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Selected Examples for the Optimization of Cutting Patterns for Textile Membranes

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1 INTRODUCTION

The cutting patterning is an essential part of the engineering process in textile architecture. In the past it was very costly and time-intensive if the waste of material was to be minimized. Nowadays we have fast or even automatic patterning tools where those problems are solved efficiently as we are willing to show in this paper.

In a short introduction we present different flattening theories, the meaning of geodesic lines in this context, the definition of constraints as angles and distances, and the final estimation of the quality of the patterning by numbers. Then the preparation of the cutting patterns for the production is shown briefly as e.g. seam allowances, welding-marks, etc. But patterning is not only a in the responsibility of the membrane-engineer, the architects are using the seam-lines as an important factor in the design of lightweight architecture. We support it with our software by so-called hierarchical cuts, by a fast extension of seam-lines in the neighbor-membrane-fields (to avoid staggered lines), etc.

Then we put the focus on procedures with respect to selected examples where cutting patterns are calculated and optimized efficiently. The optimization is almost always related to the waste of material. The widths of the patterns should be a maximum with respect to the role-width of the material by considering the quality of the patterns.

Starting from a small four-point-sail we describe precisely the process chain. Then we show the automatic patterning generation of a high point membrane and a big air-hall.

2 FLATTENING THEORIES

Formfinding of membrane structures is usually done by using finite element meshes. The result of the formfinding procedure are 3D coordinates and the mesh consisting of lines, triangles or polylines. In general we have to define seam-lines on the surface in order to receive the widths of all patterns with respect to the role-widths of the chosen material. The problem of the flattening remains unchanged, if we have to flatten total net-parts or by seam-lines separated sub-surfaces as patterns. The problem can be defined as follows: a surface consisting of points and lines has to be projected into a 2D system. This task can be seen completely independent from the mechanics of the membrane structures, if the so-called compensation is neglected. The compensation is the reduction of the pattern size depending on the material properties of the membrane due to the pre-stress in the 3D-shape. I repeat again: by neglecting the compensation our problem is treated by the map-projection theories This field is already very old since the surface of the earth (as sphere, ellipsoid or geoid) is shown in 2D maps with all problems that we face also in membrane flattening procedures. The mathematical map-projection theories were developed especially by Carl Friedrich Gauss, who was also a surveyor and responsible for the triangulation network of the Kingdom Hannover (1818-26); he also developed the Adjustment Theory, where the square-sum of residuals of observations are minimized, in order to get the 'best' unknowns [1]. We are going to use now both of his concepts for the flattening of patterns; the adjustment theory to minimize the so-called distortions energy Π in the map (2D situation) and the distortion energy itself is a function of differences between values in their 3D- and 2D-positions of the net. Generally the unknowns in our systems are the 2D coordinates of all flattened points, transformation parameters and Lagrange multipliers in case of additional constraints.

