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AIR VOLUME ELEMENTS FOR DISTRIBUTION OF PRESSURE IN AIR CUSHION MEMBRANES

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Summary. This paper describes techniques and presents examples for nonlinear finite element analysis on membranes interacting with an air volume.

1 INTRODUCTION

Extended use of air cushion membranes requires detailed analysis of pressure distribution and pressure transmission from one membrane to another. Standard finite element analysis, coupling membrane shell with real volume brick elements with specific air material properties, produces good results for semi-scaled deformations. The following figure shows a cushion loaded only on top-left side. The inner brick volume elements move to the right and distribute the pressure to the whole upper as well as to the lower membrane. A constant air pressure in the cushion volume elements (yellow) appears.

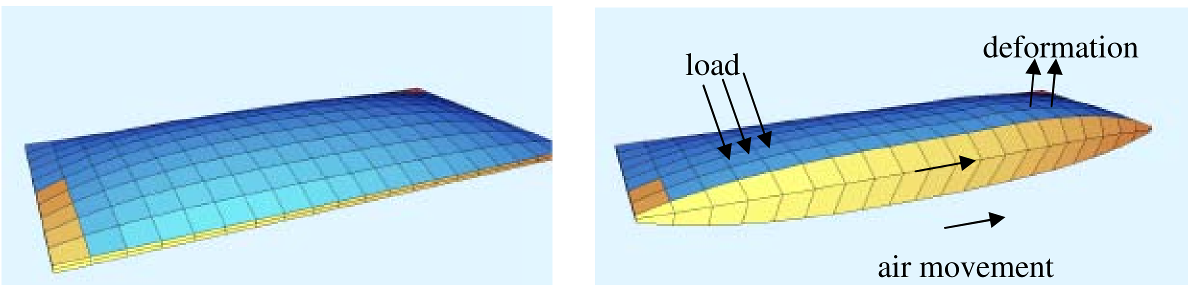

Figure 1: Air cushion with brick air volume elements

However, this technique is not applicable for larger deformations and air movements. Also the need of brick elements is not comfortable.

A technique without brick elements with a single stiffness bubble connecting all nodes of the membrane is implemented into a common software package and is presented in this paper. A similar approach is described in [1] with further references.

2 TECHNIQUE FOR AIR VOLUME BUBBLE

The finite element, connecting all nodes of the membrane with a single stiffness bubble, will be called VOLU element in this paper.

2.1 Principle of load stiffness method

Looking to a closed air volume with contact nodes on the boundary, we can easily determine the effect of the displacement of one node. If we move one node in z direction in the sketch below (v_i), we get a reduction of the total volume. This volume reduction results in an air pressure increase inside. This pressure now acts on all nodes of the boundary with forces P_{ik}.

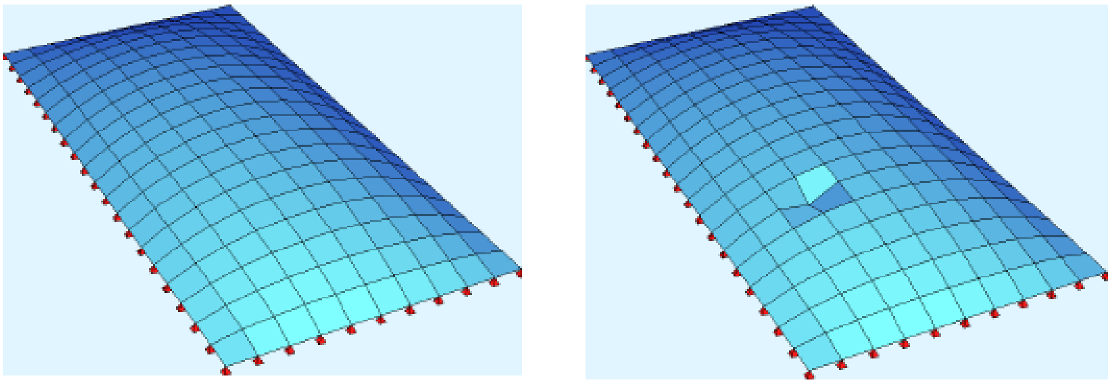

Figure 2: Load stiffness method: increased air pressure induced by deformation v_i of one node

