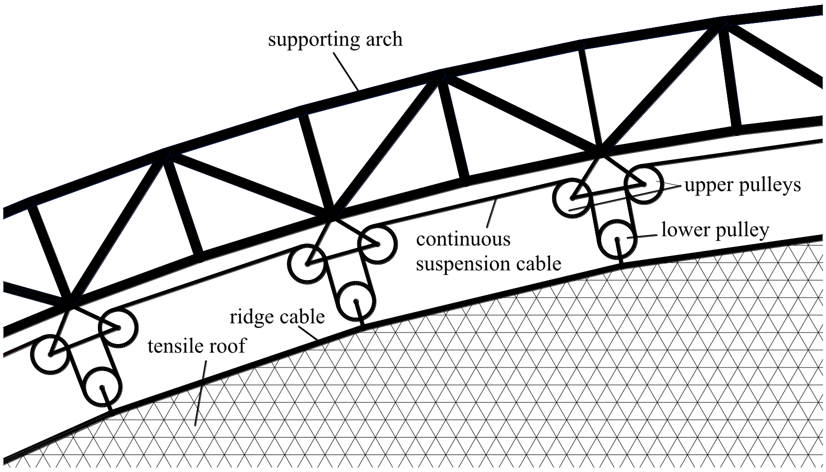

Figure 1: Side view of the block and tackle suspension system

2 THE ANALYSED MODELS

The models of four structures have been analysed. The main parts of the structures are the truss arch(es) of constant curvature, block and tackle suspension system, hyperbolic cable net and the supports. The static analysis of cable net roofs supported by 1, 2, 3 and 4 arches has been completed. Fig. 2. – Fig. 9. show the floor plan and the axonometric view of the models. The length of the diagonal of the covered area is 100 m. The free span of the supporting arches is 107.4 m. The height of the cable net roofs is approximately 18.5 m. The depth of the supporting arch(es) is 3 m, the width is 2.5 m. The arches have one lower and two upper chords and they are supported by universal hinges. The cable net is suspended at 15 points in the case of model 1. In the case of the other three models there are 14 suspension points on every arch. The ratio of the radius of the pulley (R) and the radius of its shaft (r) is $R/r=5$, the coefficient of friction (μ) is varied between 0.005 and 0.5.

The ratio of the total weight of the different element types to the covered area have been set to constant in the case of the different models to get comparable results. The ratio of the covered areas in the case of the four structures is 1 : 1 : 1.299 : 1.414. For example in the case of the different models the cross-sectional area of the chord members of the arch(es) is $A_1=500$ cm^2 (model 1), $A_1=250$ cm^2 (model 2), $A_1=216.5$ cm^2 (model 3) and $A_1=176.8$ cm^2 (model 4). The cross-sectional area of the snow cables is $A_s=14.85$ cm^2 (model 1), $A_s=12$ cm^2 (model 2), $A_s=14.68$ cm^2 (model 3) and $A_s=15.64$ cm^2 (model 4). The cross-sectional area of the wind cables is $A_w=16.98$ cm^2 (model 1), $A_w=12$ cm^2 (model 2), $A_w=9.00$ cm^2 (model 3) and $A_w=6.99$ cm^2 (model 4). The prestress in the continuous suspension cable is $P=200$ kN (model 1), $P=100$ kN (model 2), $P=86.6$ kN (model 3) and $P=70.71$ kN (model 4).