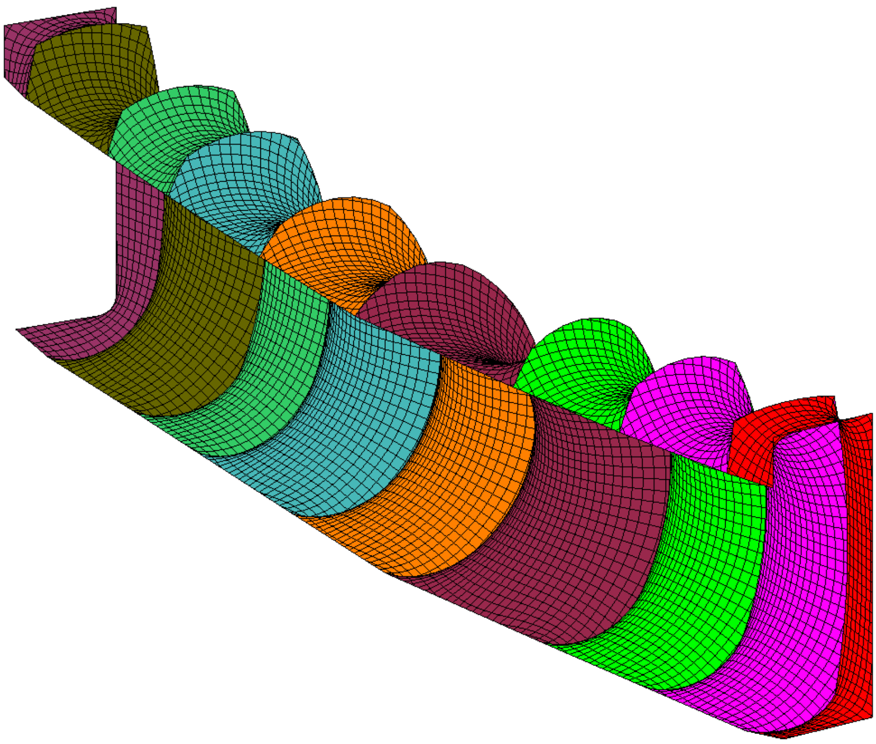

Figure 1: 3-D surface with points, links and polylines

Now we start with a simple formulation of the distortion energy Π, where the energy is only a function of the 2D-coordinates of the flattened net.

$$\Pi(x, y) = \frac{1}{2} v^t \boldsymbol{P} v \Rightarrow min. \tag{1}$$

Very generally the distortion energy Π - to be minimized – can be expressed as the square-sum of residuals *v*. The residuals are combined with so-called observations *l* having the weight *p*. All weights are ordered in a diagonal matrix ***P***. See also [2] [3] [4] .

$$l^* = l + v = f(x, y)\ with\ weight\ p \tag{2}$$

The observation *l* is corrected by a residual *v* and the adjusted observation l^* is a function of the 2D coordinates *(x, y)*. We would like to add some examples for those observations in order to clarify the situation. We assume that *l* is the 3D-distance between two neighbor points A and B. The projected points (in the flattened situation) are A^* and B^*. The distance l^* in 2D should be more or less in the same range as *l*. This range 'more or less' is called residual *v* in the adjustment theory. We can write the equation for the 2D distance as follows:

$$l^* = l + v = \sqrt{(x_{A^*} - x_{B^*})^2 + (y_{A^*} - y_{B^*})^2} \tag{3}$$

To be complete we show 3 more equations, including equation (3) again:

$$l^* = l_{2D} = l_{3D} + v = distance\ 2D\ with\ \ p\ = p_l \tag{4}$$

$$\alpha^* = \alpha_{2D} = \alpha_{3D} + v = angle\ 2D\ with\ p = p_\alpha \tag{5}$$

$$A^* = A_{2D} = A_{3D} + v = area\ 2D\ with\ \ p = p_A \tag{6}$$

On the equations (4-6) we can see that our possibilities are limited as always in map-projection theories, where the maps can be lengths-, angle- or area-preserving. The residuals can be steered a little bit by their weights *p*. In general we can say: the bigger the weight the smaller the residual, but this concept has limitations mainly because of numerical reasons. We do not recommend at all substituting a constraint by an equation (2) and a very big weight. This leads always to numerical problems. In case of real constraints we have to add the constraint equation by using Lagrange multipliers. Now the distortion energy is not only a function of the 2D coordinates, but also from the Lagrange multiplier *h*. Of course we can use any number of constraints without a change of the principle. In order to find the smallest distortion energy we have set the derivations of the energy to the unknowns to 0. We did it in equation (8) for the Lagrange multiplier *h* in order to show the constraint-equation.

$$\Pi(x, y, h) = \frac{1}{2} v^t \boldsymbol{P} v - h(g(x, y) - g_0) \Rightarrow min. \quad (7)$$

$$\frac{\vartheta \Pi}{\delta h} = g(x, y) - g_0 = 0 \quad (8)$$

An example for those constraints are lengths or angles between points. When we assume e.g. for an ETFE cushion fixed boundaries we have to make sure that the lengths of the patterns and the lengths of frame - where the foil is inputted - are identical disregarding the compensation and also 90° angles may occur, for which we have to use also additional equations with Lagrange multipliers.

