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HIGH PRECISION LARGE SPACE STRUCTURES: CHALLENGES IN CABLE NETWORKS DESGIN

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1 INTRODUCTION

Nowadays space missions such as Earth observation, telecommunication and science require large antenna reflectors in the diameter range up to 20 meters or even larger for high performances. The antenna reflector structure has to satisfy strict requirements like high surface accuracy, low mass, high packaging efficiency, high reliability/stability in hazardous space environments for a long service life among others. These requirements lead to great challenges in developing large space deployable reflector (LDR) structures. One of the challenges is related to providing the parabolic high accurate (sub-millimeter range) shape with a minimum mass. Most of the flown reflectors known use cable networks suspended circumferentially or radially over a deployable framework. The cable networks approximate a parabolic surface with flat triangular facets, creating support for reflective material, e.g. extremely flexible knitted metal mesh.

In some specific missions LDR of about 6 m shall operate in high radio-frequency band (e.g. up to 30 GHz) [2]. This indicates a need of a fine facet size of the cable network of about 100 mm with a large number of cables and a strict surface error tolerance (around 0.1 mm). Moreover, manufacturing and assembling of such complex cable network with high accurate positioning and proper tensioning in cables is a quite challenging task as well. All above mentioned aspects lead the cable network design to be a form finding and design optimization problem with large number of variables, strict goal definition and multiple complex constraints.

A mechanical architecture of a large deployable mesh reflector (Figure 1) contains a deployable peripheral structure, which supports a cable network and a reflecting metal mesh. The cable network usually consists of a front network, a rear network and tie cables between them to form a so called tension truss (see also [12]).

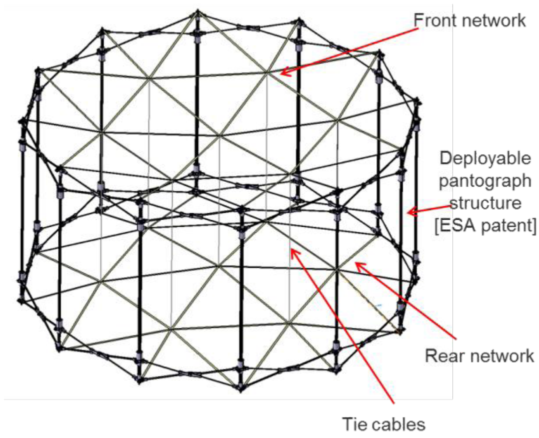


Figure 1: A mechanical architecture of a mesh reflector [17]

The cable network is a kind of flexible structures, which generally have strong interactions between their geometry and stresses. Different to conventional rigid structures, the configurations of flexible structures are more likely to be found rather than to be designed. The sustainable configurations of such structures are the equilibrium status among used materials, stresses in structures, defined boundary conditions and external loads. From energy point of view, the equilibrium status can be understood as a system with minimum total potential energy, which is the same to minimum difference between internal strain energy to external kinetic energy.

2 FORM FINDING AND STRUCTURAL OPTIMIZATION

Form finding is one of the classical problems in flexible tension structures like cable networks and membranes. Two typical approaches used in form finding are force density method (FDM) and dynamic relaxation (DR). Their basic principles are briefly explained in the following.

The force density method is initialized by Linkwitz and Schek in early 70s, which is used to design the cable-net roofs of Olympia stadium Munich (1972). The core of FDM is linearizing the system of nonlinear equations by introducing a parameter called force density (ratio of force to cable length). These linearized system equations enable an efficient and accurate analytical approach to be performed for finding good initial geometry of flexible structures. In later publications of Schek [1], he has given a more general understanding of form finding and optimization about cable networks. Cable network design is understood as a structural optimization, whose goal is finding an equilibrium initial geometry subjecting to constrains like cable lengths, cable forces and point positions. The expression of the optimization problem is:

$$g(x(q), y(q), z(q), q) = 0 \tag{1}$$

Goal:	min. system potential energy
Design variable:	force density (q) of each cable
Constraints:	cable lengths, cable forces and point positions

One of the simplest algorithms for solving the optimization problem is gradient based least squares principle. The gradients and related Lagrange factors can be readily derived through "chain rules" [1]. To improve the convergence behavior of a form finding design optimization particularly in problems with large force densities or shape changing, damping factors are added into functions as well.