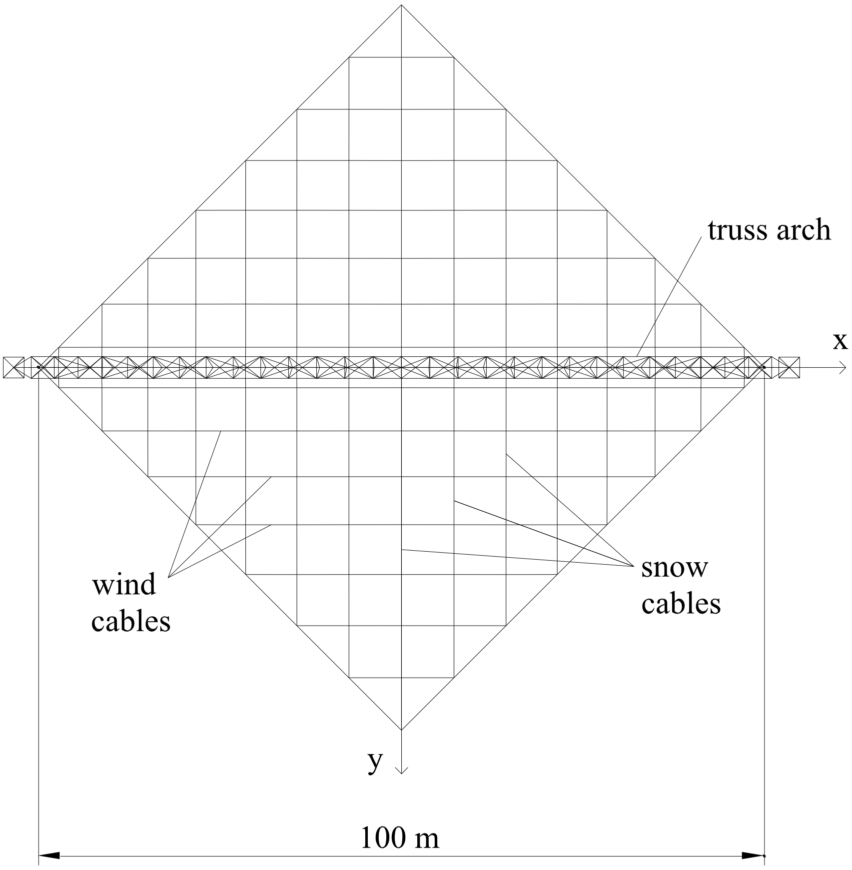

Figure 2: Floor plan of model 1, cable net supported by a single truss arch

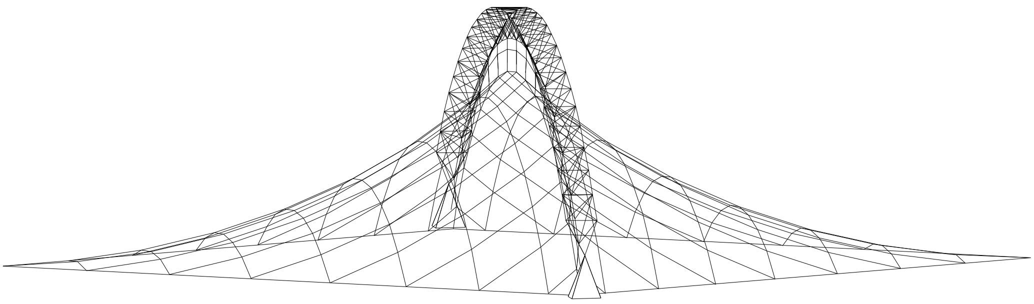

Figure 3: Axonometric view of model 1, cable net supported by a single truss arch

