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USING NURBS AS RESPONSE SURFACE FOR MEMBRANE MATERIAL BEHAVIOR

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Abstract. This work proposes a NURBS for the determination of a smooth response surface relating biaxial strains and stresses from discrete test data. These NURBS surfaces are based on two axes of strain and one axis of stress. The constitutive material tensor is calculated with the derivatives of the NURBS surfaces and curves. The motivation of the proposed work came from the use of new materials in membrane structures that requires complex material models to describe the complex material behavior. A method for the establishment of a matrix of material coefficients from these surfaces is developed aiming its application in finite element models. The response surface stress-strain relation and the material matrix derived are compared to classical hyperelastic and Mooney-Rivlin material models. The response surface approach using NURBS allows for an easy implementation in an existent FE code, requiring few changes. A similar application is found in the work of Bridgens and Gosling [1]. This approach provides a direct correlation between stresses and strains in the wide range of possible stress paths the material is subject to. Curve fitting based on least squares approximation is employed to generate NURBS surfaces for the experimental data. The advantage of this material model is that a smooth stress-strain response surface can be obtained directly from the experimental results. On the other hand, in order to generate good NURBS surfaces the experimental data should provide an adequate point distribution. This could require a large range of experimental data. We conclude that this material model is a good alternative to conventional material models for complex material behavior.

1 INTRODUCTION

Non Uniform Rational Basis Splines (NURBS) is a mathematical representation of a geometry in 3D used for curves and surfaces. This representation is widely used in Computer Aided Design (CAD) to create and modify designs offering smooth surfaces. Due to the success of the use of NURBS in CAD, it has been suggested in other applications. An example of this is the isogeometric analysis introduced by Hughes et al. [4], to solve problems governed by partial differential equations such a structures and fluids. Another application of NURBS in numerical analysis is the NURBS-enhanced finite element method (NEFEM). Sevilla et al. [8] reports that the NEFEM uses NURBS to accurately describe the boundary of the computational domain. The NURBS application proposed in this work aims the determination of a smooth response surface relating biaxial strains and stresses. These NURBS surfaces are based on two axes of strain and one axis of stress. The constitutive material tensor is calculated with the derivatives from the NURBS surfaces and curves.

A similar application is found in the work of Bridgens and Gosling [1]. In Bridgens and Gosling [1] Bezier functions, B-spline and NURBS are used to represent the bi-axial behavior of coated woven fabrics. The validity of the approach is assessed through an extensive fabrics testing program. This approach provides a direct correlation between stresses and strains in the wide range of possible stress paths the material is subject to. As pointed out in Bridgens and Gosling [1] this representation has the additional ability to represent surfaces with rapid changes in gradients and discontinuities in the data. Also, the plane stress constraint, frequently used by the analysis of films and membrane structures is not explicitly imposed. Curve fitting based on least squares approximation is employed to generate NURBS surfaces for the experimental data. The response surface methodology based on NURBS is tested on classical hyperelastic and Mooney-Rivlin constitutive models. A set of results of an aluminum testing program were used to illustrate the response surface construction procedure from test results. Aiming the application of this methodology together with a finite element non-linear analysis program for the investigation of global structure behavior the derivation from the NURBS surface of a constitutive matrix is developed. Most general purpose FE programs provide user access to add new functionalities such as user constitutive models. The response surface approach using NURBS presented in this work allows for an easy implementation in such programs.

2 NONUNIFORM RATIONAL B-SPLINE CURVES AND SURFACES

The concept of NURBS curve and NURBS surface used in the present study refers to the works of Piegl and Tiller [6] and L. Piegl [5].

The definition of NURBS curve/surface is the rational generalization of the tensor-product nonrational B-spline curve/surface. According to Rogers [7], technically, a NURBS surface is a special case of a general rational B-spline surface that uses a particular form of knot vector. For a NURBS surface, the knot vector has multiplicity of duplicate knot values equal to the order of the basis function at the ends. The knot vector may or may not have uniform internal knot values.

3 MATERIAL MODEL BASED ON NURBS FOR PRINCIPAL DIRECTIONS (PD–NURBS)

The proposed material model covers isotropic nonlinear materials under plane stress conditions. This model is based on principal directions of stress and strain. Therefore only one surface is required for its definition.

PD–NURBS is valid for isotropic materials because of the use of orthogonal transformation to calculate the response of the stress. According to Gruttmann and Taylor [3], for isotropic material response the contravariant components of the second Piola–Kirchhoff stress tensor are recovered by an orthogonal transformation of the principal stresses.

