International Conference on Textile Composites and Inflatable Structures
STRUCTURAL MEMBRANES 2011
E. Oñate, B. Kröplin and K.-U.Bletzinger (Eds)

SIMULATION OF INFLATABLE STRUCTURES: TWO PROPOSALS OF DYNAMIC RELAXATION METHODS USABLE WITH ANY TYPE OF MEMBRANE ELEMENTS AND ANY REVERSIBLE BEHAVIOR

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Key words: Inflatable Structures, Dynamic Relaxation, Form-finding

Abstract. This work deals with the numerical study of inflatable fabric structures. As implicit integration schemes can lead to numerical difficulties, such as singular stiffness matrices, explicit schemes are preferred. Since the final objective of this study is to obtain the final shape of a structure, dynamic relaxation (DR) methods are used. These methods permit to obtain the final and stable shape of the inflatable fabric structures without doing so many time increments, which is the case when using a classical explicit integration method. Han and Lee (Computers and structures, 2003, 81, pp. 1677-1688) proposed an extension of the DR method stated by Barnes (Computers and Structures, 1988, pp. 685-695) suitable for triangular elements and elastic behavior. In this work we propose a modification of the method presented by Han and Lee which permits the method to be used with any kind of membrane or volumetric finite elements and any reversible behavior. Also, we propose another formulation based on the one initially proposed by Barnes. Furthermore, these presented methods are adapted to incremental loadings, allowing this way to obtain the pseudo-equilibriums of the intermediate phases. Numerical examples from academic problems (rectangular and circular membranes) show the efficiency and the reliability of proposed methods, with linear elasticity behavior, and also with general hyperelasticity and finite deformation states.

1 INTRODUCTION

The simulation by the FE method of inflatable fabric structures, when a pressure load is applied and an implicit scheme is used, can lead to severe instabilities due to the lack of stiffness in the fabric. Explicit time schemes overcome this difficulty, but they need a huge number of time steps to obtain a realistic stable final shape. This occurs when using natural damping.

We can find several examples of this issue in civil engineering: geotechnical problems [1], prestressed coated fabric membranes [2], achitectural structures [3], and space inflatable structures [4]. There have been several solutions proposed [5, 6, 7, 8, 9] by using *dynamic relaxation* methods.

Among the existing dynamic relaxation methods, we are interested on the one proposed by Barnes [10]. He has initially applied it to the calculation of prestressed cable structures and it has been further extended by Han and Lee [5] to be used with triangular elements and a linear elastic behavior. This method combines a kinetic damping (resetting the speed to zero at each kinetic energy peak), often used in form-finding, and an optimization of the mass matrix (proposed by Han and Lee).

One application of the method is thin fabric structures loaded by pressure, which are notably unstable during loading due to the lack of flexion stiffness. The static final form does not have to depend on the inertial forces that act during the transient evolution. Considering this, the right value of the mass is supposed to have no influence on its static final form. In order to quickly reach the stable deformed state, we must first adapt the mass matrix (a correct choice leads to an optimal convergence) and then use kinetic damping.

In this paper, we will present two main formulations. Firstly, we propose an extension of that Barnes-Han-Lee method. Secondly, we propose a general expression based on the works of Barnes for the mass matrix calculus. The basis of this second expression has already been proposed in previous papers (see Underwood [11] or Barnes [10]), but to our knowledge, no systematic studies have been done concerning its applications for simulation of the inflation of unstable structures. Our methods aim to find one solution when one or more solutions exist (there can be several stable final shapes).

2 DYNAMIC RELAXATION METHOD

The dynamic relaxation method proposed by Barnes [10] uses an arbitrary mass term in order to improve the kinetic damping while keeping the numerical stability. Barnes proposes a lumped mass matrix where the elements m_i in the diagonal are:

$$[m_i] = \lambda \frac{\Delta t^2}{2} [k_i] \tag{1}$$

The optimum mass matrix is calculated by adjusting the parameter λ. In the shape-finding process of membrane structures, due to the large variations of the structures, Barnes [12] proposed to choose the largest stiffness term for the calculation of mass term. Han and Lee [5] stated, for CST (constant stress triangle) elements, that the stress k_i at node i with m members can be approximated as

$$k_{imax} = \sum_e \frac{h}{4S_0^e} \left(\frac{E}{1-\nu^2} + \sigma_x + \sigma_y + \sigma_{xy} \right) \tag{2}$$

where h is the thickness of the element e; S_0^e is the initial surface of the element e and $\sigma_x, \sigma_y, \sigma_{xy}$ are the components of the stress tensor in an orthonormal basis associated to the surface element. In reference [13], the authors propose to suppress the surface term S_0^e in order to obtain mass dimensions in equation 1. They show, particularly, that in this case the optimal value of the coefficient λ is more stable, what is advantageous when it has to be defined.

