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DEFORMATION ANALYSIS OF RECTANGULAR COMPOSITE FLEXIBLE MEMBRANE OF THE PHOTOVOLTAIC SPACE SOLAR ARRAY

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Summary. The thin flexible composite membrane stretched on the frame of space solar array should be capable of withstanding the mechanical loadings exerted on the structure during the delivery to orbit and deployment. Nonlinear analysis of the deflections of the orthotropic flexible membrane stretched over the rectangular frame cell and subjected to transverse loading is presented in this paper.

1 INTRODUCTION

Thin film photovoltaic (TFPV) solar arrays offer the potential for providing a higher level of power generation in a lightweight configuration that can be compactly stowed for a space launch [1 - 4]. A typical solar wing design is shown in Fig. 1. Advanced high-modulus, high-strength carbon fibre reinforced polymers (CFRP) are normally implemented in current designs of the frames for solar wings. The thin flexible membrane is stretched on the frame, and then the photovoltaic cells are attached to its surface. The deployable solar arrays are normally stowed folded and could be deployed in various configurations. When stowed, they are usually placed parallel to each other and compactly packaged for launch. During the delivery to orbit, the membranes are subjected to the transverse g-force. The resulting pressure is equal to the product of the weight-per-unit-area of the membrane with the photovoltaic elements attached by the g-force. As a result of this loading the flexible membrane deflects. The excessive deflection could lead to the damage of the photovoltaic cells and/or electrical circuits. For this reason, one of the design requirements is that the deflection of the membrane should be limited to some specified value.

The solution of the problem related to the non-linear deformation of the flexible membrane carrying photovoltaic elements is presented in this paper. The problem is formulated for the orthotropic flexible membrane subjected to the transverse uniform pressure and tensile in-plane forces applied to the edges of the membrane. The membrane deformation is modelled by the system of non-linear differential equations which is solved using Galerkin method. The similar approach has been applied by Lopatin et al. [5]. An analytical formula for the calculation of the membrane deflection at the central point has been derived. Using this formula, the calculation of the deflections have been performed for orthotropic flexible membranes having different geometry parameters and subjected to different levels of loads. The results have been verified using comparisons with the finite-element solutions.

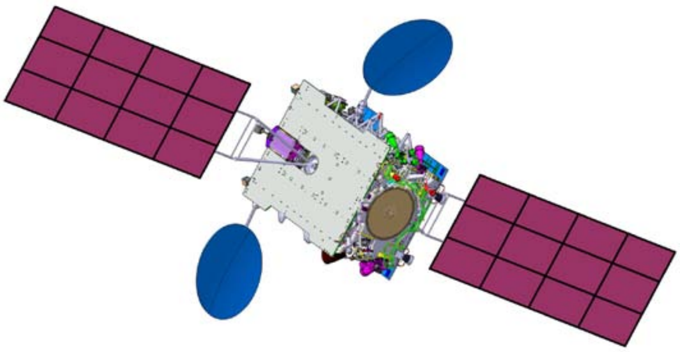

Figure 1: Spacecraft with solar arrays (Courtesy of ISS-Reshetnev Company).

2 PROBLEM FORMULATION

Consider a flat frame of the solar array shown in Fig. 1 and single out from this frame a typical representative rectangular fragment formed by the four rigid composite ribs. Refer this $a \times b$ cell to the Cartesian coordinated frame xyz as presented in Fig. 2. As shown, the rectangular flexible orthotropic membrane with the photovoltaic plates attached to its surface is stretched with the in-plane forces-per-unit-length T_x and T_y and fixed to the ribs. The membrane is subjected to the transverse g-force, n_z. It is assumed that this loading can be represented by the transverse pressure:

$$p = B_\rho n_z g \tag{1}$$

where B_ρ is the mass of the unit area of membrane material (including photovoltaic elements attached) and $g = 9.8\ m/s^2$.