The second Piola–Kirchhoff stresses and the Green–Lagrange strains in principal directions are given by:

$$\hat{\mathbf{S}} = \begin{bmatrix} S_1 & S_2 & \hat{S}_{12} \end{bmatrix} \tag{1}$$

$$\hat{\mathbf{E}} = \begin{bmatrix} E_1 & E_2 & \hat{E}_{12} \end{bmatrix} \tag{2}$$

where $\hat{S}_{12} = 0$ and $\hat{E}_{12} = 0$.

The constitutive material tensor in general directions is obtained with the rotation matrix calculated as follows:

$$\frac{d\mathbf{S}}{d\mathbf{E}} = \begin{bmatrix} \frac{dS_{11}}{dE_{11}} & \frac{dS_{11}}{dE_{22}} & \frac{dS_{11}}{2dE_{12}} \\ \frac{dS_{22}}{dE_{11}} & \frac{dS_{22}}{dE_{22}} & \frac{dS_{22}}{2dE_{12}} \\ \frac{dS_{12}}{dE_{11}} & \frac{dS_{12}}{dE_{22}} & \frac{dS_{12}}{2dE_{12}} \end{bmatrix} = \mathbf{T}^T \cdot \frac{d\hat{\mathbf{S}}}{d\hat{\mathbf{E}}} \cdot \mathbf{T} \tag{3}$$

where $\frac{d\hat{\mathbf{S}}}{d\hat{\mathbf{E}}}$ is the constitutive material tensor in principal directions

$$\frac{d\hat{\mathbf{S}}}{d\hat{\mathbf{E}}} = \begin{bmatrix} \frac{dS_1}{dE_1} & \frac{dS_1}{dE_2} & \frac{dS_1}{2d\hat{E}_{12}} \\ \frac{dS_2}{dE_1} & \frac{dS_2}{dE_2} & \frac{dS_2}{2d\hat{E}_{12}} \\ \frac{d\hat{S}_{12}}{dE_1} & \frac{d\hat{S}_{12}}{dE_2} & \frac{d\hat{S}_{12}}{2d\hat{E}_{12}} \end{bmatrix} = \begin{bmatrix} \frac{dS_1}{dE_1} & \frac{dS_1}{dE_2} & 0 \\ \frac{dS_2}{dE_1} & \frac{dS_2}{dE_2} & 0 \\ 0 & 0 & \frac{d\hat{S}_{12}}{2d\hat{E}_{12}} \end{bmatrix} \tag{4}$$

and derivatives of the NURBS surface for S_1 in directions u and v are given by

$$S_{u_1}^{NURBS}(u,v) = \begin{bmatrix} \frac{dE1}{du} & \frac{dE2}{du} & \frac{dS1}{du} \end{bmatrix} \tag{5}$$

$$S_{v_1}^{NURBS}(u,v) = \begin{bmatrix} \frac{dE1}{dv} & \frac{dE2}{dv} & \frac{dS1}{dv} \end{bmatrix} \tag{6}$$

and analogously for the derivatives of the NURBS surface for S_2 in directions u and v.

$$S_{u_2}^{NURBS}(u,v) = \begin{bmatrix} \frac{dE1}{du} & \frac{dE2}{du} & \frac{dS2}{du} \end{bmatrix} \tag{7}$$

$$S_{v_2}^{NURBS}(u,v) = \begin{bmatrix} \frac{dE1}{dv} & \frac{dE2}{dv} & \frac{dS2}{dv} \end{bmatrix} \tag{8}$$

and the rotation matrix $\mathbf{T}$ is given by:

$$\mathbf{T} = \begin{bmatrix} cos^2\phi & sin^2\phi & cos\phi sin\phi \\ sin^2\phi & cos^2\phi & -cos\phi sin\phi \\ -2cos\phi sin\phi & 2cos\phi sin\phi & cos^2\phi - sin^2\phi \end{bmatrix} \tag{9}$$