2.1 Proposal 1: extension of the formulation of Barnes-Han-Lee

We propose an extension of the previous formulation, on one side to other type of elements and on other side to other material's behavior. The aim is therefore to study the feasability of this extension. Thus, the following expression would replace Han-Lee's [5] :

$$k_{imax} = \sum_e \frac{l_e}{4} \left(\alpha K + \beta \mu + \gamma \frac{I_\sigma}{3} + \frac{\theta}{2} \sigma_{mises} \right) \tag{3}$$

Comparing it with the expression 2, the term $\frac{E}{1-\nu^2}$ can be considered as controlling the shape changing or the element volume changing. It could be replaced by a linear combination of the average compressibility modulus K and shear modulus μ, available for all elastic and hyperelastic laws: $\alpha K + \beta \mu$. Concerning the second part of the equation 2 proposed by Han-Lee, the term $\sigma_x + \sigma_y + \sigma_{xy}$ can be considered as representing the stress state in the material (cumulating the spheric and deviatoric parts). For our proposal, and in order to extent the use of the formula to other geometries than triangular elements, we replace this term by an invariants' combination: $\gamma \frac{I_\sigma}{3} + \frac{\theta}{2} \sigma_{mises}$, where $I_\sigma = \sigma_k^k$ is the trace of the Cauchy stress tensor and σ_{mises} is the Mises stress. These two quantities are tensor invariants so they could be calculated for any type of element.

The parameters α, β, γ and θ in the expression 3 permit to control the influence of each entity. And finally, l_e represents a geometrical characteristic length, suitable for 2D elements (thickness) and for 3D elements (cubic root of the volume).

2.2 Proposal 2: alternative formulation for the mass matrix

Our second proposal refers to the theoretical elements proposed by the early work of Underwood [11] by using the theorem of Gerschgorin which permits to obtain an upper bound to the eigenvalue "i" of the stiffness matrix "K" of the system:

$$\rho_i \leqslant \sum_j |K_{ij}| \tag{4}$$

The mass matrix is then built to satisfy the stability condition with a unitary time step.

$$m_i = \frac{\lambda}{2} MAX_{k=1}^{3} \ \left(\rho_{3(i-1)+k} \right) \tag{5}$$

Unlike the physical masses, we can expect a variation of the mass matrix built this way during the calculation. Given that on one side we choose the maximum value over the 3 axes (loop over k in 5) and on the other side the stiffness of the initial material behavior is generally more important than during deformation, it has been proved in our simulations that the mass matrix calculated at the beginning was enough to "guide" the whole simulation, i.e. the update of the mass matrix along the calculation of our simulations did not provoke any time gainings.

The method presents as a disadvantage that it needs at least the calculation of one stiffness matrix, what implies the need of being able to calculate the tangent behavior. Generally, at the beginning of the loading process, the evolution is mainly elastic, so a priori the stiffness belonging to the tangent behavior should be enough if we consider that the material tends to soften.

2.3 Incremental scheme and convergence criterion

In the case of an incremental law of behavior, a priori not totally reversible, when the loading leads to big deformation-stress final states, the final-form finding procedure in one step is not correct anymore. The final state depends indeed on the loading path which in the case of DR can be very different to the real path. A solution is to use an incremental loading procedure. Assuming that increments are small enough, the procedure then guarantees a succession of points of static physical equilibrium that allows to be close to the real response of the structure during the loading.