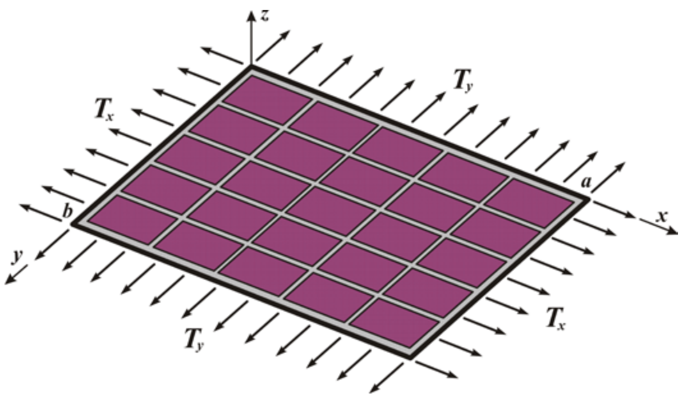

Figure 2: Typical rectangular fragment of the solar array with the stretched membrane and photovoltaic elements attached.

Deformation of the membrane is modelled by the following geometrically non-linear equations including the equations of equilibrium

$$\frac{\partial N_x}{\partial x}+\frac{\partial N_{xy}}{\partial y}=0 \qquad \frac{\partial N_{xy}}{\partial x}+\frac{\partial N_y}{\partial y}=0$$

$$N_x\frac{\partial \omega_x}{\partial x}+N_{xy}(\frac{\partial \omega_x}{\partial y}+\frac{\partial \omega_y}{\partial x})+N_y\frac{\partial \omega_y}{\partial y}-p=0 \tag{2}$$

constitutive equations

$$N_x=B_{11}\xi_x+B_{12}\xi_y+T_x\ , \qquad N_y=B_{21}\xi_x+B_{22}\xi_y+T_y\ , \qquad N_{xy}=B_{33}\xi_{xy} \tag{3}$$

and strain-displacements relationships

$$\xi_x=\varepsilon_x+\frac{1}{2}\omega_x^2\ ,\quad \xi_y=\varepsilon_y+\frac{1}{2}\omega_y^2\ ,\quad \xi_{xy}=\varepsilon_{xy}+\omega_x\omega_y$$

$$\varepsilon_x=\frac{\partial u}{\partial x},\qquad \varepsilon_y=\frac{\partial v}{\partial y}\ ,\qquad \varepsilon_{xy}=\frac{\partial u}{\partial y}+\frac{\partial v}{\partial x} \tag{4}$$

$$\omega_x=-\frac{\partial w}{\partial x},\qquad \omega_y=-\frac{\partial w}{\partial y}$$

in which N_x, N_y, and N_{xy} are the membrane stress resultants; ξ_x, ξ_y, and ξ_{xy} components of the finite strain; ε_x, ε_y, and ε_{xy} components of the infinitesimal strain; u and v in-plane displacements in the x- and y-directions, respectively; w is the transverse deflection; ω_x and ω_y are the angles of rotations of the lines tangent to the coordinate axes x and y; B_{11}, B_{12}, B_{22}, and B_{33} (B_{12}=B_{21}) are the membrane stiffness coefficients.

Equations Eqs. (2) – (4) are derived based on the geometrically nonlinear equations for the orthotropic plate given in [6] in which the bending stiffness coefficients are neglected. Substituting Eq. (4) into Eq. (3) and subsequently into Eq. (2) yields the following governing system of equations:

$$B_{11}\frac{\partial^2 u}{\partial x^2}+B_{33}\frac{\partial^2 u}{\partial y^2}+(B_{12}+B_{33})\frac{\partial^2 v}{\partial x\partial y}+B_{11}\frac{\partial w}{\partial x}\frac{\partial^2 w}{\partial x\partial y}+$$

$$+(B_{12}+B_{33})\frac{\partial w}{\partial y}\frac{\partial^2 w}{\partial x\partial y}+B_{33}\frac{\partial w}{\partial x}\frac{\partial^2 w}{\partial y^2}=0$$

$$(B_{12}+B_{33})\frac{\partial^2 u}{\partial x\partial y}+B_{33}\frac{\partial^2 v}{\partial x^2}+B_{22}\frac{\partial^2 v}{\partial y^2}+B_{33}\frac{\partial w}{\partial y}\frac{\partial^2 w}{\partial x^2}+$$
$$+(B_{12}+B_{33})\frac{\partial w}{\partial x}\frac{\partial^2 w}{\partial x\partial y}+B_{22}\frac{\partial w}{\partial y}\frac{\partial^2 w}{\partial y^2}=0$$