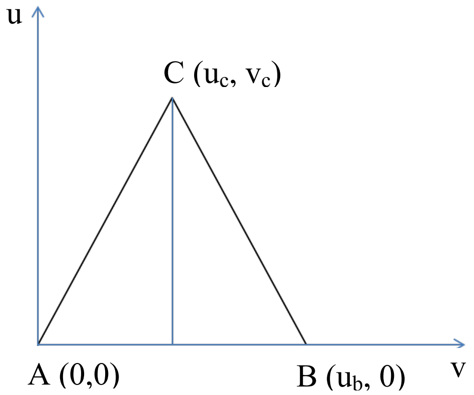

Figure 2: Local triangle u, v coordinates as observations

Now we would like to extend the possibilities of the flattening procedures by very general coordinate-transformations. As we defined at the beginning of our paper the surface to be flattened is described by 3D coordinates of points and an additional net (or a mesh) (see Figure 1). When we assume now that our 3D surface is described by triangles; so we can calculate rectangular u,v coordinates for all triangles in their 3D position as shown in Figure 2. In order to flatten the surface we write the residuals by assuming observations having the value 0 as follows in (9). The transformation matrix $\boldsymbol{T}$ is here describing a rectangular transformation with an identical scale in both directions (Helmert-Transformation) [8].

$$\begin{pmatrix} v_x + 0 \\ v_y + 0 \end{pmatrix} = T \begin{pmatrix} u \\ v \end{pmatrix} + \begin{pmatrix} x_0 \\ y_0 \end{pmatrix} - \begin{pmatrix} x_{2D} \\ y_{2D} \end{pmatrix} = \begin{pmatrix} a & b \\ -b & a \end{pmatrix} \begin{pmatrix} u \\ v \end{pmatrix} + \begin{pmatrix} x_0 \\ y_0 \end{pmatrix} - \begin{pmatrix} x_{2D} \\ y_{2D} \end{pmatrix} \quad (9)$$

The transformation matrix $\boldsymbol{T}$ is here describing a rectangular transformation with an identical scale in both directions (Helmert-Transformation). There are other possibilities for the setting of the transformation matrix $\boldsymbol{T}$ as e.g. an orthogonal transformation with the scale 1. In this case the transformation parameter a is substituted by $cos(\alpha)$ and b by $sin(\alpha)$. The advantage of the used transformation is the linear equations in (9). By using this formulation we can find easily in one linear step all 2D coordinates. We simply have to fix 2 points in 2D and the 2D coordinates of all points are together with the transformation parameters a, b, x_0, y_0 for each triangle the solution of the linear system. The advantage is obvious: a flattened surface is quickly obtained. The disadvantage of the described method is as follows. By the fact that the equations (9) are depending on a coordinate system, we end up with results depending on the chosen 2 fixed points. For a high quality patterning we cannot accept it, and therefore we are going to a free network adjustment. This means we do not use fixed points at all, and therefore we have to add 3 equations, in order to fix the 3 degrees of freedom in the 2D space. Those equations are not simple to use, because it needs Pivot-strategies to find the solution.

$$\sum_{i=1}^{n} x_i = 0 \qquad \sum_{i=1}^{n} y_i = 0 \qquad \sum_{i=1}^{n} x_i \cdot y_i = 0 \qquad (10), (11), (12)$$

Our flattening procedure has to make sure, that the flattened area and the 3dimensional area are in the same range. Therefore we add nonlinear observations to our system as follows. For all transformations (e.g. 1 system per triangle) one more equation to maintain the scale 1 as good as possible.

$$1 + v = \sqrt{a^2 + b^2}\ with\ p = p_s \qquad (13)$$

In the Figure 3 we can see a membrane project consisting of 9 parts. All of those parts are patterned individually, here I would like to describe the flattening procedure for the whole part in one piece; the reason is only to get a flat boundary for a good mesh-generation (very similar as we do it for cable-nets!) We see already the flattened boundaries for all parts. We would like to count the unknowns for the patterning of the biggest part (dark red). This part has 1624 points, 1372 polylines (meshes). The unknowns are so 3240 coordinates and 5488 transformation-parameters, All those meshes have at least 4 corner points, for all systems (polylines) we have an equation (13) and at least for 4 points equation (9). Therefore the number of residual rows is at least 9*1372=12348. The number of unknowns is (3240 + 5488) 8728. So we still have a very good redundancy of (12348-8728) 3620. We forgot to mention, that we have 3 more unknowns by the constraints (10-12).