The convergence criterion we use in the calculations is the following, being ε the instruction value:

$$Max\left(\frac{||Residual||_\infty}{||Reactions||_\infty}, \frac{Kinetic\ Energy}{Internal\ Energy}\right) \leqslant \varepsilon \tag{6}$$

3 NUMERICAL CASE STUDIES

We will present in the following subsections several numerical case studies on the formulae 3 and 5. We use the C++ academic finite elements software Herezh++ [14], and for the meshing and postprocessing, we use the software Gmsh [15]. Calculations are made on an Apple computer (Processor: 2x2.93 GHz Quad-Core Intel Xeon, Memory: 16 Gb 1066 MHz DDR3) with just one processor .

3.1 Membranes: complex meshes

We study here 2D meshes, with triangular and rectangular elements, and with linear and quadratic interpolations. Also, two different qualities of mesh are considered: a grid of 25×25 elements and another one of 50×50. For the first proposal, we use the parameters $\alpha = 0.9022557, \beta = 0.9022557, \gamma = 1, \theta = 1$. The calculation is carried out in linear elasticity, $E = 125MPa$ and $\nu = 0.41$, which are coherent with the parameters of behavior of a usual thin fabric.

This first numerical test is a classical one, which has already been studied, for example, in reference [6]: the inflation of a rectangular shaped cushion. It consists in two membranes joined at their periphery, with dimensions $500mm \times 500mm \times 0.27mm$. Due to the symmetries, just 1/8 of the cushion is studied. The cushion is loaded with an instantaneous internal pressure of 0.015 MPa. The mesh is constituted of a ruled triangular division, $25 \times 25 \rightarrow 625$ elements. The convergence criterion 6 is set to: $\varepsilon = 1.e - 3$.

For the different tested samples, we use the notation indicated in table 1. Thus, as an example, the notation RL1 means: test made with a mesh with 25×25 Rectangular elements, and using Linear interpolation.

T: Triangular Elements	**R:** Rectangular Elements
L: Linear Interpolation	**Q:** Quadratic Interpolation
1: Mesh with 25x25 elements	**2:** Mesh with 50x50 elements

Table 1: Notation

The figure 1 shows an example of inflated membrane. For each geometry, the table 2 shows the obtained results with an optimum λ.

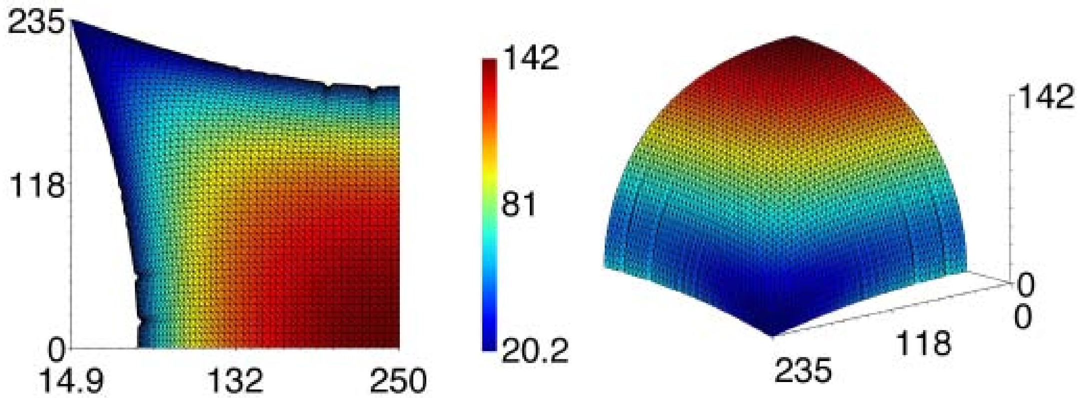

Figure 1: Inflated squared cushion: representation of 1/8 of the cushion, displacement isovalues

It can be observed that convergence is reached in all the cases. Particularly, the quadratic interpolation does not induce a particular difficulty.

3.2 Membrane: circular mesh

Inflation of a circular cushion, with a diameter or 400mm, where the mesh, Figure 2, includes both triangular and quadrilateral linear elements. The other material, geometric, etc, characteristics are identical to the squared cushion's ones, and also the methods.