The dynamic relaxation (DR) is an explicit solution technology based on finite difference method. Day [13] introduced this method in 60s to tidal flow computation and later Barnes extended it to solve design problems of flexible structures [15]. It traces step-by-step for small time increments of the motion of each node of the structure until it reaches the static equilibrium due to damping. Since the objective is to find the equilibrium status rather than trace the real dynamic behavior, the mass is fictitious here and is set to optimal convergence. In addition, better numerical stability of the analysis can be reached by taking kinetic damping instead of viscous damping. According to Newton's second law, the residuals between external loads and internal loads cause motion of a system and the velocity and displacement of each node can be obtained by integrating time steps. The update of geometry is performed on each node step-by-step based on center finite difference method. So their computation costs in DR are significantly less than stiffness matrix inversion calculations in FDM. In kinetic damping method, once the maximal kinetic energy of a system is detected, the analysis has to be restarted at the current geometry configuration with zero residuals and acceleration for the next iteration (Figure 2).

$$R_{ix}^{t} = M_i \cdot \dot{V}_{ix}^{t} \quad with\, \dot{V}_{ix}^{t} = (V_{ix}^{t+\frac{\Delta t}{2}} - V_{ix}^{t-\frac{\Delta t}{2}})/\Delta t \tag{2}$$

$$V_{ix}^{t+\frac{\Delta t}{2}} = V_{ix}^{t-\frac{\Delta t}{2}} + \frac{\Delta t}{M_i} \cdot R_{ix}^{t} \tag{3}$$

$$x_i^{t+\Delta t} = x_i^{t} + \Delta t \cdot V_{ix}^{t+\frac{\Delta t}{2}} \tag{4}$$

$$R_{ix}^{t+\Delta t} = P_{ix}^{t+\Delta t} + \sum (\frac{T}{l})_m^{t+\Delta t} \cdot (x_j - x_i)^{t+\Delta t} \tag{5}$$

$R: residuals$
$M: lump\ mass\ at\ node$
$V, \dot{V}: velocity, acceleration$
$\Delta t: time\ increasement$
$P: external\ loads$
$T: tension\ in\ cables$
$l: cable\ length$
$x: node\ position\ in\ x\ component$
$Equations\ are\ similar\ in\ corresponding\ y\ and\ z\ components.$

There are three aspects of requirements to the cable network design. First is the high surface accuracy requirement. Generally the surface shape accuracy is characterized by systematic and random errors. Triangular faceted approximation of the parabola determines the extent of systematic errors, which can be controlled by the facet size. The random error is the deviation of the achieved point positions from the nominal positions, which depends on the design and manufacturing quality. The second aspect of requirements is related to tensions in the cables. The cables are to be tensioned and, for the reason of manufacturing simplification, to be uniform in magnitude. Last but not least, the cable network must be in equilibrium configuration with satisfying the above two aspects of requirements simultaneously.