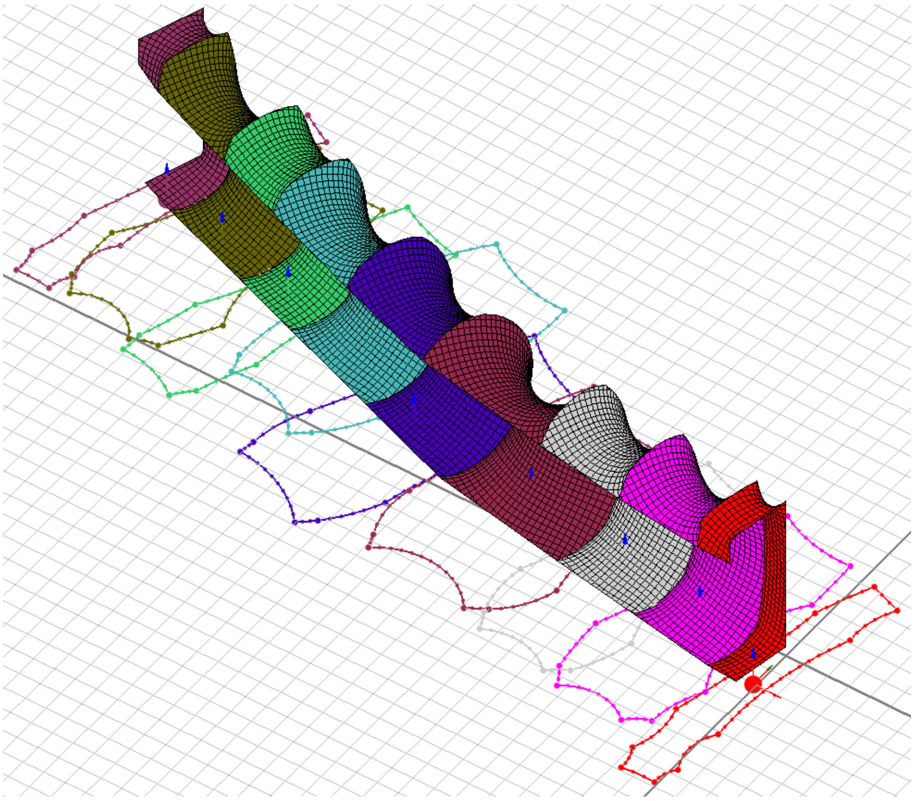

Figure 3: Flattened boundaries for the mesh generation

3 GEODESIC LINES

Geodesic lines play an important role in the patterning procedure. Before explaining the reason why we use geodesic lines I would like to give 3 definitions for geodesic lines.
A geodesic line is defined as

1. the shortest line between 2 points on a surface.
2. whose bi-normal vector in all points of the line is identical to the normal vector of the surface.
3. a line with a constant force on a surface (without any friction).

The reason for using geodesic lines as seam-lines is very simple and it can be seen directly on point 1 of the 3 definitions. The geodesic lines help to minimize the waste of material. Figure 4 shows the situation. The red strip has to geodesic lines as seam-lines and the violet strip 2 vertical cuts as seams. In the flattening procedure the red strips automatically becomes 'straight' and the violet one 'curved'. The reason is as follows: geodesic lines in the 2D space are straight lines and therefore the strip becomes as straight as possible.