$$(5)$$

$$B_{11}\frac{\partial u}{\partial x}\frac{\partial^2 w}{\partial x^2}+B_{12}\frac{\partial u}{\partial x}\frac{\partial^2 w}{\partial y^2}+\frac{1}{2}B_{11}\left(\frac{\partial w}{\partial x}\right)^2\frac{\partial^2 w}{\partial x^2}+\frac{1}{2}B_{12}\left(\frac{\partial w}{\partial x}\right)^2\frac{\partial^2 w}{\partial y^2}+$$
$$+B_{12}\frac{\partial v}{\partial y}\frac{\partial^2 w}{\partial x^2}+B_{22}\frac{\partial v}{\partial y}\frac{\partial^2 w}{\partial y^2}+\frac{1}{2}B_{12}\left(\frac{\partial w}{\partial y}\right)^2\frac{\partial^2 w}{\partial x^2}+\frac{1}{2}B_{22}\left(\frac{\partial w}{\partial y}\right)^2\frac{\partial^2 w}{\partial y^2}+$$
$$+2B_{33}\frac{\partial u}{\partial y}\frac{\partial^2 w}{\partial x\partial y}+2B_{33}\frac{\partial v}{\partial x}\frac{\partial^2 w}{\partial x\partial y}+2B_{33}\frac{\partial w}{\partial x}\frac{\partial w}{\partial y}\frac{\partial^2 w}{\partial x\partial y}+$$
$$+T_x\frac{\partial^2 w}{\partial x^2}+T_y\frac{\partial^2 w}{\partial y^2}+p=0$$

This system provides the governing equations for the pre-stretched membrane under consideration in terms of displacements u, v, and w.

2 SOLUTION PROCEDURE

Galerkin procedure is employed for the solution of the system of equations given by Eqs. (5). Taking into account that the displacements of the membrane u, v, and w are equal to zero at the edges supported by the ribs, and that for the given loading the lines $x = a/2$ and $y = b/2$ are the lines of symmetry, the approximating functions are selected in the following form:

$$u(x,y)=U\sin\frac{2\pi x}{a}\sin\frac{\pi y}{b}\ ,\quad v(x,y)=V\sin\frac{\pi x}{a}\sin\frac{2\pi y}{b}\ ,\quad w(x,y)=W\sin\frac{\pi x}{a}\sin\frac{\pi y}{b} \qquad (6)$$

where U, V, and W are unknown coefficients. Following Galerkin procedure, i.e., substituting Eq. (6) into Eq. (5) the corresponding errors are presented as follows:

$$R_x(x,y)=-(B_{11}\frac{4\pi^2}{a^2}+B_{33}\frac{\pi^2}{b^2})U\sin\frac{2\pi x}{a}\sin\frac{\pi y}{b}+$$
$$+(B_{12}+B_{33})\frac{\pi}{a}\frac{2\pi}{b}V\cos\frac{\pi x}{a}\cos\frac{2\pi y}{b}-$$
$$-\frac{\pi}{a}\left[(B_{11}\frac{\pi^2}{a^2}+B_{33}\frac{\pi^2}{b^2})\sin^2\frac{\pi y}{b}-(B_{12}+B_{33})\frac{\pi^2}{b^2}\cos^2\frac{\pi y}{b}\right]W^2\sin\frac{\pi x}{a}\cos\frac{\pi x}{a}$$

$$R_y(x,y) = (B_{12} + B_{33})\frac{2\pi}{a}\frac{\pi}{b}U\cos\frac{2\pi x}{a}\cos\frac{\pi y}{b} -$$

$$-(B_{22}\frac{4\pi^2}{b^2} + B_{33}\frac{\pi^2}{a^2})V\sin\frac{\pi x}{a}\sin\frac{2\pi y}{b} -$$

$$-\frac{\pi}{b}\left[(B_{22}\frac{\pi^2}{b^2} + B_{33}\frac{\pi^2}{a^2})\sin^2\frac{\pi y}{a} - (B_{12} + B_{33})\frac{\pi^2}{a^2}\cos^2\frac{\pi y}{a}\right]W^2\sin\frac{\pi y}{b}\cos\frac{\pi y}{b}$$

$$R_z(x,y) = \frac{2\pi}{a}\left[-(B_{11}\frac{\pi^2}{a^2} + B_{12}\frac{\pi^2}{b^2})\cos\frac{2\pi x}{a}\sin\frac{\pi x}{a}\sin^2\frac{\pi y}{b} + \right.$$