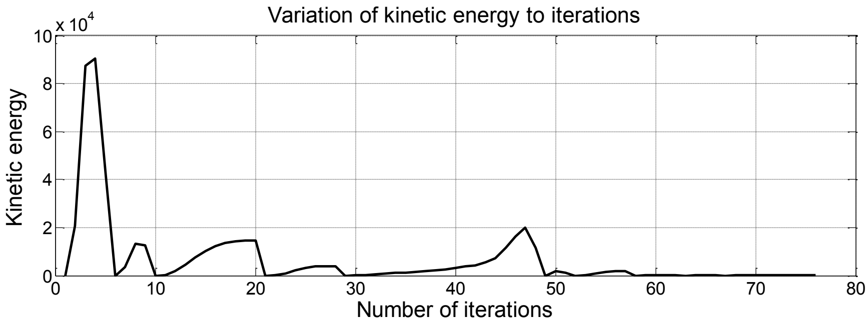


Figure 2: A typical curve of variation of kinetic energy to iterations

The above mentioned two approaches of form finding are integrated into a structural optimization code of cable networks. Two types of examples are used as testing benchmarks to compare these two design approaches. One uses the frame structure as in Figure 1 and has a front to rear symmetric networks. The other uses a conical ring structure (Figure 3) and has a front to rear asymmetric networks.

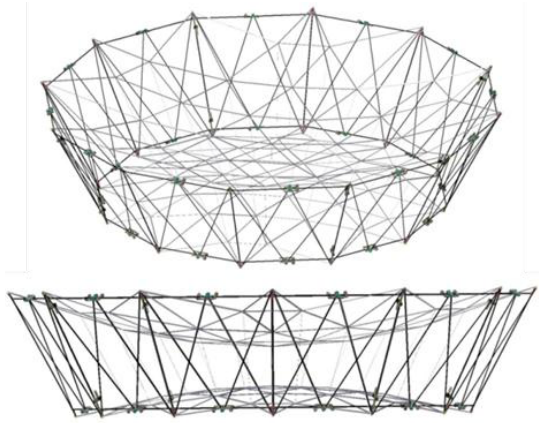


Figure 3: Conical ring structure [ESA patent 568]

3 DESIGN OF CABLE NETWORKS USING FORCE DENSITY METHOD

In the design approach using FDM, FDM is used for form finding and gradient based algorithm (least square) is used for structural optimization. The performance and features of this method are discussed through design examples.

There are lots of design examples of symmetric cable networks in references [4, 6, 8, and 9]. Since it has a symmetric construction, simplifications e.g. using half of cable network, using vertical forces to instead of tie cables are taken in designs. Further simplifications like considering only the internal regular cable network while ignoring the interface cables, frame structures as well as their interactions are taken in these examples. In [9], Morterolle presented a simple design approach using FDM to design parabolic and uniform tensioned cable networks. In his approach, Z coordinates of points are directly given by analytical parabolic functions according to their in-plane positions and tie forces are calculated by relating to tensions in cables and points' positions. Therefore a 3D design problem is simplified to be a 2D design problem with only tension constraints. In his later publication [14], he demonstrated this method works also for offset parabolic configurations and other facets like square and hexagon.

Generally, those methodologies in literature have number of simplifications, which in fact have to be followed more accurately. For example, behavior of an interface between internal cable network to interface cables (Figure 4) is better to be taken into account. The over simplified problem leads to difficulties in applying this method to practice applications directly. Therefore in [14], a stepwise approach is performed to design the interface cables based on designed internal cable networks. But actually, the interface cables belong to the same structure as well, whose design can be implemented similarly and simultaneously to the internal cable network.