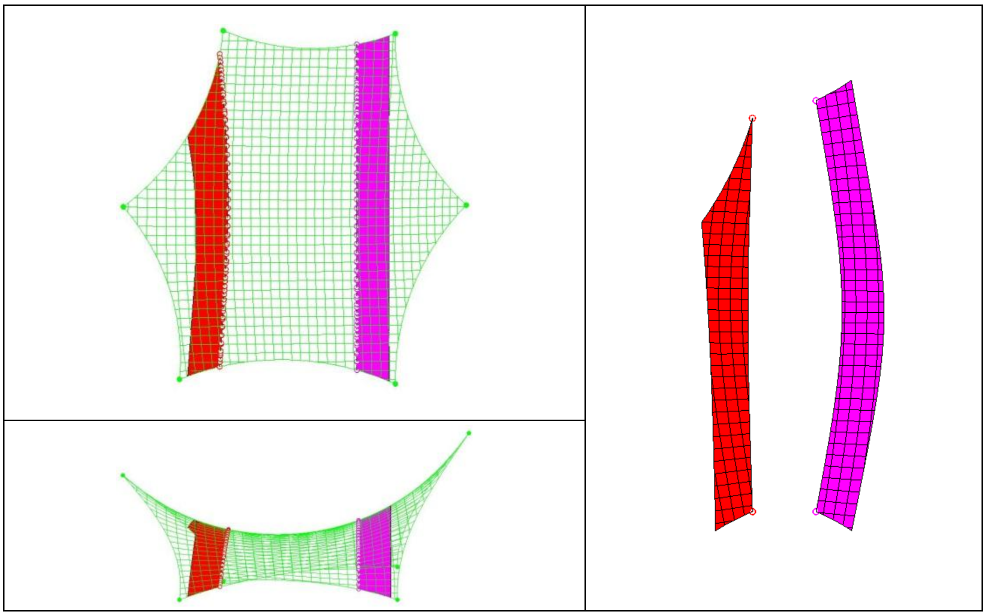

Figure 4: Geodesic lines and vertical cuts

3 SELECTED EXAMPLES

In order to optimize the widths of patterns automatically we need optimization variables. We all know that the widths of our patterns depend on the position of the seam-lines (mainly geodesic lines). In order to achieve appropriate widths for all patterns we have to modify the position of those seam lines until the desired widths of the patterns are reached. Therefore we want to change the position of the geodesic lines depending on one value. We define this value to be a coordinate (for regular meshes), an arc-length for pneumatic systems or an angle for high point membranes [5], [6].