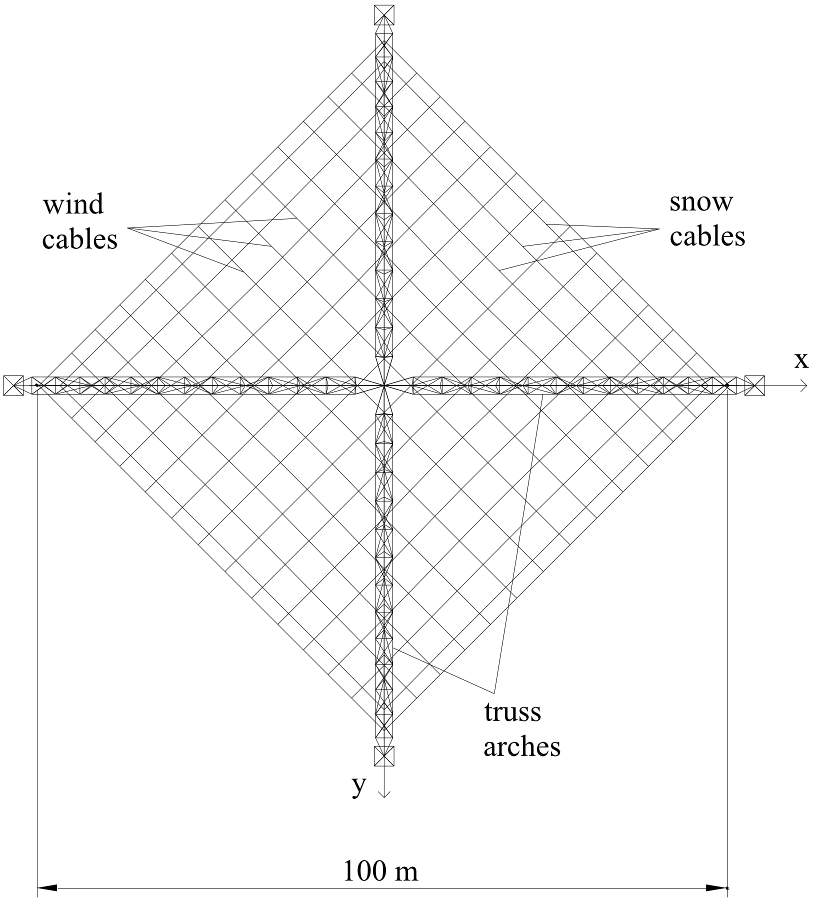

Figure 4: Floor plan of model 2, cable net supported by two truss arches

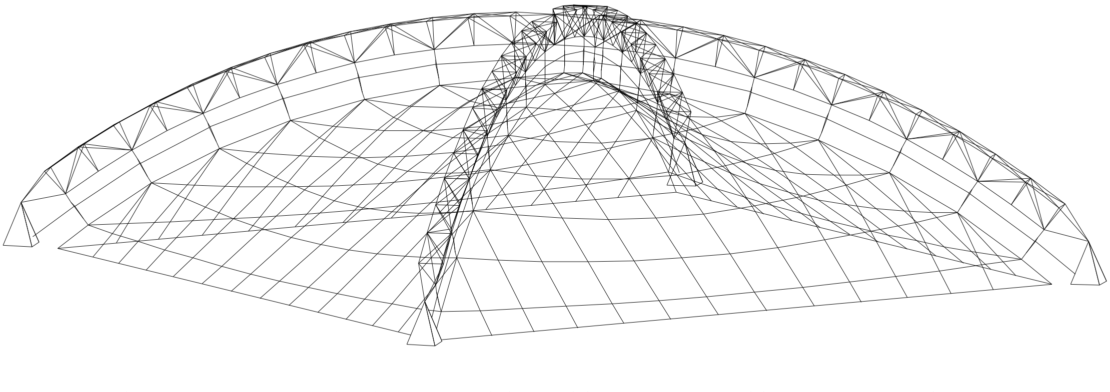

Figure 5: Axonometric view of model 2, cable net supported by two truss arches

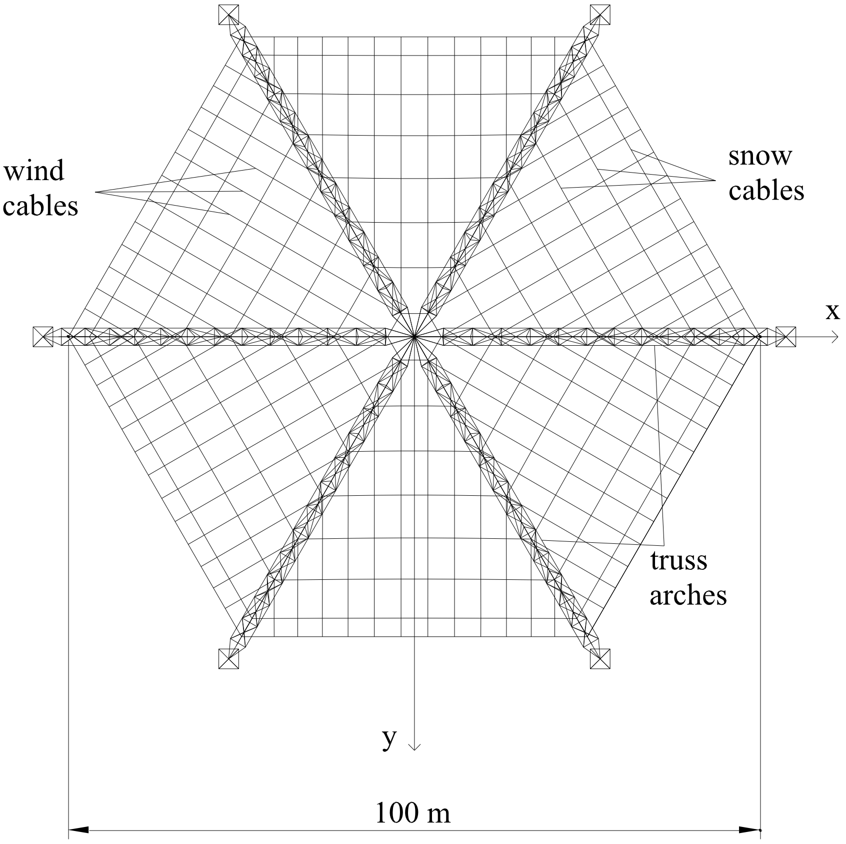

Figure 6: Floor plan of model 3, cable net supported by three truss arches

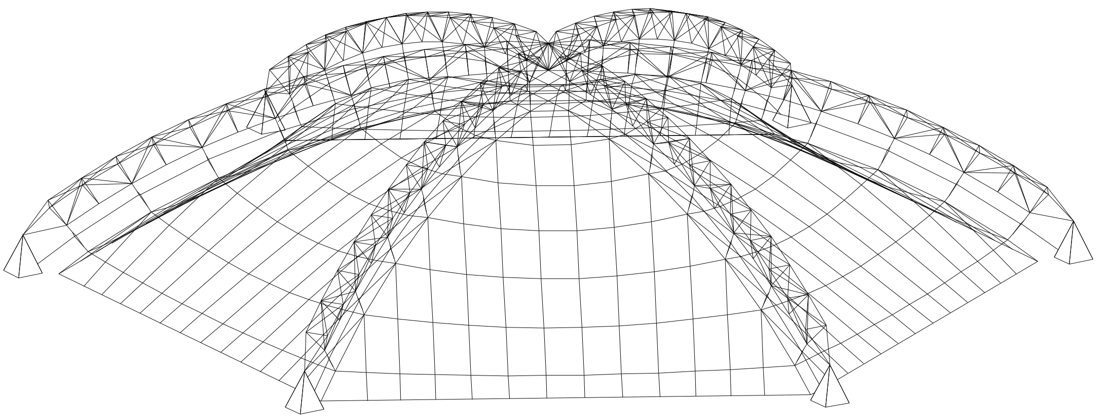

Figure 7: Axonometric view of model 3, cable net supported by three truss arches

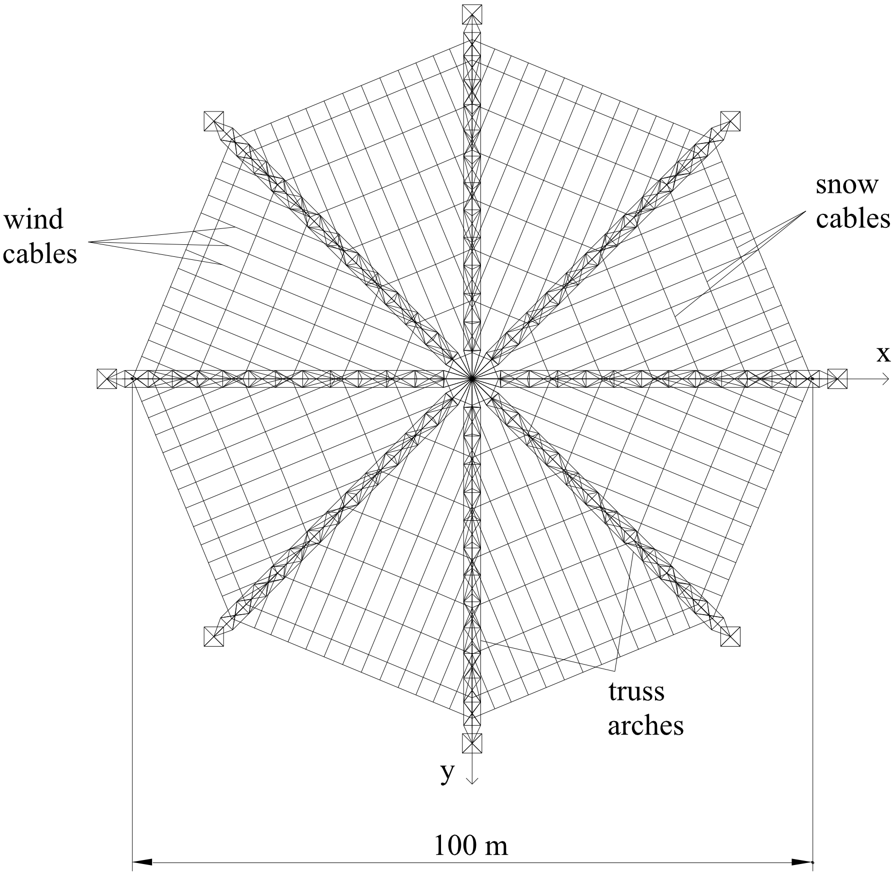

Figure 8: Floor plan of model 4, cable net supported by four truss arches

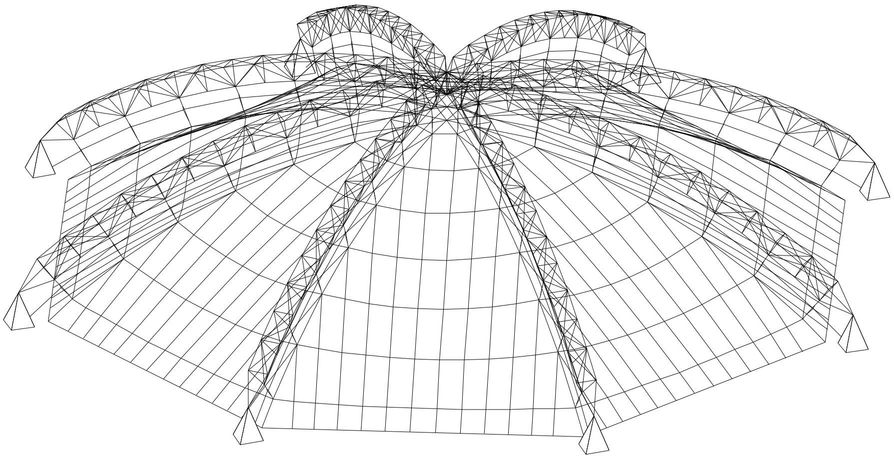

Figure 9: Axonometric view of model 4, cable net supported by four truss arches

All models have been analysed under six load cases:

- self weight and prestress: the construction shape without any external loads,
- total snow load: 1 kN/m^2 load on the whole roof,
- partial snow load 1: 1 kN/m^2 load on the flat part of the roof, where the slope is less than 30°,
- partial snow load 2: 1 kN/m^2 load on the half of the roof, where $x>0$,
- wind load x: parallel with direction x, the dynamic pressure is 1 kN/m^2,
- wind load xy: the angle between the wind direction and direction x is 45°, the dynamic pressure is 1 kN/m^2.

During the wind analysis fictitious, simplified pressure coefficients have been taken into account, calculated from the angle (α) between the wind direction and the normal vector of the roof, pointing into the roof, on the basis of the following relations:

0.8 when $\alpha \leq 30°$,
$-0.6+1.4(75°-\alpha)/45°$ when $30°<\alpha\leq 75°$,
-0.6 when $\alpha>75°$.

3 NUMERICAL RESULTS