This means that a displacement v_i in node i produces forces P_{ik} at all other nodes or one row in the air volume stiffness bubble S:

$$\begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ \cdots \\ \cdots \\ \cdots \\ P_n \end{bmatrix} = \begin{bmatrix} \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ & \cdot & \cdot & \cdot & \cdot & S_{ik} & \cdot \\ & & \cdot & \cdot & \cdot & \cdot & \cdot \\ & & & \cdot & \cdot & \cdot & \cdot \\ & & & & \cdot & \cdot & \cdot \\ & & & & & \cdot & \cdot \\ & & & & & & \cdot \end{bmatrix} * \begin{bmatrix} v_1 \\ v_2 \\ v_3 \\ \cdots \\ \cdots \\ v_k \\ v_n \end{bmatrix}$$

$$\boxed{\vec{P} = S * \vec{v}}$$

Figure 3: A single displacement v_i creates one row in the volume stiffness bubble S

As the volume reduction is v_i multiplied with the corresponding area a_i of the node, the pressure increase is $k*v_i*a_i /V0$, with V0 = total volume and k = compression modulus (100 kN/m2 for air). So S_{ik} is then pressure*area: $S_{ik} = k*v_i*a_i /V0$.

Of course in 3D the surface normal vector direction has to be taken into account. For the coarse finite element mesh in 3.1 (tennis court), we have 361 nodes with 3*361=1083 unknowns and get a triangular stiffness matrix containing n*(n+1)/2 = 586986 entries.

2.2 Global finite element analysis

Using the system of figure 2 we now get:

- 10*20 = 400 quad elements for the membrane with each 3*4=12 unknowns
- 1 VOLU air volume element with 3*10*20=600 unknowns

With these 401 elements the FE program can now solve the equation system for external loads. The quad elements create membrane forces while the VOLU yields only one single result – the air pressure inside.

2.3 Nonlinear effects

Geometric nonlinear behavior leads to the effect that the volume increase is not linear with the deformation because the quad element size may change as extremely shown in the balloon example 3.5. These effects must be balanced in a nonlinear iteration. The nonlinear iteration is necessary anyway for the geometric nonlinear membrane and the wrinkling effects. Also an air pressure on the membrane changes the direction with increasing deformations and must be updated (nonlinear effect).

2.4 Similar problem on slip cables

A similar approach is also used for slip cables in the SOFiSTiK software. Using a load stiffness method, all cables form one single slip cable stiffness. In the following example a displacement in one node of the slip cable ensemble (yellow) results in a length change and thus in a force change in all cable parts. Using the stiffness bubble technique this also works in just one single linear equation step. All cable parts work as one element!

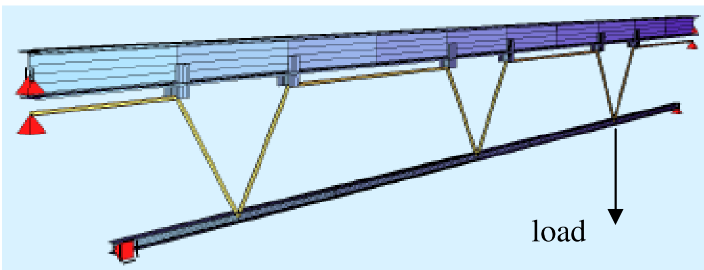

Figure 4: One single slip cable element (yellow) connecting the upper and lower beams (blue)

3 EXAMPLES

3.1 Water bed

Starting with a plain mesh we first apply a prestress on the membrane. Then we apply a partial area load as shown in figure 5. We compare the analysis of the pure membrane with the analysis of the membrane plus a VOLU air element with a starting volume of 5000 m3. Input: VOLU NO 1 GRP 1 V0 5000. GRP 1 just defines the quad elements used for the air volume boundary.