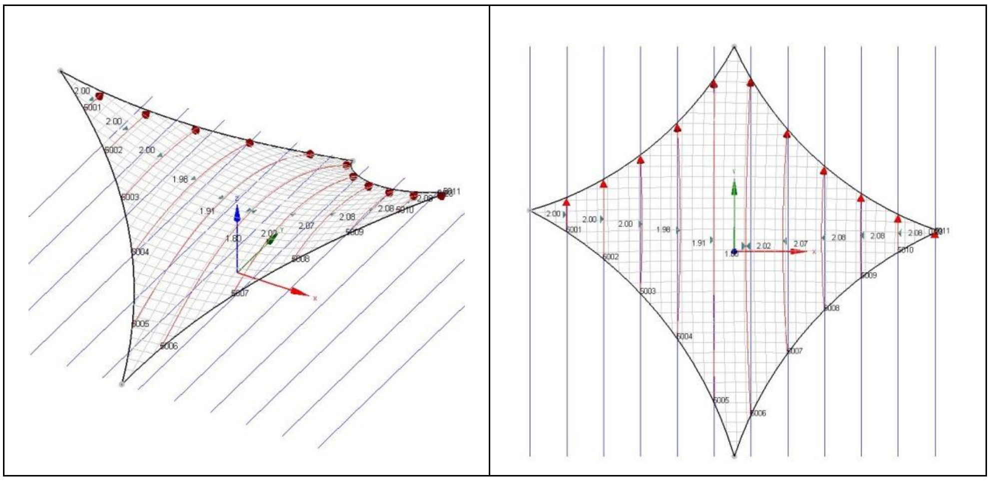

Figure 5: Geodesic line Start and Endpoint on one x-value

Figure 5 shows how the widths are optimized for regular nets. Parallel lines with the desired widths are starting values for an iterative procedure .With those starting lines geodesic lines are produced automatically by a start-and end-point. Then the flattening of all patterns is performed to get real widths for all patterns. With this information new gridlines can be calculated in a way to get widths being closer to the desired width, etc. After some iteration loops all widths are perfect. Let me refer again to Figure 5 to explain it precisely by this example. The distance of the gridlines is 1.80 m in the 1st iteration. We want all patterns to be 1.80 m wide. Now the flattening at the end of the 1st iteration shows us, that that real width of the first pattern (left side) is 2.01 m (0.21 m wider than desired). So we know the gridline have to start in the 2nd iteration with 1.59 m (=1.80 m-0.21 m) in order to get 1.80 m. This procedure converges very fast and after some iterations all widths are exactly 1.80 m except the last one the right hand side.

In Figure 6 we find the same principle with a guideline and automatic produced geodesic perpendicular to the guideline; therefore a widths optimization can be done automatically also for air-halls and cushions.

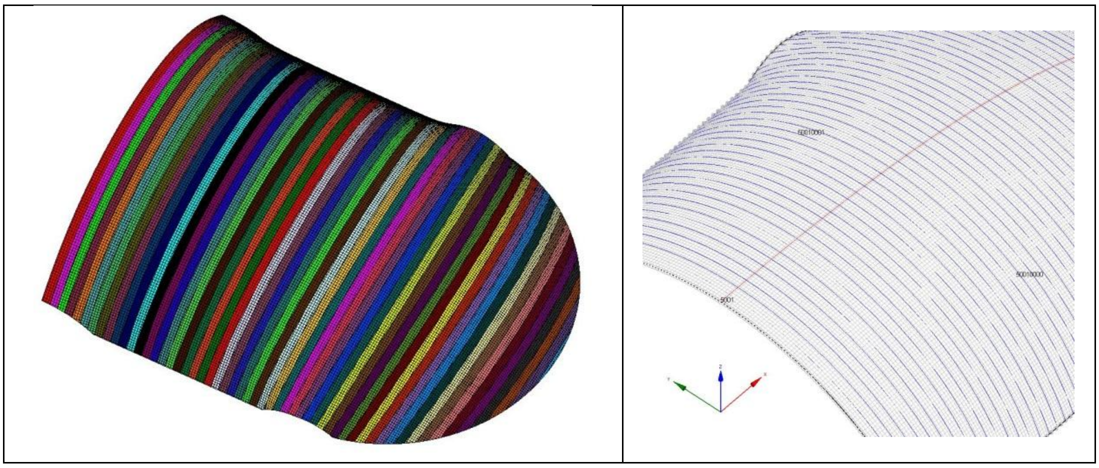

Figure 6: Geodesic lines (blue) perpendicular to a guideline (red)

Also radial patterns can be arranged and optimized automatically by using an angle in order to set a geodesic line. (see Figure 7)