$$\left. + B_{33}\frac{\pi^2}{b^2}\sin\frac{2\pi x}{a}\cos\frac{\pi x}{a}\cos^2\frac{\pi y}{b}\right]UW +$$

$$+\frac{2\pi}{b}\left[-(B_{12}\frac{\pi^2}{a^2} + B_{22}\frac{\pi^2}{b^2})\sin^2\frac{\pi x}{a}\cos\frac{2\pi y}{b}\sin\frac{\pi y}{b} + \right.$$

$$\left. + B_{33}\frac{\pi^2}{a^2}\cos^2\frac{\pi x}{a}\sin\frac{2\pi y}{b}\cos\frac{\pi y}{b}\right]VW +$$

$$+\left[-\frac{1}{2}\frac{\pi^2}{a^2}(B_{11}\frac{\pi^2}{a^2} + B_{12}\frac{\pi^2}{b^2})\cos^2\frac{\pi x}{a}\sin\frac{\pi x}{a}\sin^3\frac{\pi y}{b} - \right.$$

$$-\frac{1}{2}\frac{\pi^2}{b^2}(B_{12}\frac{\pi^2}{a^2} + B_{22}\frac{\pi^2}{b^2})\sin^3\frac{\pi x}{a}\cos^2\frac{\pi y}{b}\sin\frac{\pi y}{b} +$$

$$\left. + 2B_{33}\frac{\pi^2}{a^2}\frac{\pi^2}{b^2}\cos^2\frac{\pi x}{a}\sin\frac{\pi x}{a}\cos^2\frac{\pi y}{b}\sin\frac{\pi y}{b}\right]W^3 -$$

$$-(T_x\frac{\pi^2}{a^2} + T_y\frac{\pi^2}{b^2})W\sin\frac{\pi x}{a}\sin\frac{\pi y}{b} + p \tag{7}$$

The orthogonality conditions for the errors, Eq. (7) and the basis functions, Eq. (6) have the form

$$\int_0^a\int_0^b R_x(x,y)\sin\frac{2\pi x}{a}\sin\frac{\pi y}{b}dxdy = 0$$

$$\int_0^a\int_0^b R_y(x,y)\sin\frac{\pi x}{a}\sin\frac{2\pi y}{b}dxdy = 0 \tag{8}$$

$$\int_0^a\int_0^b R_z(x,y)\sin\frac{\pi x}{a}\sin\frac{\pi y}{b}dxdy = 0$$

Integrating yields the following system of non-linear algebraic equations:

$$-a_{11}U - a_{12}V = b_1W^2 \ , \qquad -a_{21}U - a_{22}V = b_2W^2 \tag{9}$$

$$-c_{11}W + c_{12}UW + c_{21}VW - c_{22}W^3 + p\frac{1536}{\pi^2} = 0 \tag{10}$$

in which

$$a_{11} = 9(4B_{11}\frac{\pi^2}{a^2} + B_{33}\frac{\pi^2}{b^2}) \ , \qquad a_{12} = a_{21} = \frac{64}{ab}(B_{12} + B_{33}) \ , \qquad a_{22} = 9(4B_{22}\frac{\pi^2}{b^2} + B_{33}\frac{\pi^2}{a^2})$$

$$b_1 = \frac{6}{a}[2B_{11}\frac{\pi^2}{a^2} - (B_{12} - B_{33})\frac{\pi^2}{b^2}] \qquad b_2 = \frac{6}{b}[2B_{22}\frac{\pi^2}{b^2} - (B_{12} - B_{33})\frac{\pi^2}{a^2}]$$

$$c_{11} = 96(T_x\frac{\pi^2}{a^2} + T_y\frac{\pi^2}{b^2}) \ , \quad c_{12} = \frac{128}{a}[2B_{11}\frac{\pi^2}{a^2} + (2B_{12} + B_{33})\frac{\pi^2}{b^2}] \tag{11}$$

$$c_{21} = \frac{128}{b}[2B_{22}\frac{\pi^2}{b^2} + (2B_{12} + B_{33})\frac{\pi^2}{a^2}]$$

$$c_{22} = 3[3B_{11}\frac{\pi^4}{a^4} + 2(3B_{12} - 2B_{33})\frac{\pi^2}{a^2}\frac{\pi^2}{b^2} + 3B_{22}\frac{\pi^4}{b^4}]$$