During the nonlinear analysis the forces in the different elements and the displacements of the joints have been calculated. The normal and shear forces and the bending moments of the arches have been calculated between the suspension points on the basis of the forces in the truss members. All models have been analysed besides different coefficients of friction.

The analysis of the internal forces of the arches due to different load cases shows that the maximum normal force of the arches can be detected in the case of total snow load. The maximum shear force, the maximum bending moment and the maximum displacement of the roof can be detected in the case of partial snow load 2.

Fig. 10 shows the maximum normal force in the arches of model 2 due to different load cases, besides different coefficients of friction. The results show that the smaller coefficient of friction results in larger maximum normal force in the arches, in the case of $\mu=0.005$ the maximum normal force is approximately 15% larger than in the case of $\mu=0.5$.

Fig. 11 and Fig. 12 show the maximum shear force and maximum bending moment in the arches of model 2 due to different load cases. The results show that the smaller coefficient of friction results in significantly smaller maximum shear force and bending moment. In the case of $\mu=0.005$ the maximum shear force is 69%, the maximum bending moment is 82% smaller than in the case of $\mu=0.5$.

Since the aim of the block and tackle suspension system is to decrease the weight of the supporting arches by decreasing the bending moment, one of the most important questions is the effect of the coefficient of friction on the normal stress in the chord members of the truss arches. Fig. 13 presents the maximum normal stress in the chord members under partial snow load 2. The results show that the maximum normal stress is more than 30% smaller in the case of $\mu=0.005$ than in the case of $\mu=0.5$ (for every model).