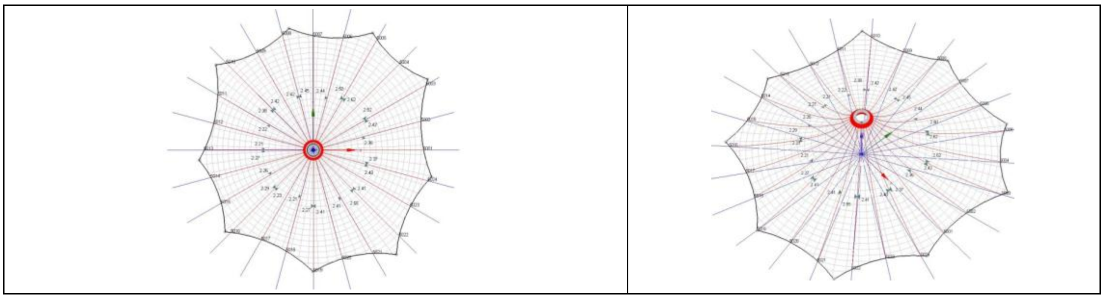

Figure 7: Guidelines for radial nets

Our system allows to flatten membrane patterns very fast and width-optimized. We also add seam-allowances and welding marks to simplify the production process [7]. Our clients get also information about the quality of the patterns by the simple numbers as length-, angle and area-differences (and the distortion energy itself) see Figure 8. Hierarchical cuts are also possible as we see in Figure 9.

```
Cloth  110                                           Stausn    07/31/201320:34:01
 No               Center of Gravity            Area   Middle Width     Max. Width       Area (3D)
 110         -0.0001          -0.0029       31.2692         1.0853          1.6977         31.2648
 No       From          To Mode Distance STORD   Distance EIN      Difference Difference EIN
 449  60025110  60025001    B         0.1448         0.1448          0.0000
 450  60025001  60015001    G        35.9184        35.9184          0.0000          0.0000
 451  60015001  60015110    B         3.2000         3.2000          0.0000
 452  60015110  60025110    G        38.5972        38.5972          0.0000          0.0000

StRep:  Area of all strips (2D):       3279.0986
StRep:  Area of all strips (3D):       3278.7374
StRep:  Delta Area (3D-2D) in %:         -0.0110
StRep:  Geodesic Lengths (all):        7934.6202
StRep:  Boundary Lengths (all):         228.5237
StRep:  Number of Polygons:                2
StRep:  Polygon, No. of links:             1        122

   No       From         To   Distance EIN   Distance(3D)
    1  99000008  99000009        20.0238        20.0238
    1  99000009  99000010        20.0696        20.0696
    1  99000010  99000011        18.0667        18.0666
    1  99000011  99000012        15.9067        15.9066
    1  99000012  99000001        18.2559        18.2558
    1  99000001  99000002        18.5037        18.5037
    1  99000002  99000003        16.8273        16.8273
    1  99000003  99000004        16.8254        16.8255
    1  99000004  99000005        17.1556        17.1558
    1  99000005  99000006        18.6435        18.6434
    1  99000006  99000007        15.4922        15.4922
    1  99000007  99000008        15.9479        15.9479
StRep:  Polygon, No. of links:             2        110

   No       From         To   Distance EIN   Distance(3D)
    2  60025001  60025001        16.8054        16.8070
StRep: Time of calculation:         0.10s
StRep: Version 11.0.03 done.
```

Figure 8: Radial net with welding marks and quality report

4 CONCLUSION

It has been shown that by applying the adjustment theory for the flattening of membrane patterns, in a first step linear and then - after having got approximation values - nonlinear, a powerful system is obtained; its basis are coming from map-projection doctrine.

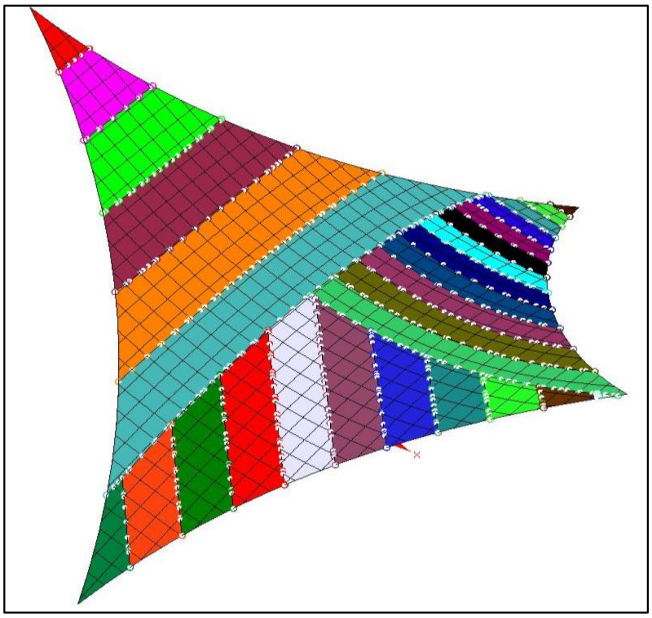

Figure 9: Hierarchical cuts

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