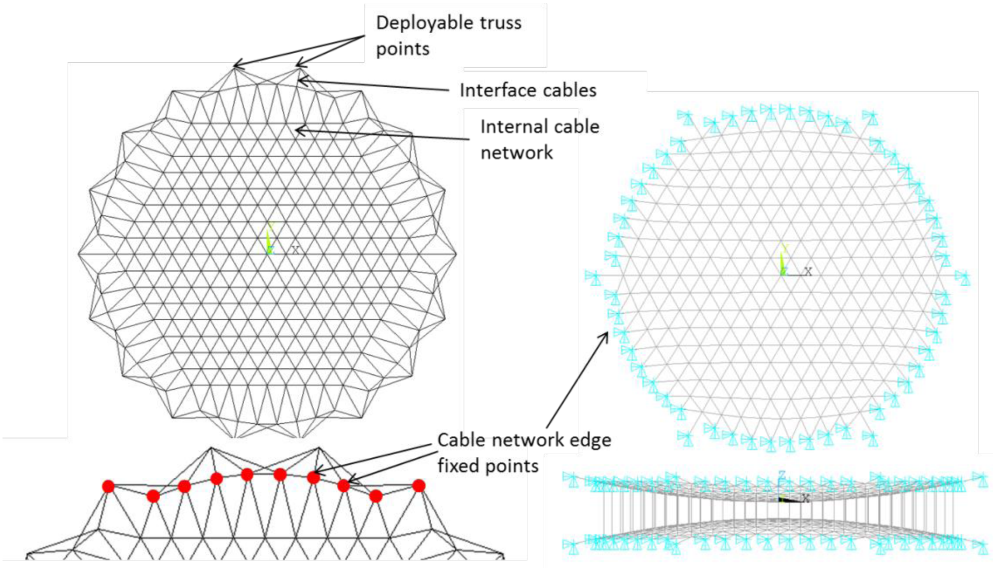


Figure 4: A design example of a symmetric cable network with uniform tension

The fact that the number of support truss points is usually less than the number of the edge points of the internal cable network determines the need in having interface cables. The interface cables connect several internal network points to a single truss point. As proved by numerical tests, it is infeasible to find an equilibrium form with identical tensions in all internal and interface cables. Therefore, modification of constraints with respect to the interface cables has to be implemented. It can be done in two ways. One is maintaining uniform tension in interface cables while allowing varying their positions. Then the tensions in interface cables can be allowed to be larger than in the internal cables. The other is maintaining the rim points' positions of the internal network while varying tensions in interface cables. Examples of using the first modification can be found in [16] and examples of using the second modification are explained in the following.

The simplified method of Morterolle is not capable to handle these multi-types constraints. Therefore the general method described by Schek [11], which is a gradient based optimization, is used. Different gradients from position constraints (internal cable network edge points) and tension constraints (internal cables) are linearly combined by weighting factors for iterative update of the force densities. Four design examples using edge point position constraints are demonstrated in Figure 5. The first two have the same number of points and cables while different edge points' positions. The latter two contain large number of members and complex interface cable connections.

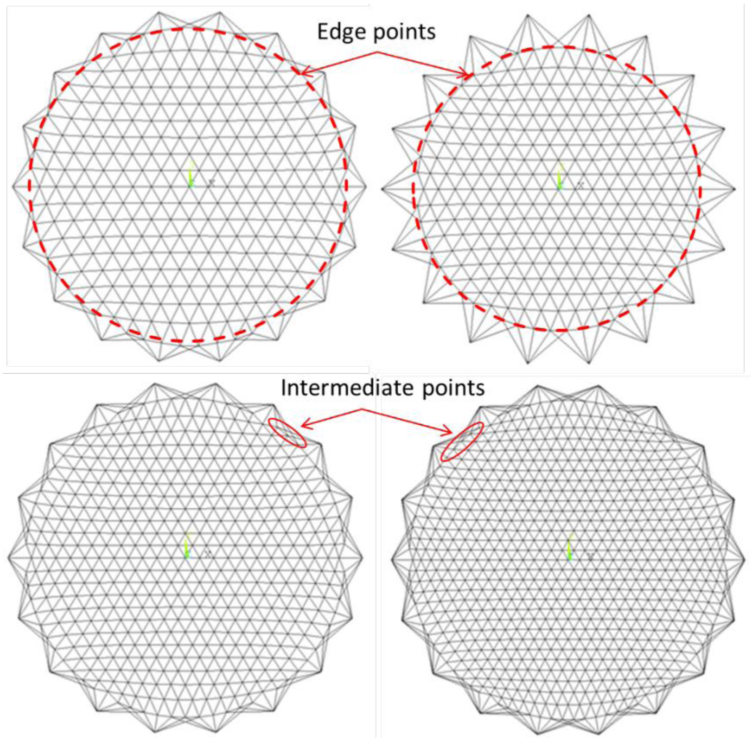


Figure 5: Design examples of using edge points' positions constraints

All these four designs are achieved with uniform tension in internal cables and predefined positions of the edge points are maintained. One highlight of these designs is the narrow varied facet size, which is preferred for surface accuracy reason. The tensions in interface cables vary adding some complexities in manufacturing and assembling.