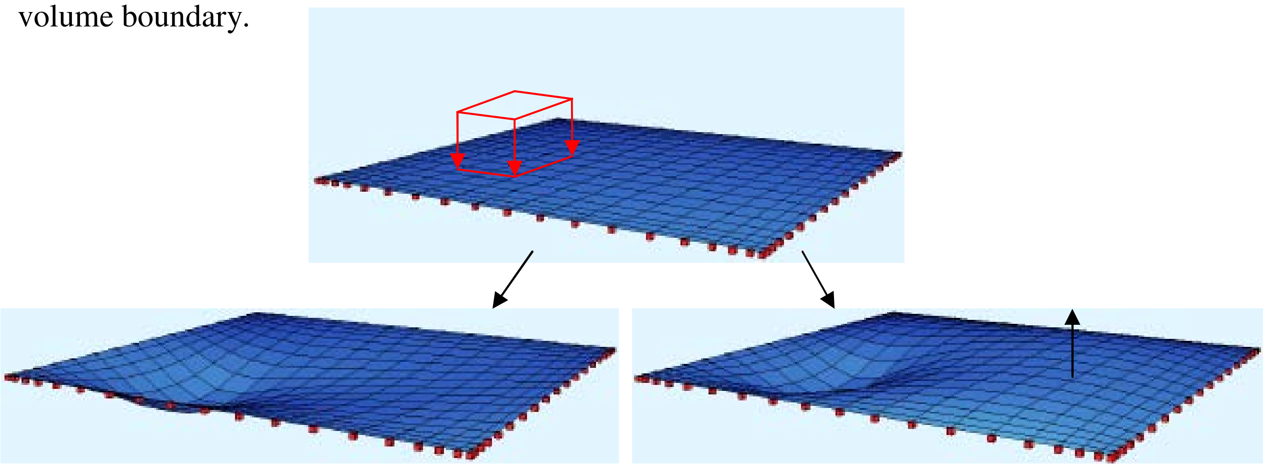

Figure 5: Straight membrane: left without VOLU; right with VOLU air volume: uplift water bed effect

With the air volume element VOLU, the load induces vertical deformation and thus an air pressure increase in the volume below, uplifting the right part of the water bed. This already happens in a first linear equation step!

3.2 Tennis court

With the VOLU element we can also blow up the membrane over a tennis court. Defining a fixed pressure we now do not need the VOLU stiffness, because we do not want a pressure change due to the deformations. But the pressure load has to be updated during the nonlinear iteration because the load direction as well as the load area changes – the membrane area and so the volume surface increases. The membrane itself can be defined without stress change leading to a perfect formfinding with given membrane prestress.

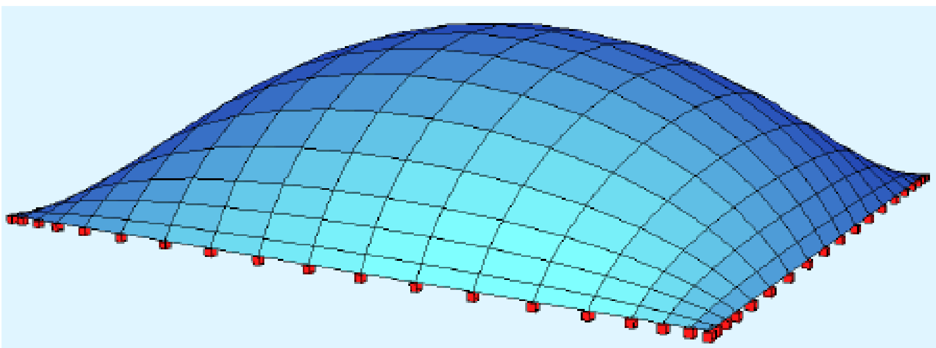

Figure 6: Membrane formfinding with given membrane prestress and given VOLU air pressure

Using this formfinding shape and stress state we can now activate the real stiffness of the membrane as well as of the VOLU element and apply additional external loads as wind or snow. Also an inner air volume increase or an air temperature increase can be applied on the VOLU air volume element. The following result shows the effect of such a temperature loadcase in the VOLU element:

```
QUAD VOLUME RESULTS (VOLU)
Loadcase    24    +Temperatur Luft +20 Gra
NO        V0     V-PLC     V-now     P-PLC     P-now area-PLC area-now
        [m3]      [m3]      [m3]   [kN/m2]   [kN/m2]      [m2]      [m2]
 1 5000.000 11164.30 11388.47      0.40      0.67 1536.901 1555.427
 P: positive values = pressure
 P-start = P-PLC + k*DP/V-PLF + k*DT*alphaT   with k=compression modulus
```

The starting volume defined for the initial mesh was 5000 m3 (4 m height under the straight membrane). The formfinding increases this initial volume to 11164 m3, the additional air temperature increases it further to 11388 m3. The initial formfinding air pressure of 0.40 kN/m2 increases to 0.67 N/m2, of course corresponding to a stress increase in the membrane. The actual geometric nonlinear surface of the VOLU element reaches 1555 m2 (plain mesh at the beginning was 37.5*33.5 = 1256 m2).