Solving Eqs. (9), for *U* and *V* the latter can be presented as follows:

$$U = \frac{-b_1a_{22} + b_2a_{12}}{a_{11}a_{22} - a_{12}^2}W^2 \qquad V = \frac{-b_2a_{11} + b_1a_{12}}{a_{11}a_{22} - a_{12}^2}W^2 \tag{12}$$

Substituting these expressions into Eq. (10) yields the following cubic equation:

$$c_{33}W^3 + c_{11}W - p\frac{1536}{\pi^2} = 0 \tag{13}$$

in which

$$c_{33} = c_{22} + \frac{b_1(a_{22}c_{12} - a_{12}c_{21}) + b_2(a_{11}c_{21} - a_{12}c_{12})}{a_{11}a_{22} - a_{12}^2} \tag{14}$$

Equation, Eq. (13), is the governing equation for the problem of nonlinear deformation of the orthotropic membrane under consideration subjected to the transverse uniformly distributed pressure and pre-stretching tensile in-plane loads. This equation links the forces T_x , T_y , pressure *p*, membrane dimensions *a* and *b*, membrane stiffness coefficients B_{11}, B_{12}, B_{22}, B_{33}, and deflection at the centre of the membrane *W*, and it can be transformed into the form

$$W^3 + SW - R = 0 \tag{15}$$

where

$$S = \frac{c_{11}}{c_{33}} \ , \qquad R = \frac{1536}{\pi^2 c_{33}} \tag{16}$$

The discriminant of Eq. (15) is calculated as

$$Q = \left(\frac{S}{3}\right)^3 + \left(\frac{R}{2}\right)^2 \tag{17}$$

In this case, the discriminant is positive so one root of Eq. (15) is real and another two are complex conjugates. According to Cardano's formula the real root is given by

$$W = \sqrt[3]{\frac{R}{2} + \sqrt{Q}} - \sqrt[3]{\sqrt{Q} - \frac{R}{2}} \tag{18}$$

Using this equation, the deflection can be found for various combinations of stiffness parameters of the orthotropic membrane subjected to the pressure and in-plane tensile loads.

The equation, Eq. (13) can be used when designing the membrane with the voltaic elements attached. Based on this equation, the membrane parameters and in-plane stretching loads, delivering the specified deflection W, can be found for given mass of the unit area of membrane material and g-force**.**

Note that if the pre-tensioning loads $T_x = T_y = 0$, than the coefficient $c_{11} = 0$ and it follows from Eq. (13) that

$$W = 8\sqrt[3]{\frac{3p}{\pi^2 c_{33}}} \tag{19}$$

For given W, the values of U and V are calculated using Eqs. (12) and the displacements at any point of the membrane are determined by Eqs. (6).
Furthermore, substituting Eqs. (6) into Eqs. (4) and the resulting equations into Eqs. (3), the stress resultants can be calculated as follows

$$\begin{aligned}
N_x &= T_x + 2(B_{11}\frac{\pi}{a}U\cos\frac{2\pi x}{a}\sin\frac{\pi y}{b} + B_{12}\frac{\pi}{b}V\sin\frac{\pi x}{a}\cos\frac{2\pi y}{b}) + \\
&\quad + \frac{1}{2}(B_{11}\cos^2\frac{\pi x}{a}\sin^2\frac{\pi y}{b} + B_{12}\frac{\pi^2}{b^2}\sin^2\frac{\pi x}{a}\cos^2\frac{\pi y}{b})W^2 \\
N_y &= T_y + 2(B_{12}\frac{\pi}{a}U\cos\frac{2\pi x}{a}\sin\frac{\pi y}{b} + B_{22}\frac{\pi}{b}V\sin\frac{\pi x}{a}\cos\frac{2\pi y}{b}) + \\
&\quad + \frac{1}{2}(B_{12}\frac{\pi^2}{a^2}\cos^2\frac{\pi x}{a}\sin^2\frac{\pi y}{b} + B_{22}\frac{\pi^2}{b^2}\sin^2\frac{\pi x}{a}\cos^2\frac{\pi y}{b}) \\
N_{xy} &= B_{33}(U\frac{\pi}{b}\sin\frac{2\pi x}{a}\cos\frac{\pi y}{b} + V\frac{\pi}{a}\cos\frac{\pi x}{a}\sin\frac{2\pi y}{b} + \\
&\quad + \frac{1}{4}W^2\frac{\pi}{a}\frac{\pi}{b}\sin\frac{2\pi x}{a}\sin\frac{2\pi y}{b})
\end{aligned} \tag{20}$$