Different to the symmetric design, the simplifications due to symmetric conditions are not valid and design approaches are to be updated correspondingly in case of asymmetric network designs (Figure 3) [16].

As the number of points and cables increase (the largest number of points is 733 and cables is 2214 in Figure 5) in designs, the computation cost increases dramatically. Besides this, the convergence behavior is not stable due to multiple types of constraints. Damping factors must be appropriately selected to find a compromise between computation robustness and cost. For instance, the last example with proper selected damping factors took around 3 hours for computation on a high performance computer. To find these proper damping factors, several trial and error tests are needed, which consumes extra time as well.

For high RF applications, the cable networks of medium class LDR require around 4000 to 6000 cable elements for achieving an acceptable level of systematic shape errors. The gradient based optimization with FDM seems quite complicated to accomplish this challenging task due to the following main drawbacks. The first is related to the stiffness matrix inversion in each iteration in the gradient calculation, which is generally time consuming and inaccurate. The second is related to the multiple types of constraints in optimization, which generate multiple gradients and they are difficult to be combined properly for updates. Both drawbacks belong to the used FDM but they can be avoided using DR method addressed in the following.

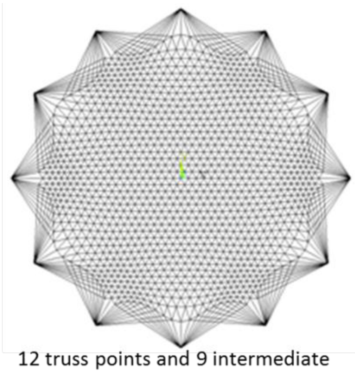
4 DESIGN OF CABLE NETWORKS USING DYNAMIC RELAXATION

Form finding with dynamic relaxation has significantly lower computation cost especially when using the kinetic damping method, which ensures a rapid and stable convergence of the analysis. DR method for form finding and structural optimization is implemented and tested through different examples discussed below.

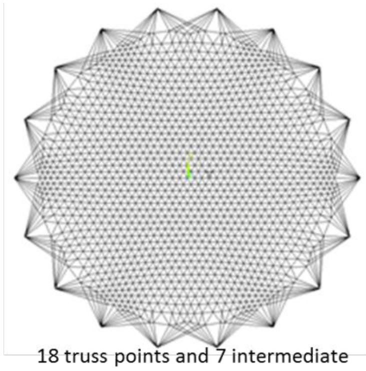
Found shapes for several examples with front and rear symmetric networks are presented in Figure 6. Optimization constraints of internal cable network edge points are set to maintain their positions in a very narrow range while varying tensions in interface cables. Given examples in the figure contain different number of support truss points and intermediate points as well as the number of cables, which varies between 4000 and 6000.

Different to the iteratively updated design parameters to approach the goal using FDM, the targeted cable forces and positions are directly assigned to the initial network to cause oscillations (geometry changes) until reaching the equilibrium due to damping. The convergence criterion is set to force residuals on nodes, which determines the accuracy of results and computation cost.

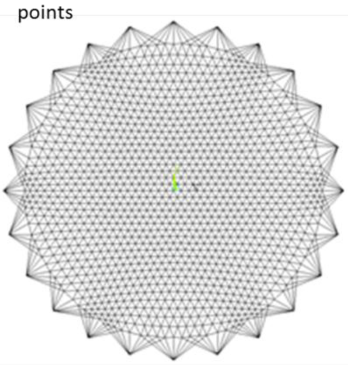
Computation cost of the both design methods discussed has been compared using the same cable network with the same convergence criterions (tension residuals). This particular investigation shows that the FDM needs around 180 minutes for computation, while using the DR 10 minutes are sufficient. Additionally, the point position error in using DR is significantly smaller than it in using FDM (refer to the similar constraints). The design method using DR has shown attractive features of computation efficiency and robustness, which is appropriate to accomplish tasks of designing cable networks for high precision LDR for high radio frequencies.