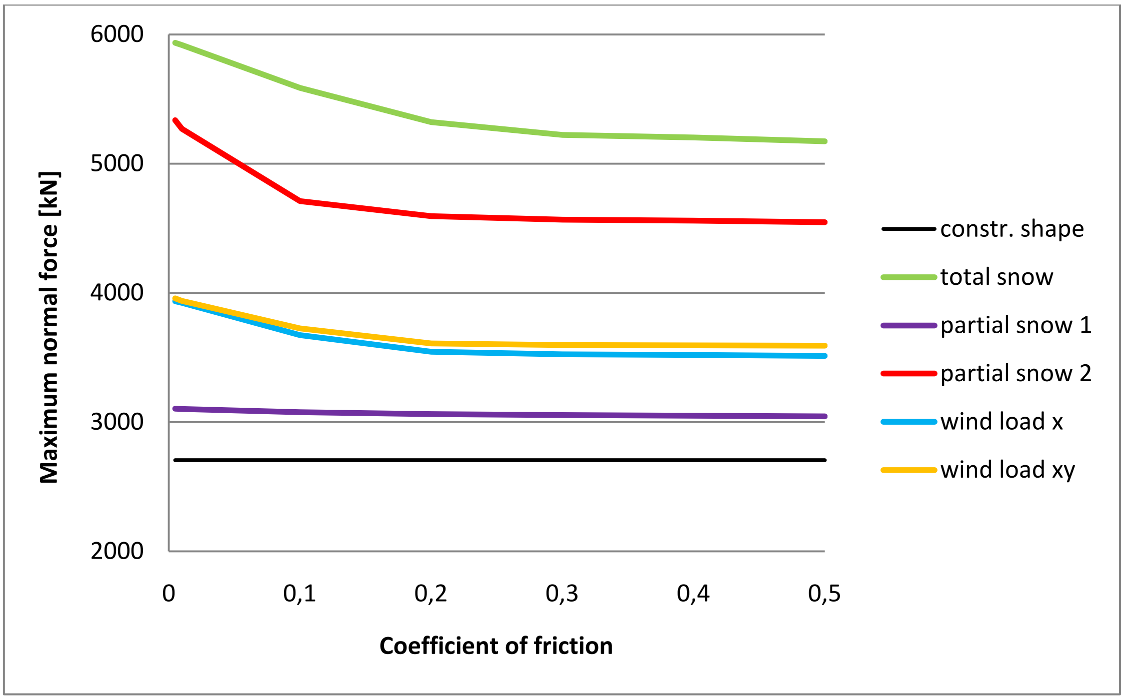

Figure 10: The maximum normal force in the arches of model 2 due to different load cases

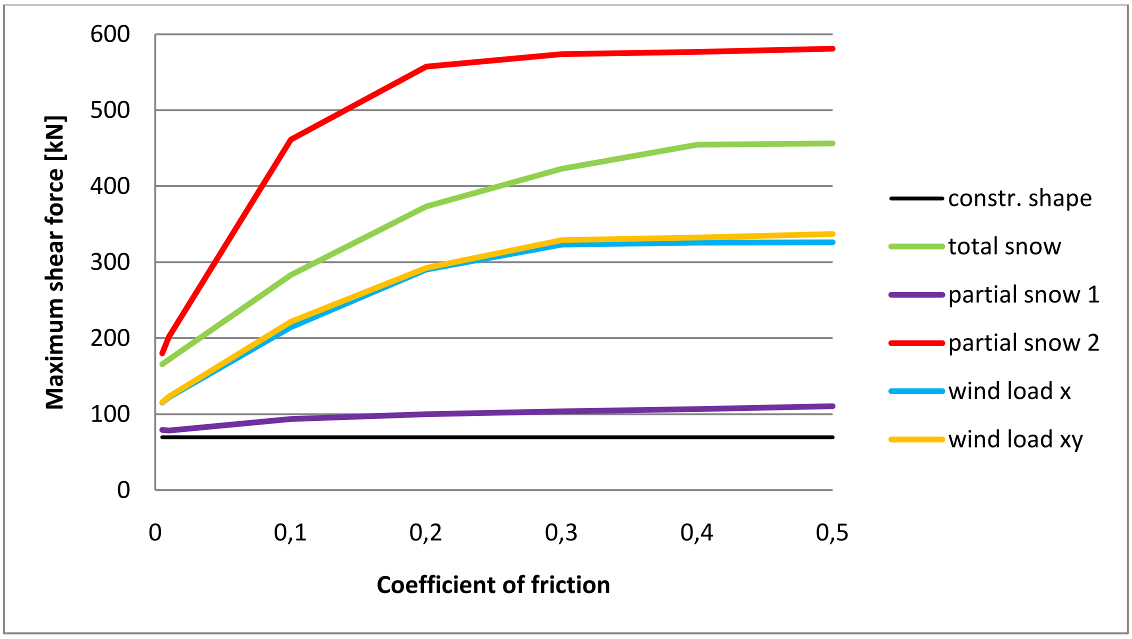

Figure 11: The maximum shear force in the arches of model 2 due to different load cases