The constitutive material tensor in principal directions is computed with the NURBS surface derivatives:

$$\begin{bmatrix} \frac{dS_1}{dE_1} \\ \frac{dS_1}{dE_2} \end{bmatrix} = \left(\begin{bmatrix} \frac{dE_1}{du} & \frac{dE_2}{du} \\ \frac{dE_1}{dv} & \frac{dE_2}{dv} \end{bmatrix} \right)^{-T} \cdot \begin{bmatrix} \frac{dS_1}{du} \\ \frac{dS_1}{dv} \end{bmatrix} \tag{10}$$

$$\begin{bmatrix} \frac{dS_2}{dE_1} \\ \frac{dS_2}{dE_2} \end{bmatrix} = \left(\begin{bmatrix} \frac{dE_1}{du} & \frac{dE_2}{du} \\ \frac{dE_1}{dv} & \frac{dE_2}{dv} \end{bmatrix} \right)^{-T} \cdot \begin{bmatrix} \frac{dS_2}{du} \\ \frac{dS_2}{dv} \end{bmatrix} \tag{11}$$

The algorithm of the material model based on NURBS for principal directions is presented in the following box:

1. Update the strain tensor.
$$\mathbf{E}_{n+1} = \mathbf{E}_n + \nabla^S u$$
2. Calculate the strains in principal directions
$$\hat{\mathbf{E}}_{n+1} = \mathbf{T}^T \mathbf{E}_{n+1}$$
3. Calculate the local parameter u and v from the strains.
4. Obtain the stress values $S_1(u,v)$, $S_2(u,v)$.
5. Calculate the derivatives $\frac{dS_1}{dE_1}$, $\frac{dS_1}{dE_2}$, $\frac{dS_2}{dE_1}$, $\frac{dS_2}{dE_2}$, and $\frac{d\hat{S}_{12}}{2d\hat{E}_{12}}$ (equations 10, 11 and **??**).
6. Constitutive material tensor is obtained:
$$\frac{d\mathbf{S}}{d\mathbf{E}} = \mathbf{T}^T \cdot \begin{bmatrix} \frac{dS_1}{dE_1} & \frac{dS_1}{dE_2} & 0 \\ \frac{dS_2}{dE_1} & \frac{dS_2}{dE_2} & 0 \\ 0 & 0 & \frac{d\hat{S}_{12}}{2d\hat{E}_{12}} \end{bmatrix} \cdot \mathbf{T}$$
7. Calculate the stress tensor.
$$\mathbf{S} = \mathbf{T}^T \cdot \hat{\mathbf{S}}$$

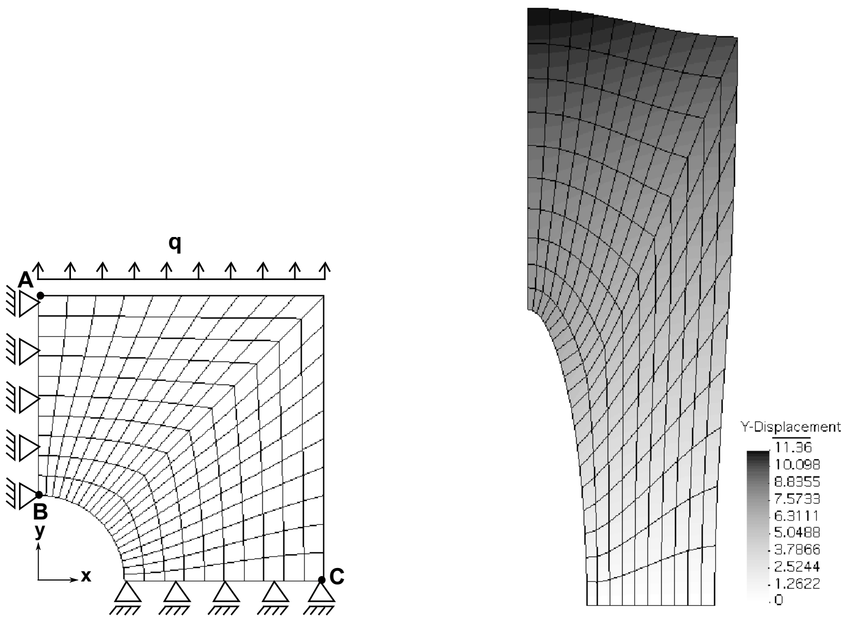
4 VALIDATION EXAMPLES

The PD–NURBS material model is applied to examples with different material responses to validate the proposed material model. Attention is given to materials with large strains.

Data fitting based on least-squares aproximation is used to generate NURBS surfaces for the experimental data. For more details see the works of Piegl and Tiller[6] and L. Piegl [5]. An alternative approach for the generation of NURBS surfaces is the use of a CAD software.