3.3 Air cushion

The same technique is now applied on two membranes. The VOLU element uses both membranes as surface. The starting volume is just the membrane area 4m*8m multiplied with the initial 10 mm distance = 0.32 m3.

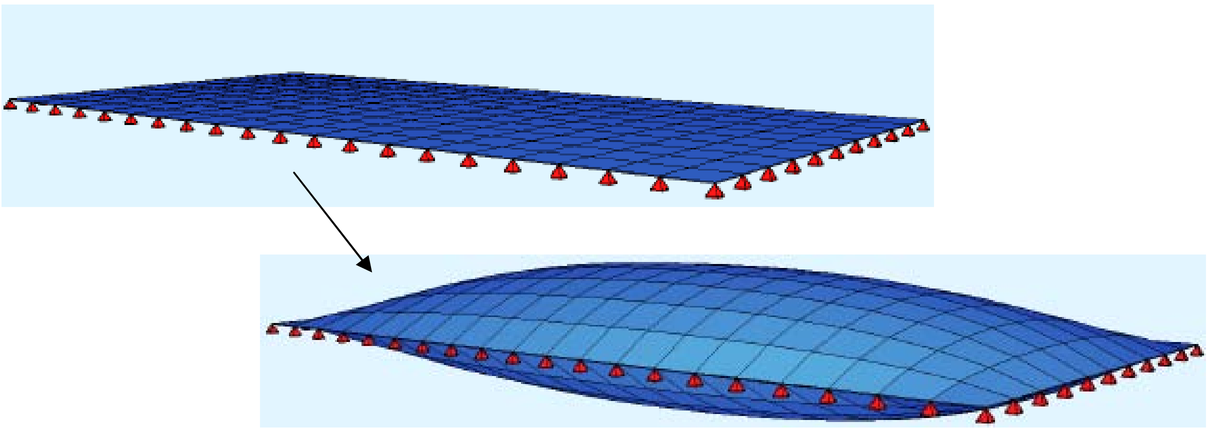

Figure 7: Air cushion formfinding with given membrane prestress and given VOLU air pressure

In the formfinding step with 0.2 kN/m2 air pressure and 0.6 kN/m2 membrane prestress we get a volume of 21.472 m3:

```
QUAD VOLUME RESULTS (VOLU)
Loadcase     1   Formfindung
NO        V0     V-now        P0     P-now    area-0 area-now
        [m3]      [m3]   [kN/m2]   [kN/m2]      [m2]      [m2]
 1     0.320    21.472      0.00      0.20    64.000    67.616
 P: positive values = pressure
```

This formfinding system is now frozen and can be loaded with real external actions. A wind gust loading applied only to the upper left part of the membrane, as shown in the brick example in figure 1, now behaves similar (higher load used). In comparison to figure 1, the analysis is now done without brick elements but with a not visible VOLU element.

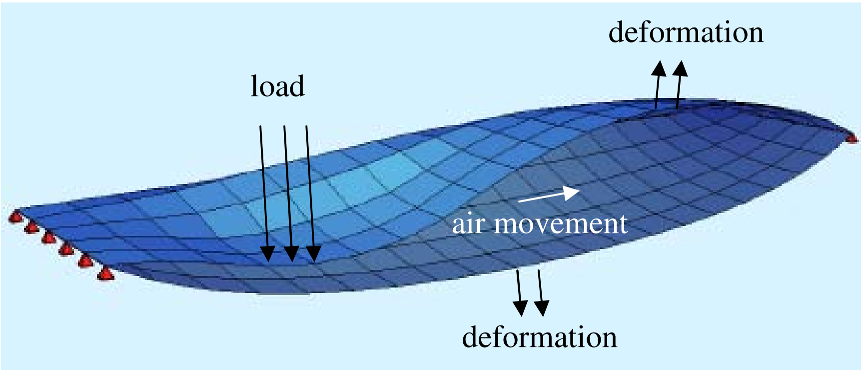

Figure 8: Air pressure effect in an air cushion – air volume element VOLU acts but is not visible

The air pressure increases up to 0.44 kN/m2 and as in a real air cushion. Also the lower membrane is a part of the flexible system and acts with a stress increase. The air volume only decreases slightly from 21.472 to 21.420 m3 due to the given air compressibility:

```
QUAD VOLUME RESULTS (VOLU)
Loadcase    11   Wind gust on upper folio
 NO        V0     V-PLC     V-now     P-PLC   P-start     P-now area-PLC area-now
        [m3]      [m3]      [m3]   [kN/m2]   [kN/m2]   [kN/m2]      [m2]      [m2]
 1     0.320    21.472    21.420      0.20      0.20      0.44    67.616    67.921
 P: positive values = pressure
 P-start = P-PLC + k*DP/V-PLF + k*DT*alphaT   with k=compression modulus
```

3.4 Independent air surfaces

In the example above the upper and lower membranes are only connected by the VOLU element. This behavior can be shown more crass, if the two membranes are really separated widely. In the following system only the left membrane is loaded but also the right membrane reacts due to the given air volume inside the building.