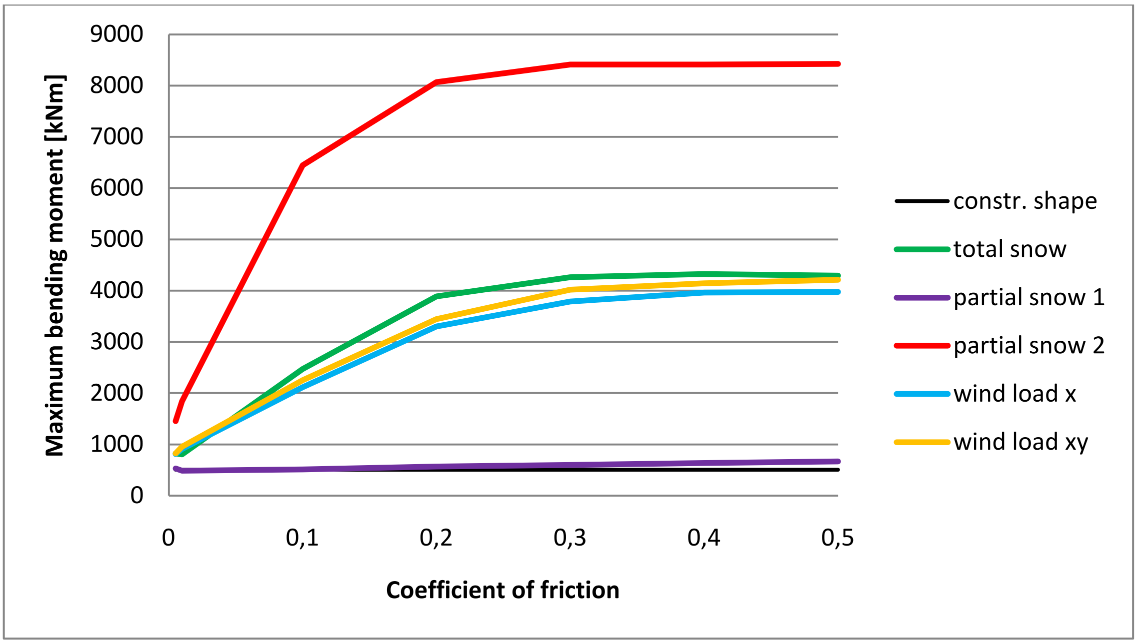

Figure 12: The maximum bending moment in the arches of model 2 due to different load cases

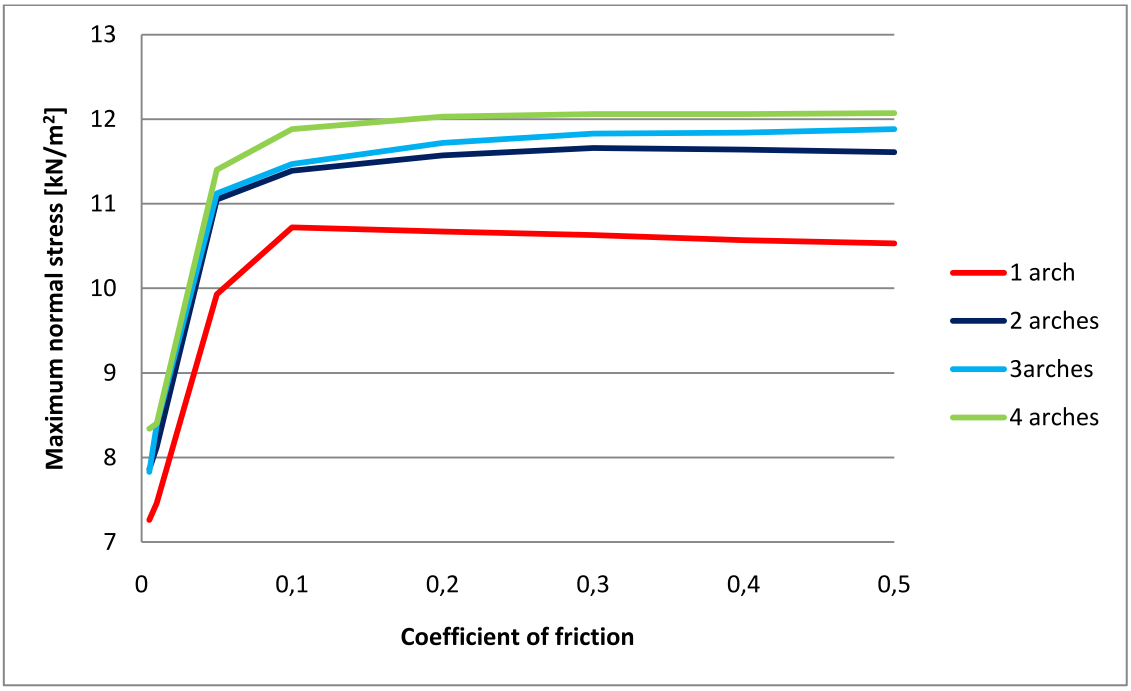

Figure 13: The maximum normal stress (compression) in the chord members under partial snow load 2 in the case of the different models

Fig. 14 and Fig. 15 show the maximum displacements of the cable net roof in the case of model 2 and model 3 due to different load cases. The results show that the smaller coefficient of friction results in larger displacements. The displacements of the joints of the cable net consist of two effects, the displacements of the lower pulleys (the "supports" of the cable net) and the deformation of the snow and wind cables. Since the smaller coefficient of friction results in larger displacements of the pulleys and the ridge cables, it results in larger maximum displacements of the cable net also. The difference between the maximum displacements for μ=0.005 and for μ=0.5 in the case of the different models are 33% (model 1), 39% (model 2), 87% (model 3), 139% (model 4). The results show that there are significant differences in the behaviour of the models.