4.1 Hyperelasticity – Mooney-Rivlin

This example consists of the stretching of a square sheet with a circular hole. This example is found in Gruttmann and Taylor [3] and in Souza Neto et al. [9]. The length of the sheet is $20m$, the radius of the circle is $3m$ and the thickness is $1m$. Due to problem symmetry, one quarter of the sheet was analyzed and the mesh with 200 linear quadrilateral membrane elements is presented in figure 1(a). The material used is on of the Mooney-Rivlin type with the constant values of $C1 = 25MPa$ and $C2 = 7MPa$. Thus the Ogden material constants are $\mu_1 = 50MPa$, $\mu_2 = -14MPa$ and $\alpha_1 = 2$, $\alpha_2 = -2$. The analysis was performed under load control conditions in three steps.



(a) (b)

Figure 1: Square sheet with a circular hole (a) undeformed sheet mesh with applied load (b) diplacement result in y direction with deformed sheet in a scale of 1:1.

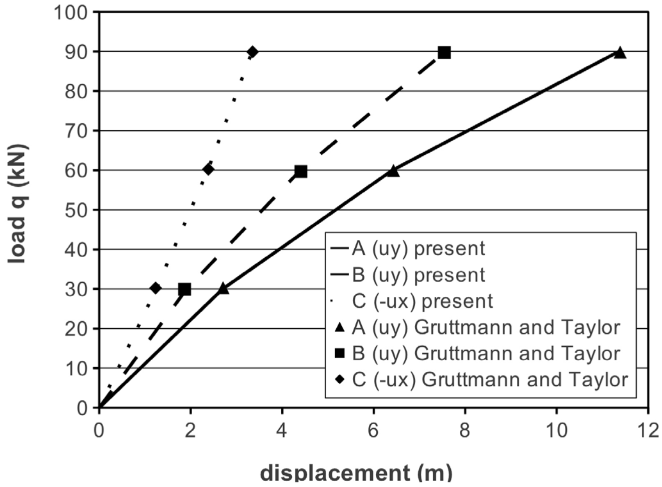


Figure 2: Load–displacement curves of stretching of a square sheet

The results obtained are compared with the nonlinear material model based on NURBS surfaces. Figure 3 shows the NURBS surfaces used in these examples. This surfaces are composed by a net of control points 120(u) x 120(v) and degree 3 ($p = 3$ and $q = 3$).