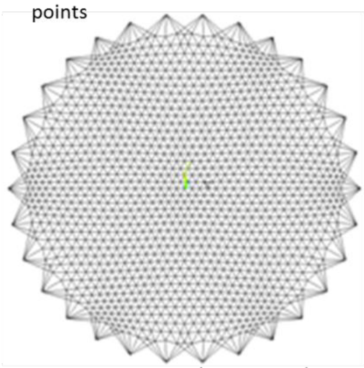
12 truss points and 9 intermediate points



18 truss points and 7 intermediate points



24 truss points and 4 intermediate points



30 truss points and 3 intermediate points

Figure 6: Design examples using DR

One of the key points in asymmetric network design process using DR is the way of determination of the tensions in tie cables. The tensions in tie cables are updated in each iteration to be the difference between the actual tensions and out-of-plane residuals in front network points. The purpose of this action is adjusting the tensions in tie cables to set residuals in front network points to zero. So their pre-defined positions can be maintained. The rear network points have almost no position constraints and their geometry is simply updated according to updated residuals.

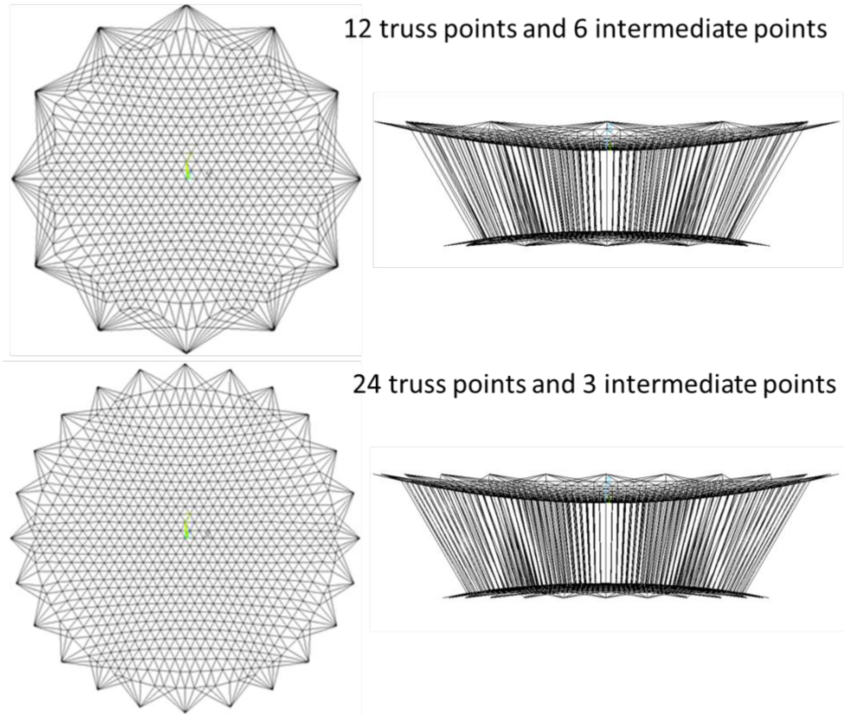


Figure 7: Asymmetric cable networks designs using DR

Various cable networks designs have been investigated using the described method. Results of these investigations show that computation cost is usually higher in designing asymmetric cable networks than symmetric ones due to configuration differences. The front networks of asymmetric cases have no significant configuration difference from the symmetric ones however their facet sizes vary wider than in symmetric cases for smaller rear network constructions.