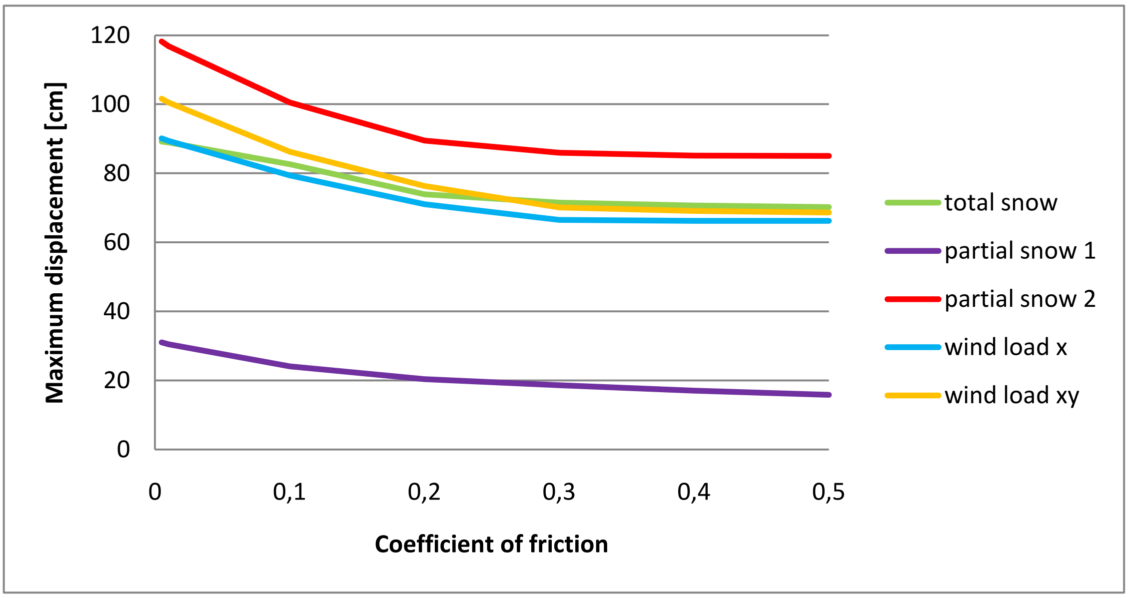

Figure 14: The maximum displacements of the roof supported by 2 arches due to different load cases

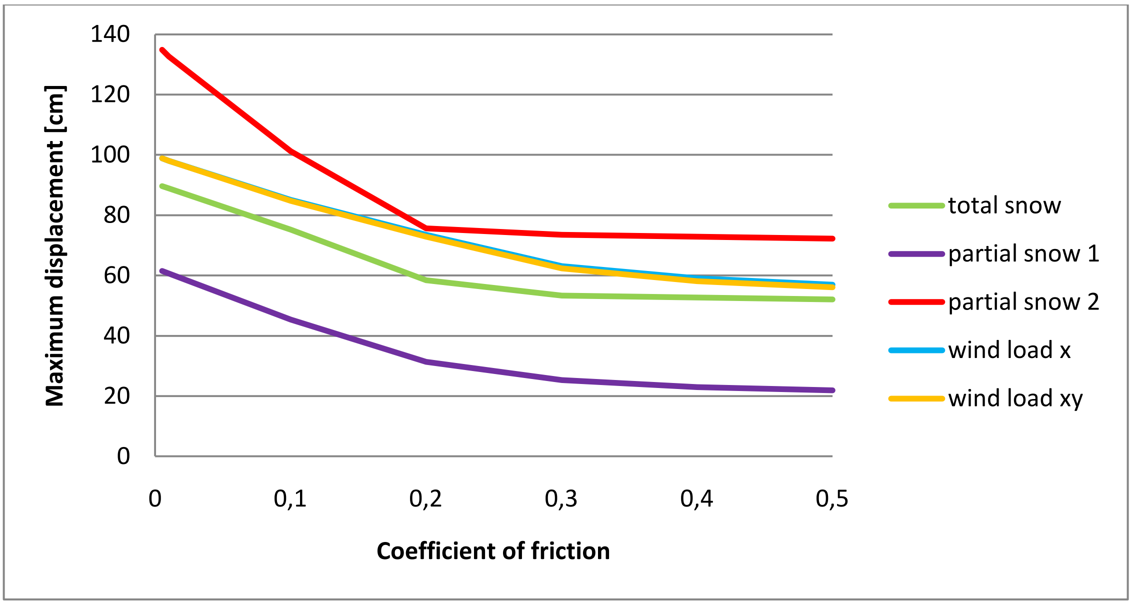

Figure 15: The maximum displacements of the roof supported by 3 arches

Fig. 16 shows the maximum displacements of the different models under partial snow load 2. In the case of large coefficient of friction the motion of the pulleys is less significant than the deformation of the snow cables. Therefore the length of the snow cables is determinant, the longer snow cables result in larger maximum displacements, the smallest maximum displacement has been detected in the case of model 4. In the case of smaller coefficient of friction the effect of the motion of the pulleys is more significant. On the other hand the motion of a pulley depends on the stiffness of the cable net in the suspension point of the ridge cable in radial direction. The increasing of the number of supporting arches results in smaller stiffness, because of the less significant, smoother ridge. In the case of μ=0.005 the motion of the pulleys and the stiffness of the cable net in radial direction at the ridge is determinant, the maximum displacement is detected in the case of model 4.

