

# FINITE ELEMENT MODELLING IN INTEGRAL DESIGN STRATEGIES OF FORM- AND BENDING-ACTIVE HYBRID STRUCTURES

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## Summary

*This paper discusses form-finding and simulation strategies for form- and bending-active hybrid structures, with practical feedback from two realised projects. Next to some general aspects of computational form-finding approaches with focus on finite element methods (FEM), the influence of changing mechanical properties of elastic beams on the resultant form-found hybrid system will be discussed on an umbrella structure with integrated bending-active beam elements. Alongside the question of simulation strategies comes the search for a practical design setup to establish an FEM environment that is cross integrating information from various other modelling environments. This is discussed through the case study project M1 where physical form-finding and vector-based spring methods are utilised to generate input data for the FEM simulation.*

## 1 INTRODUCTION

The flexibility and lightness inherent to bending-active structures [1] integrates well with form-active membrane structures that are themselves flexible and adjust to applied loads [2]. Their particular reciprocal dependency of mechanical properties, pre-stress and form makes them an interesting field of study for exploring computational form-finding techniques and thereby developing new kinds of structure systems.

Functionally, the integration of bending-active elements within a pre-stressed membrane surface offers the possibility of short-cutting tension forces and creating free corner points. The system is stabilised solely by the elastic beams which, in turn, are restrained by the membrane surface. Since buckling of the beam is prevented, slender elastic beam profiles may be used. Formally, adding the elastic beam to the limited catalogue of basic membrane types generates a vast extension to the possibilities of shape generation. The introduction of the elastic beam hybridizes the clearly separated membrane-only types as it may, for example, transform from a curved boundary edge directly into a ridge within the surface. In this hybrid

approach of combining form- and bending-active structures, a new scope of formal possibilities presents itself. We classify this interdependence of form and force of mechanically pre-stressed membranes and bending-active fibre-reinforced polymers as a *textile hybrid*.

Independent of the exact simulation technique, the form-finding process of such textile hybrid structures may be split into several steps, usually starting with the elastic deformation of the beam elements. The membrane surfaces are then generated on the updated boundary conditions of the beam elements leading to a second form-finding of the combined system. In some cases of low topological complexity, a completely simultaneous form-finding of both systems may also be possible. In this paper both approaches are shown in the discussion of two case study projects.

While standard routines for the form-finding of membrane structures are available in commercial FEM software, the necessity of incremental deformation for form-finding large elastic deformations in the beam elements is accomplished through custom programming.

## 2 VARIABLES IN THE FORM-FINDING PROCESS

The design process of bending-active structures can be summed up as the alignment of mechanical and geometrical variables to generate a structurally and architecturally functioning result from a physically informed deterministic form-finding process. Similar to membrane structures, the built geometry is a result of the erection process, where the structure is tied to its boundary points. The self-defining shape of the structure is previously discretised by a cutting-pattern in the case of a membrane and the unrolled geometry in the case of a bending-active structure. In contrast to membrane structures, the form-finding result does not automatically define a structurally optimised geometry which may be linked to the fact that bending-active structures offer far more variables to influence the form-finding result.

Both form- and bending-active systems undergo large deformations during form-finding, yet there is a fundamental difference in the simulation of their stress states. For form-active membrane elements, the biaxial tension pre-stress is a command variable set prior to the form-finding, whereas stresses in the bending-active beam elements are computed from the deformation resultant strains. This leads to considering material properties differently during form-finding. In the modified stiffness method used in FEM form- for pre-stressed membranes, one simply considers the fact that a membrane only serves to carry tension forces by simulating a surface under pure tension. The actual mechanical material properties of the membrane are, however, not considered since the form-finding is purely based on the equilibrium of tension stresses and only geometrical stiffness is considered while elastic stiffness is temporarily set to zero. In contrast to this, the form-finding of bending-active structures is largely influenced by the mechanical material behaviour of the beam or shell elements. While the geometry of a single and homogeneous elastica curve is independent of size and material, the structurally necessary coupling of several bending-active components

results in a material dependent geometry.

In form-active structures, the surface dimensions are the minimal result defined by the stress state and boundary conditions which are independent of the input geometry. In contrast to this, the form-finding of bending-active structures is largely influenced by the length of a beam or dimension of a surface that is bent as a result of the constraining boundary conditions as well as the mechanical material behaviour of the beam or shell elements. Consequentially, the two fundamental differences in the form-finding of form- and bending-active structures lie in the definition of length and surface dimension and the simulation of material behaviour.

Having factually doubled the amount of input variables in the form-finding process of bending-active structures, as opposed to form-active structures, noticeably complexifies the general design process. Putting these input variables into a functioning relation, which satisfies both mechanical behaviour and architectural specifications, becomes the challenge of this form-finding and the general design process. Because of this unique combination of freedom and complexity, it was found that one computational simulation technique alone does not offer necessary tools for developing textile hybrid structures. The combination and integration of various modelling techniques into a design process is necessary to successfully develop complex textile hybrid structures. These may include physical, behaviour-based, computational and finite element based modelling.

### 3 FEM FORM-FINDING APPROACHES

#### 3.1 FEM Form-finding of textile hybrids

The form-finding of membrane structures with bending-active support systems necessitates a combined form-finding of the form- and bending-active elements. There are three principal approaches that can be followed to achieve such a combined equilibrium system:

**Additive:** The form-finding of the bending-active and form-active structures are separated. The two systems are coupled together once the separate entities are form-found. A subsequent equalising calculation of the coupled system, where stress is referenced but no additional loads are applied, will find the system's final equilibrium shape. This approach is possible for systems where the membrane has a small or predictable influence on the bending-active structure.