5. INFLUENCE OF COMPLIANCE OF FRAME STRUCTURES

In above test examples, cable networks are attached to idealized rigid frame structures. A simplified truss frame structure, which is made from carbon fiber reinforced plastic (CFRP) tubes, is implemented to figure out the influence of its compliance to cable networks (Figure 8). The used cable network contains 1670 points and 5815 cables and its convergence criterion for design optimization is set to residual to be less than 1% of tension in cables.

Subject to an idealized rigid frame, the point position error of the designed network in FEM analysis is insignificant (6.2e-5 mm under 1% residuals). With considering the compliance of the frame structure under radially symmetric and statically determinate constraints in FEM, the point position error is increased significantly (Table 1).

The compliance of the frame structure can be reduced by increasing its bending stiffness. For instance, increasing the CFRP tubes diameter by about 25%, the maximal deformation of the frame structure is reduced by 30% and the surface error is also marginally reduced. This example has shown increasing bending stiffness of the frame structure is a useful but not an efficient method.

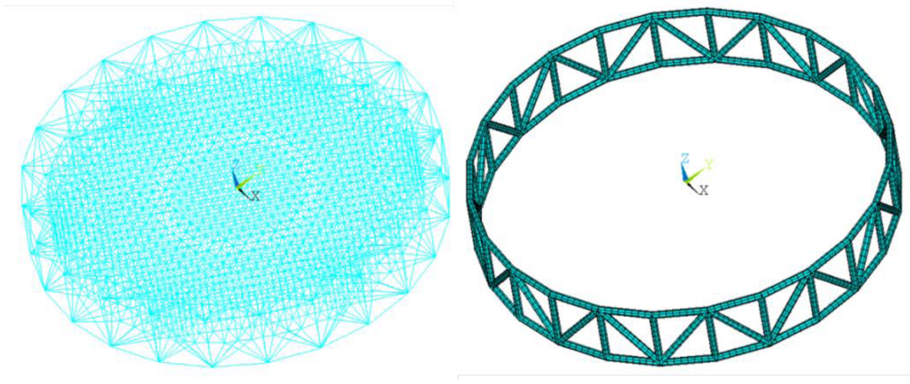


Figure 8: A cable network and its frame structure

On the other hand, the compliance problem can be solved by an iterative method. The deformation of the frame structure indicates the boundary conditions of the cable network have been changed. Therefore an update design of the cable network according to updated boundary conditions is required. The resulting acting forces due to cable network updates change deformation of the frame and in the end influences are reflected back to the cable network design. The iterative procedure is continued until either change of the reaction forces or deformations of the frame are small enough. Using this iterative method, the point position error is close to the value of rigid frame model.

Table 1: Methods and effect of solving the compliance problem of the frame structure

	Rigid frame model (reference)	Compliant frame model	Increasing bending stiffness of the frame structure on 25%	Iterative update frame structure and cable network
Relative point position error	1	114	92	1.1

Though the relative change of point position error due to compliance of frame structures is large, its absolute value can be not critical and be accepted in applications. With a sufficient high stiff frame structure, the compliance influence can be ignored even. These investigations have shown another fact of the designed networks, they are quite sensitive to their positioning and small deviations in points locally may cause a relatively large overall point position error.

6. CONCLUSIONS

In this paper, two approaches of cable networks design optimization for high precision LDR were discussed and compared. The design method using DR with kinetic damping is much more efficient and robust as compared to the FDM. This is especially the case when networks contain large number of members and have complex configurations.

A negative influence of the compliance of the frame structure on surface accuracy can be maintained acceptable, if setting the convergence criteria sufficiently small. On the other hand, relative error increases at least on two orders of magnitude as compared to the assumed rigid boundary. This can be still reduced by the established iterative procedure of taking into account the frame deformations during the optimization process.

In practice, manufacturing and assembling of such high accurate structures is another challenging task. The cable networks accuracy is quite sensitive to deviations of point positioning, which indicates difficulties in implementing the high accurate designs. As a part of future work, the uncertainty study of used material properties, points' positioning and tensions in cables will be investigated.

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