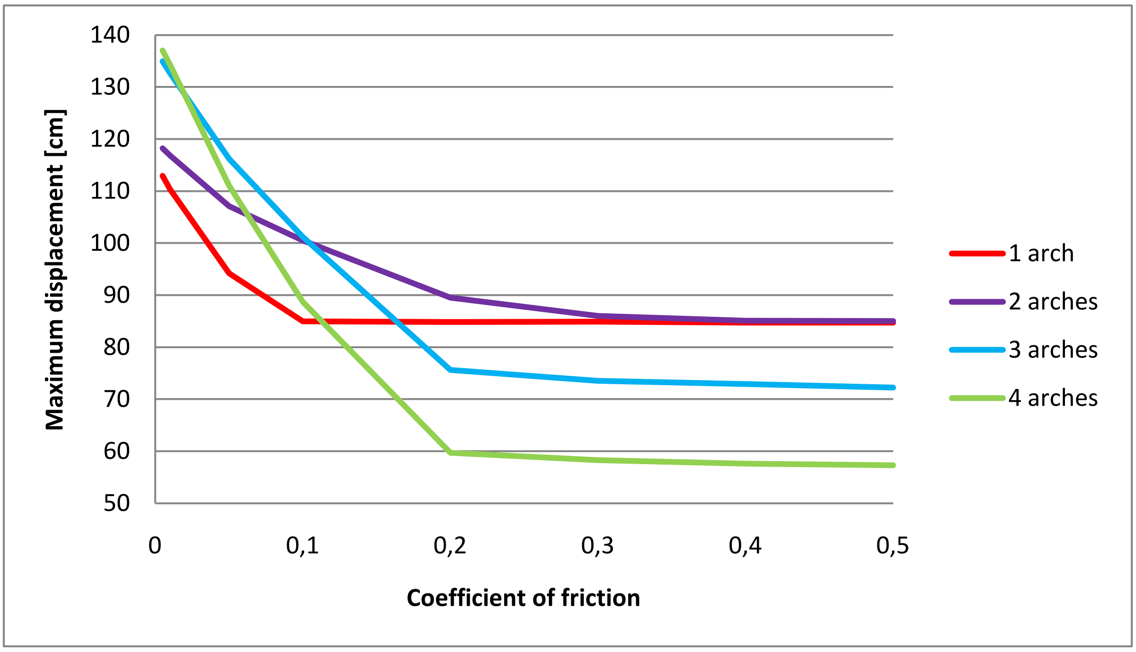

Figure 16: The maximum displacements of the structure under partial snow load 2 in the case of different number of supporting arches

On the other hand more supporting arches results in shorter snow cables and smaller forces in the snow cables. Fig. 17 presents the maximum stress in the snow cables due to different snow loads in the case of the different models.

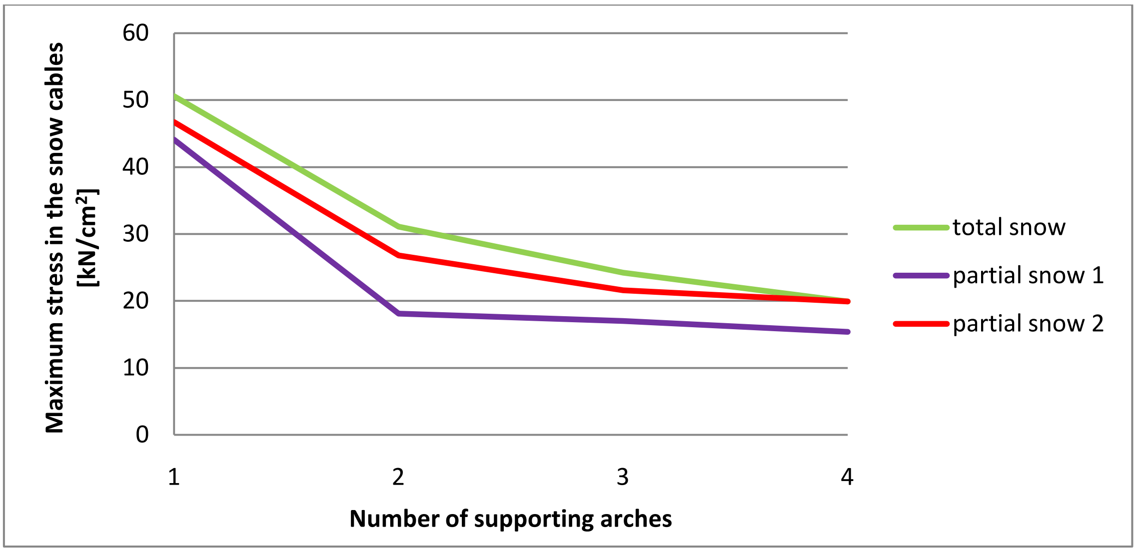

Figure 17: The maximum stress in the snow cables due to different load cases

4 CONCLUSIONS

The numerical analysis of cable net roofs supported by different number of truss arches is presented. The results show that the use of block and tackle suspension system can decrease the shear force and the bending moment in the arches significantly. The decreasing of the normal stress in the chord members and the efficiency of the block and tackle suspension system depends on the friction of the pulleys and almost unrelated to the number of supporting arches. On the other hand the results show that the number of supporting arches has a strong effect on the stiffness and the displacements of tensile roofs.

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