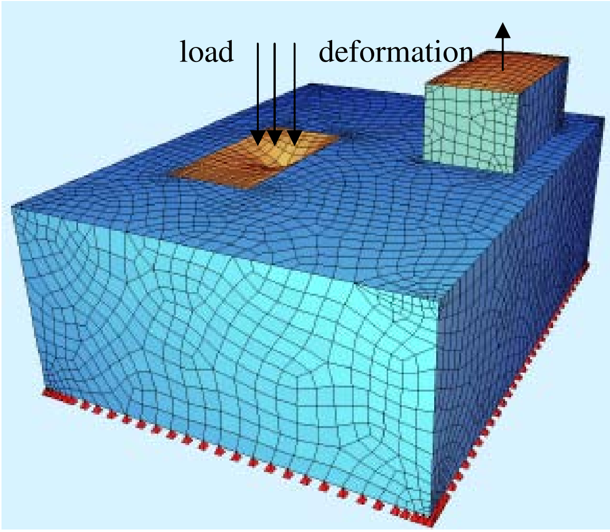

Figure 9: Air pressure effect in a closed building – right membrane goes up on loading the left membrane

3.5 Blown up balloon

In a last example the VOLU element is used to blow up a membrane. In a normal membrane analysis without volume control this causes problems, because with increasing radius the necessary inner air pressure does not increase anymore. Here we get:

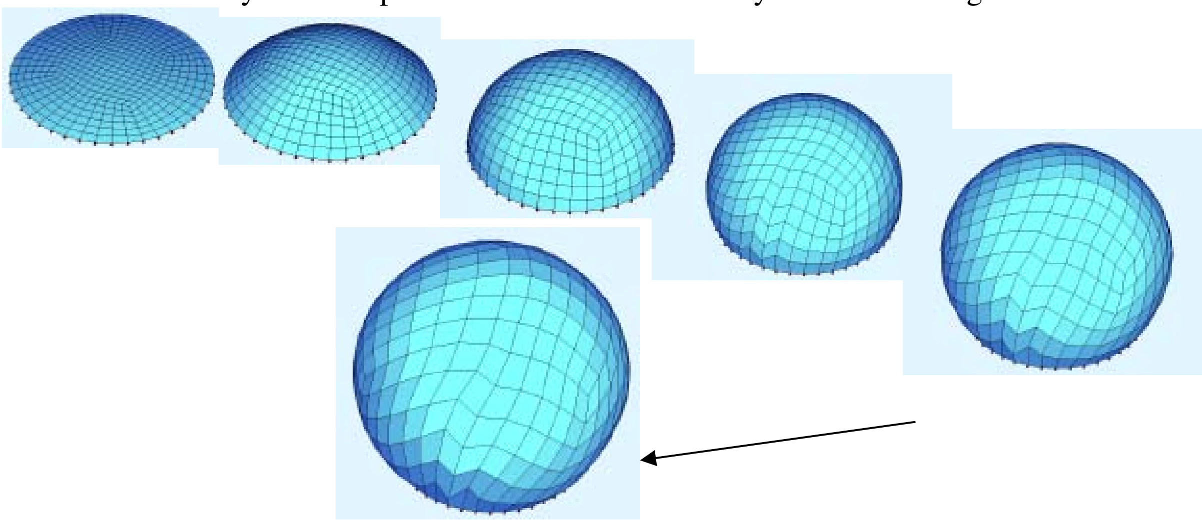

Figure 10: Balloon

4 CONCLUSIONS

The presented air volume technique allows numerous applications on membranes and inflatable structures. Simple examples explain the use of this exotic finite element.

REFERENCES

[1] Amphon Jrusjrungkiat, *Nonlinear Analysis of Pneumatic Membranes: "From Subgrid to Interface"*, Lehrstuhl für Statik der Technischen Universität München (2009)
[2] SOFiSTiK manuals: www.sofistik.com