	Proposal 1			***Proposal 2***		
Mesh	λ_{opt}	Iterations	Time [s]	λ_{opt}	Iterations	Time [s]
TL1 (2028 dof)	10	546	14,1	0,6	565	13,8
TL2 (7803 dof)	10	923	101,5	0,7	1081	111,8
TQ1 (7803 dof)	13	1128	118,4	0,6	1185	119,8
TQ2 (30603 dof)	14	2158	943,3	0,7	2358	970,1
RL1 (2028 dof)	6	422	23,7	0,5	423	22,6
RL2 (7803 dof)	6	671	150,4	0,6	841	183,9
RQ1 (7803 dof)	10	1015	159,8	0,5	970	148,8
RQ2 (30603 dof)	9	1688	1085,6	0,5	1889	1552,1

Table 2: Inflation of 1/8 of cushion in just one loading step, for different meshes

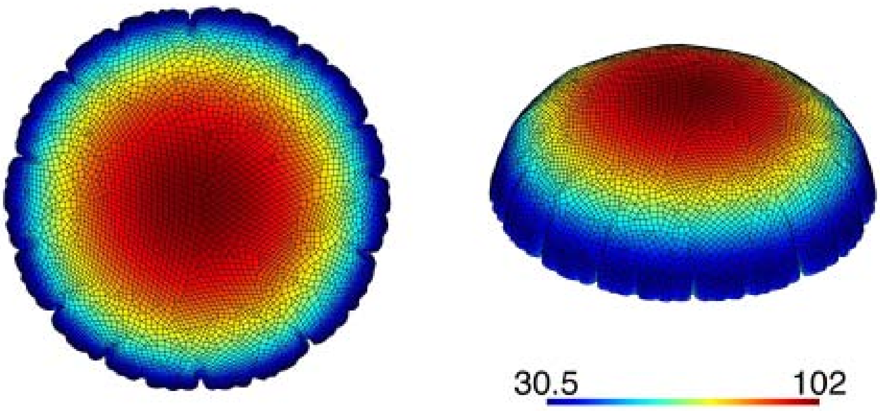

Figure 2: Inflated circular cushion: displacement isovalues

The Table 3 shows that the number of necessary iterations for convergence is coherent with those obtained for squared geometries. The mix of elements does not seem to alter the convergence. The proposal 2 is here more interesting, because even with the same previously used value of $\lambda = 0.6$, which is not the optimum for this case study, we obtain a very good convergence.

3.3 Incremental calculations

We introduced an incremental version of both proposals 1 and 2 in order to use them with incremental laws of behavior. The dynamic relaxation method is used here to find the steady state at the end of each loading step. The method is thus analogous to a classic iterative one, with the difference that it does not need the determination of a tangent evolution; but in return it needs a larger number of iterations.

We observed that the method works for all kind of elements. Figures 3 and 4 are a

	Proposal 1			***Proposal 2***		
Mesh	λ_{opt}	Iterations	Time [s]		Iterations	Time [s]
Circular	10	2096	1068,3	λ_{opt}=0,4	1322	616,745
17856 dof				λ=0,6	1703	792,6

Table 3: Inflation of a half of a circular cushion, with a mix of linear triangular and quadrangular elements

sample. They show the different steps of loading constituting the result of the intermediate pseudo-steady states resulting of the multi-step loading. To improve the clarity of the figures, not all the increments are shown.

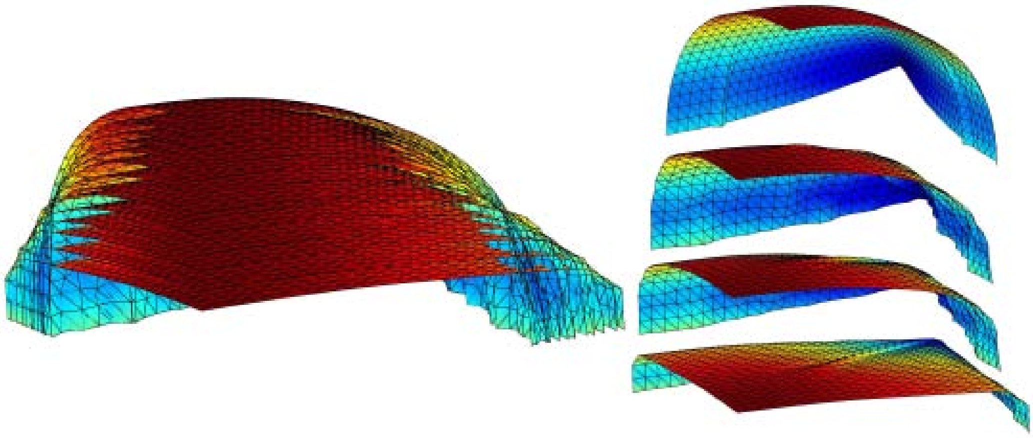

Figure 3: Incremental inflation of a cushion, representation of 1/8 of the cushion, 2D membrane mesh