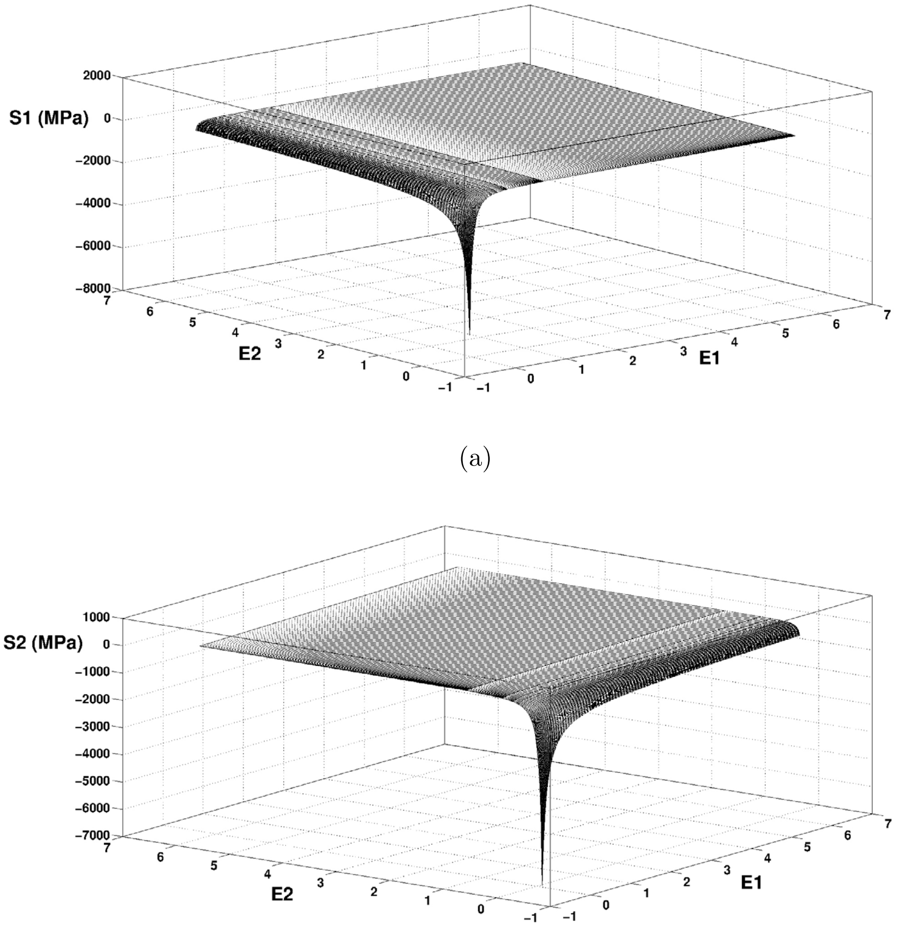
4.1.1 Results

Figure 2 show the load-displacement curves, of three points on the mesh (A, B and C highlighted in figure 1), for the work of Gruttmann and Taylor [3] and the results obtained with the proposed material model based on NURBS. The results show good accuracy.

4.2 ETFE-Foil modeled with PD-NURBS

This example shows the application of PD-NURBS to model a material making use of the available experimental data. The experimental results used to generate the NURBS surfaces are those of the biaxially loaded ETFE–foil under two loading programs ratios of applied force: 1:1 and 2:1 presented in the work of Galliot and Luchsinger [2]. The available experimental data is not enough to generate good NURBS surfaces. In order to obtain a point cloud data necessary for the generation of the NURBS surface, data points based on the von Mises elastoplatic material formulation will be used. Figure 4 shows the experimental data points represented by the filled circles and the artificial ones by hollow squares. In this figure the gap between the points of the experimental test is observed. With this data points, NURBS surfaces in principal directions for stress and strain are generated and figure 6 shows the NURBS surface in conjunction with the experimental data points.

There is a dependence of the material model formulation with the size of the NURBS surfaces, in other words, input strains outside the NURBS surface, do not generate output



(a)

(b)

Figure 3: NURBS surfaces with stresses and strains in principal directions for the Mooney-Rivlin material: (a) stresses in direction 1, and (b) stresses in direction 2.

stress results. In these regions artificial data is used to supply the stresses and strains information.

In figure 6 it is observed that the experimental data points are on the NURBS surfaces.

The test is carried out for two load ratios 1:1 and 2:1. The mesh used is a rectangular membrane presented in figure 5. This mesh has 441 nodes and 400 quadrilateral linear elements. In figure 5 the boundary conditions and the applied loads for this model are presented. These examples are symmetric, therefore one quarter of the problem is modeled.

The analysis is carried out with the arclength control method and an equivalent nodal force is applied on the edges.