Obviously, for the orthotropic membrane loaded with a uniform pressure, the largest values of N_x and N_y are reached in the centre of membrane with $N_{xy} = 0$. Thus, taking $x = a/2$ and $b = y/2$ in Eqs. (20), the maximum values of N_x and N_y are

$$N_x = T_x - 2(B_{11}\frac{\pi}{a}U + B_{12}\frac{\pi}{b}V) \quad , \qquad N_y = T_y - 2(B_{12}\frac{\pi}{a}U + B_{22}\frac{\pi}{b}V) \tag{21}$$

Substituting for the values U and V in this equation their expressions given by Eqs. (12), the stress resultants N_x , N_y can be expressed in terms of deflection W as follows:

$$\begin{aligned} N_x &= T_x + 2\frac{B_{11}\frac{\pi}{a}(b_1 a_{22} - b_2 a_{12}) + B_{12}\frac{\pi}{b}(b_2 a_{11} - b_1 a_{12})}{a_{11}a_{22} - a_{12}^2} W^2 \\ N_y &= T_y + 2\frac{B_{12}\frac{\pi}{a}(b_1 a_{22} - b_2 a_{12}) + B_{22}\frac{\pi}{b}(b_2 a_{11} - b_1 a_{12})}{a_{11}a_{22} - a_{12}^2} W^2 \end{aligned} \tag{22}$$

It should be noted that the solution of the nonlinear problem under consideration was obtained under assumption that the membranes analysed should not be too long, i.e. their aspect ratio *a/b* should not be excessively large. If the membrane is too long, the deformed shape would resemble the cylindrical surface and the use of the approximations as per Eqs.(6) could lead to noticeable errors. In practice, the aspect ratio *a/b* for the cells of solar arrays normally does not exceed 2. So the solution presented in this work can be efficiently applied at the early stages of the design of flexible-membrane stiffened space solar arrays [7].

3 NUMERICAL EXAMPLES

The solution obtained in this work has been applied to the analyses of membranes made from orthotropic material. Consider non-linear deformation of a membrane made of an orthotropic material with the moduli of elasticity E_x and E_y , shear modulus G_{xy} , and Poisson's ratios ν_{xy} and ν_{yx} . The stiffness coefficients of the membrane are given by

$$B_{11} = \overline{E}_x h \ , \quad B_{12} = B_{21} = \overline{E}_x \nu_{xy} h \ , \quad B_{22} = \overline{E}_y h \ , \quad B_{33} = G_{xy} h \tag{23}$$

where

$$\overline{E}_x = \frac{E_x}{1 - \nu_{xy}\nu_{yx}} \quad , \quad \overline{E}_y = \frac{E_y}{1 - \nu_{xy}\nu_{yx}} \tag{24}$$

Substituting Eq. (23) into Eq. (11), the latter are transformed into the following form:

$$\begin{gathered} a_{11} = \frac{\pi^2}{a^2} h\overline{a}_{11} \quad a_{12} = a_{21} = \frac{h}{a^2}\overline{a}_{12} \quad a_{22} = \frac{\pi^2}{a^2} h\overline{a}_{22} \\ b_1 = \frac{\pi^2}{a^3} h\overline{b}_1 \quad b_2 = \frac{\pi^2}{a^3} h\overline{b}_2 \end{gathered} \tag{25}$$

$$c_{11} = T_x \frac{\pi^2}{a^2} \bar{c}_{11} \quad c_{12} = \frac{\pi^2}{a^3} h\bar{c}_{12} \quad c_{21} = \frac{\pi^2}{a^3} h\bar{c}_{21} \quad c_{22} = \frac{\pi^4}{a^4} h\bar{c}_{22}$$

where

$$\bar{a}_{11} = 9\left(4\bar{E}_x + G_{xy}c^2\right), \quad \bar{a}_{12} = 64c\left(\bar{E}_x \nu_{xy} + G_{xy}\right), \quad \bar{a}_{22} = 9\left(4\bar{E}_y c^2 + G_{xy}\right)$$