3.4 Complex law of behavior

This last case study is exploratory. It consists in observing the influence of a complex law of behavior, preferably incremental. For that, we consider the inflation of a squared membrane, but now meshed with 3D quadratic hexahedral elements (showing therefore that our proposals work also with 3D elements). The geometric dimensions are $250mm \times 250mm \times 6mm$, the mesh is constituted of a grid of $10 \times 10 \times 1$ and the used λ is 3 (bigger than before, to be sure to overcome nonlinearities). The loading is quasi-static, so the speed effects are negligible. The material is considered an elastomer Vitton where the law is modeled by assembling an additive hyperelastic stress and a stress hysteresis. For more details of the law, see [16].

The calculation converges despite the complex behavior (see figure 4). We observe in the table 4 a number of iterations much higher for the first increment, and then a big regular decreasing of the number of iterations, in opposition to the case of linear elasticity. The reason is that the weak initial stiffness of the material leads to a very big displacement

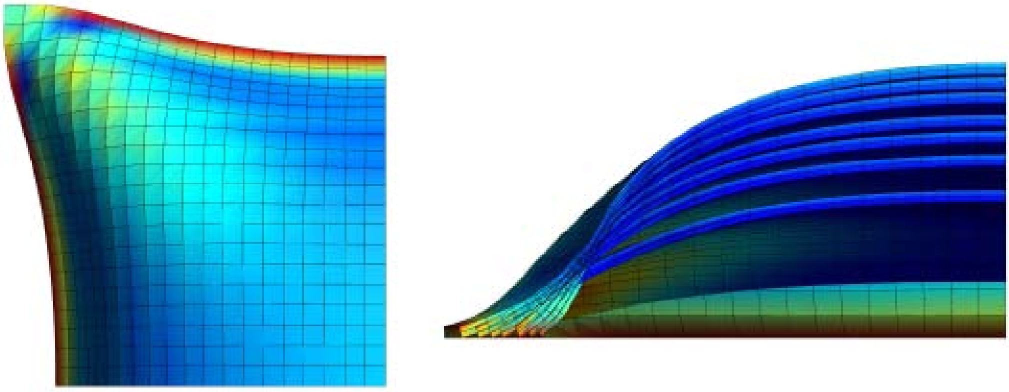

Figure 4: Incremental inflation of a cushion, representation of 1/8 of the cushion, 3D mesh

Mesh	λ	**Inc 1**	**Inc 2**	**Inc 3**	**Inc 4**	**Inc 5**
3969 dof	3	32020	8470	6540	4730	2830
		Inc 6	**Inc 7**	**Inc 8**	**Inc 9**	**Inc 10**
		2910	970	2590	910	690

Table 4: Inflation of an elastomeric plate: Needed iterations per loading step

at the first increment. Then, the material rigidifies and the displacements per increment decrease importantly. The observed evolution of the number of necessary iterations in function of the loading step is therefore logical.

4 Conclusions and discussion

We presented two proposed formulae to extend Barnes-Han-Lee's dynamic relaxation method with kinetic damping. Barnes-Han-Lee's method was limited to the particular case of linear triangular elements and elastic behavior. Our proposed formulae allow for applications beyond the original limitations. This is our main contribution.

Furthermore, we have numerically demonstrated several other advantages of our formulae. We showed our proposals are effective for 2D and 3D elements, with linear and quadratic interpolation. We showed the formulae are compatible with an incremental formulation, which minimizes the influence of the loading path. Our exploratory work showed that the method works with a complex incremental law of behavior.

Seen all these results, we present dynamic relaxation with kinetic damping, using the incremental formulation, as an useful alternative to the classic Newton's method in the cases where instabilities are found.

This work covered structural instabilities. In future work, the study will continue with material instabilities.

5 Acknowledgements

This work is financed by *Région Bretagne*, and in collaboration with the Navimo Society. We thank both of them their support and collaboration, that made it possible.

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