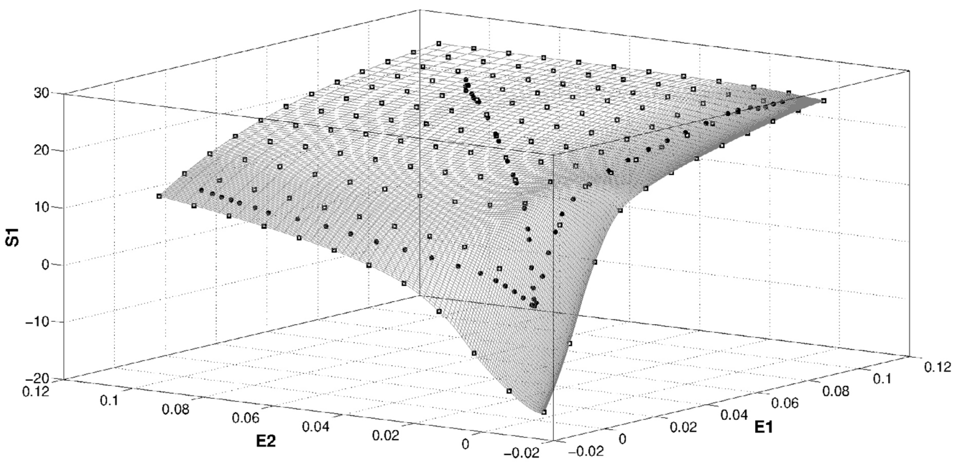


Figure 4: NURBS surface with experimental data

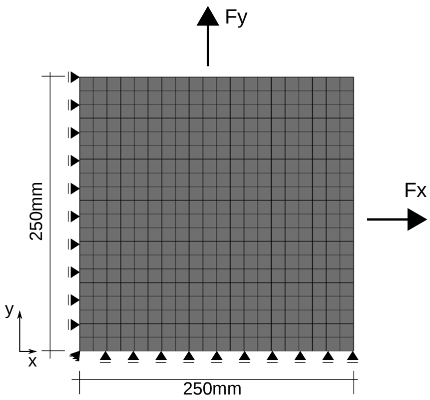


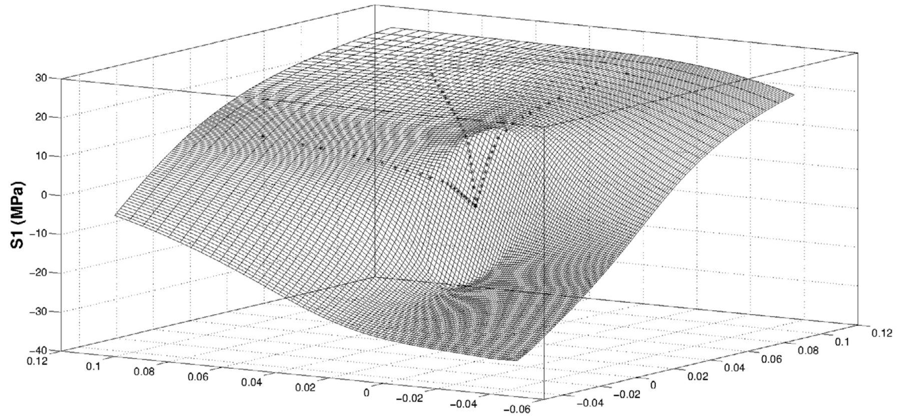
Figure 5: Mesh, geometry and boundary conditions for the biaxial test

4.2.1 Results

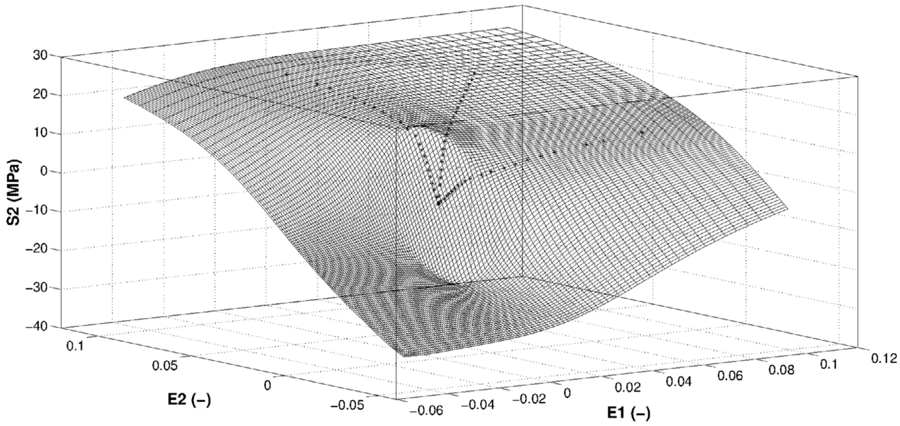
For both load ratios, the results are compared with the experimental results of Galliot and Luchsinger [2]. Table 1 shows the relative error of the numerical model with PD–NURBS material for stress and strain results.

Table 1: Relative error of biaxial test for the PD–NURBS material

Error (%)					
Biaxial 1:1		Biaxial 1:1			
Strain	Stress	Strain	Stress	Strain	Stress
		direction 2		direction 1	
0.42	1.99	0.95	0.32	1.57	1.63



(a)



(b)

Figure 6: NURBS surfaces of stress and strain in principal directions for von Mises material: (a) stresses in direction 1 and (b) stresses in direction 2.

Table 1 shows that the error with the PD–NURBS material for the biaxial test for load ratios of 1:1 and 2:1 is small compared to the experimental results.

5 CONCLUSIONS

The present work presents a material model, which use NURBS surfaces as response surfaces for material behavior. The material behavior is defined with NURBS surfaces with stresses and strains in principal directions. These NURBS surfaces are generated with the results from biaxial tests. The advantage of this material model is that from results of experimental tests, a material model can describe the material behavior. On the other hand, the experimental data should provide a such point distribution as to generate good NURBS surfaces. This point distribution could result in a necessity for a large range of experimental data.

The results obtained for the perforated square membrane with Mooney–Rivlin material model are compared with the results from literature. The results obtained are in complete accuracy.

Numerical analysis with the finite element method using the PD–NURBS material model are applied to model the ETFE material. The error obtained is small and the results can be improved with the optimization of the NURBS surface.

With respect to computational time for the analysis no significant difference between the PD–NURBS material and conventional material was observed.

We conclude that this material model is a good alternative to conventional material models.

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