$$\bar{b}_1 = 6\left[2\bar{E}_x - \left(\bar{E}_x \nu_{xy} - G_{xy}\right)c^2\right], \quad \bar{b}_2 = 6c\left[2\bar{E}_y c^2 - \left(\bar{E}_x \nu_{xy} - G_{xy}\right)\right]$$

$$\bar{c}_{11} = 96\left(1 + \alpha c^2\right) \tag{26}$$

$$\bar{c}_{12} = 128\left[2\bar{E}_x + \left(2\bar{E}_x \nu_{xy} + G_{xy}\right)c^2\right], \quad \bar{c}_{21} = 128c\left[2\bar{E}_y c^2 + \left(2\bar{E}_x \nu_{xy} + G_{xy}\right)\right]$$

$$\bar{c}_{22} = 3\left[3\bar{E}_x + 2\left(3\bar{E}_x \nu_{xy} - 2G_{xy}\right)c^2 + 3\bar{E}_y c^4\right]$$

and

$$c = \frac{a}{b}, \qquad \alpha = \frac{T_y}{T_x} \tag{27}$$

are the membrane aspect ratio and the ratio of the in-plane stretching tensile forces, respectively.

The coefficient c_{33} in Eq. (13) is calculated using Eq. (14) after substitution of coefficients determined by Eq. (25):

$$c_{33} = \frac{\pi^4}{a^4} h\bar{c}_{33} \tag{28}$$

where

$$\bar{c}_{33} = \bar{c}_{22} + \frac{1}{\pi^2} \frac{\bar{b}_1\left(\bar{a}_{22}\bar{c}_{12} - \dfrac{\bar{a}_{12}\bar{c}_{21}}{\pi^2}\right) + \bar{b}_2\left(\bar{a}_{11}\bar{c}_{21} - \dfrac{\bar{a}_{12}\bar{c}_{12}}{\pi^2}\right)}{\bar{a}_{11}\bar{a}_{22} - \dfrac{\bar{a}_{12}^2}{\pi^4}} \tag{29}$$

Substituting c_{11}, Eq.(25) and c_{33}, Eq. (28) into Eq. (13) yields the governing equation for the problem under consideration in the following form:

$$\frac{\pi^4}{a^4} h\bar{c}_{33} W^3 + T_x \frac{\pi^2}{a^2} \bar{c}_{11} W - p\frac{1536}{\pi^2} = 0 \tag{30}$$

Dividing this equation by $\pi^4 h\bar{c}_{33} / a^4$, the latter can be transformed into the form

$$W^3 + SW - R = 0 \tag{31}$$

in which

$$S = \frac{T_x}{h} \frac{\bar{c}_{11}}{\bar{c}_{33}} \frac{a^2}{\pi^2}, \qquad R = \frac{1536}{\pi^6} \frac{pa^4}{\bar{c}_{33} h} \tag{32}$$

The solution of Eq. (31) is given by Eqs. (17) and (18). Substituting coefficients defined by Eq. (25) into Eq. (22) and taking into account equations for the stiffness coefficients, Eq.(23), the stress resultants N_x and N_y acting at the centre of membrane are calculated as follows:

$$N_x = T_x + \frac{\pi h}{a^2} F_x W^2, \qquad N_y = T_y + \frac{\pi h}{a^2} F_y W^2 \tag{33}$$

where

$$F_x = 2\frac{\overline{E}_x\left(f_1 + \nu_{xy} f_2\right)}{\overline{a}_{11}\overline{a}_{22} - \dfrac{\overline{a}_{12}^2}{\pi^4}}, \quad F_y = 2\frac{\overline{E}_y\left(\nu_{yx} f_1 + f_2\right)}{\overline{a}_{11}\overline{a}_{22} - \dfrac{\overline{a}_{12}^2}{\pi^4}}$$

$$f_1 = \overline{b}_1\overline{a}_{22} - \overline{b}_2\frac{\overline{a}_{12}}{\pi^2}, \qquad f_2 = c\left(\overline{b}_2\overline{a}_{11} - \overline{b}_1\frac{\overline{a}_{12}}{\pi^2}\right) \tag{34}$$

Consider analysis of the orthotropic membrane with typical for the solar array dimensions: $a = 1$ m , $b = 0.8$ m , and $h = 0.5$ mm . The membrane is made of a material based on the glass-fibre fabric with the following elastic properties: $E_x = E_y = 0.8$ GPa , $G_{xy} = 0.15$ GPa, $\nu_{xy} = \nu_{yx} = 0.35$ and density $\rho = 1800\ \mathrm{kg/m^3}$. The mass of the unit area of membrane material with photovoltaic elements attached $B_\rho^{cell} = 1.8\ \mathrm{kg/m^2}$. Assume that $T_x = T_y = T = 0, 250, \text{and } 500\ \mathrm{N/m}$. The resulting pressure exerted on the membrane is determined as follows:

$$p = \left(\rho h + B_\rho^{cell}\right) n_z g \tag{35}$$

If $n_z = 1, 5, \text{and } 10$, than the pressure $p = 26.5,\ 132.4, \text{ and } 264.9\ \mathrm{N/m^2}$, respectively. Using Eqs.(17), (18) and (32), the membrane deflection W have been calculated. The results of calculations for different values of n_z and T are presented in Table 1. Analysis of the data shows that the increase in the stretching force T leads to a reduction of the membrane deflection. This effect is most noticeable for $n_z = 1$.

Table 1: Deflection W (mm) for different values of n_z and T.

n_z	T (N/m)		
	0	250	500
1	10.92	5.78	3.30
5	18.67	15.30	12.22
10	23.53	20.83	18.21

The results of the calculations were verified by a finite-element analysis. The non-linear analysis has been performed using the COSMOS/M module NSTAR [8]. The results (W_{FEM}) are presented in Table 2 and deformed shape of the membrane is shown in Fig. 3. Comparison of the results presented in Tables 1 and 2 shows that the maximum difference between W and W_{FEM} is -6.75% for $n_z = 1$ and $T = 0$ which is acceptable for the approximate analytical solution.

Table 2: Deflection W_{FEM} (mm) for different values of n_z and T.

n_z	T (N/m)		
	0	250	500
1	10.92	5.78	3.30
5	18.67	15.30	12.22
10	23.53	20.83	18.21

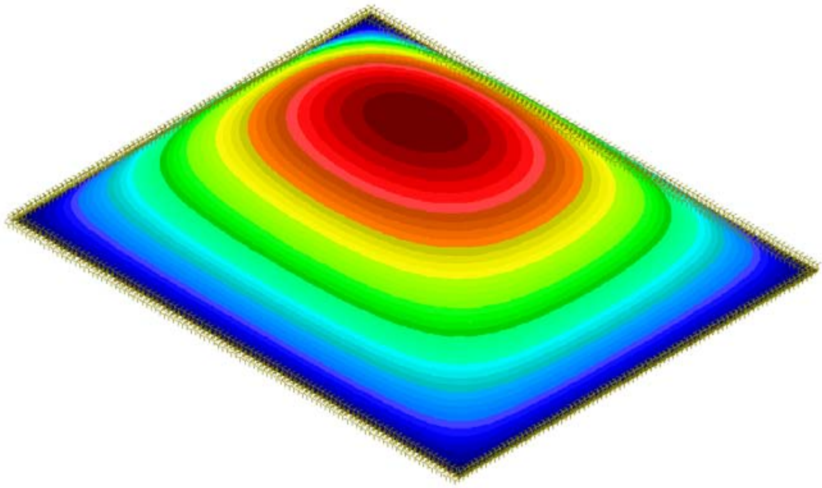

Figure 3: Shape of the deformed orthotropic membrane.

3 CONCLUSIONS

The solution of the problem of nonlinear deformation of the orthotropic flexible membrane stretched on a rectangular frame and subjected to the transverse uniform pressure was developed in this work. The system of the nonlinear differential equations written in terms of in-plane displacements and deflection was solved using Galerkin method. The membrane displacements and deflection were approximated by the trigonometric functions satisfying the boundary conditions. The problem has been reduced to the algebraic cubic equation and the analytical formula providing the value of deflection at the centre of membrane was derived.

The deflections of the membranes made from the orthotropic flexible glass-fibre fabric have been calculated and the effects of the in-plane stretching loads on the deflection and internal stress resultants have been investigated. The accuracy of the analyses has been verified by comparison with the results obtained using the finite-element method. It has been shown that the analytical solution developed in this work provides an accurate estimate of the deflection at the centre of membrane and can be successfully applied to the design of composite flexible-membrane stiffened space solar arrays.

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