**Successive:** The process is separated into first, the form-finding of an elastically bent beam structure and second, the form-finding of the membrane attached to the beams. Here, the second form-finding step serves to generate an intricate equilibrium system which is based on further deformations in the beam structure.

**Simultaneous:** Some scenarios also allow simultaneous form-finding of bending-active beam elements and pre-stressed membrane elements. For numerical form-finding, the bending of beam elements requires out of plane forces on the beam; this may be achieved by eccentricities and/or three-dimensional input of the membrane-mesh.

In the projects presented in this paper, the FEM software Sofistik® was used for the form-finding of textile hybrids, which has the advantage that the form-finding and patterning routines for tensile membranes are already included in the software. These routines could easily be combined with any of the above mentioned approaches and a custom programmed load increment loop for the bending-active elements. Therefore, any of these approaches may be chosen depending on the individual nature of the form-finding problem. In addition, a physics-based particle –spring computational modelling environment was used for informed digital form explorations with bending-active and textile hybrid systems in particular.

### 3.2 Elastic cable approach

As a practical approach for form-finding coupled bending-active systems in FEM, the first author developed a new strategy using contracting cable elements to pull associated points from an initially planar system into an elastically deformed configuration [3]. These cable elements work with a temporary reduction of elastic stiffness which enables large deformations under constant pre-stress. This method was originally developed for the form-finding of tensile membrane structures using, for example, the transient or modified stiffness method [4] and [5]. For the form-finding of coupled bending-active systems, the great advantage is that the cables allow complete freedom of the equilibrium paths that are followed during the deformation process. The pre-stress independent of the change in element length also allows the simultaneous use of several cable elements in the different positions of the system.

This approach enables the form-finding of topologically highly complex systems as represented by the M1 Project discussed below.

### 3.3 Continuous mechanical description

Form-finding in FEM requires system updates before structural analysis with external loads is performed on the form-found system. In this update, both geometry as well as inner stress states are stored in the stiffness matrices of the model to create an updated reference state of the system that includes all mechanical information. This mechanical continuity of the FEM model becomes particularly important in the aforementioned successive form-finding approach of textile hybrid systems. The fact that the form-found equilibrium state of such systems is only satisfied if both updated coordinates and stored elastic stresses are included in the subsequent static calculations was also discussed by Philipp et.al. [6].

Fig. 1 shows the continuous mechanical description of the structural FEM Model from form-finding to patterning of the M1 Project introduced below. In this example, the form-finding is

*successive*, starting with the bending-active system where the hybrid with form-active elements is form-found afterwards. For the validity of the equilibrium state form-found with this approach, it is essential that an update of the model always references both coordinates and elastic stresses. This process is continued all the way through to patterning of the membrane, in which compensation is based on the stress state of the membrane surfaces.

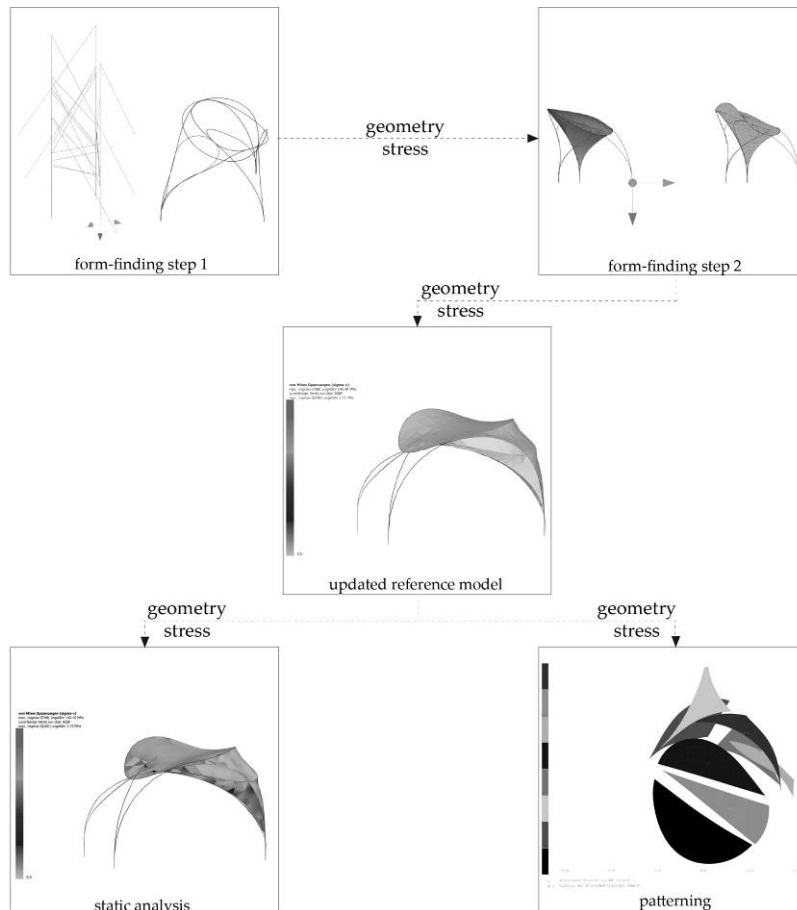


Figure 1: Continuous mechanical description of the structural FEM Model from form-finding to patterning

#### 4 SPRING BASED MODELLING

Spring-based methods have been developed to simulate and explore complex material behaviours, utilizing vector-based methods combining mass (particle) and momentum (spring) forces within a time-based solver [7]. In the research developed by the second autor, this has been employed for enacting both tensile and bending stiffness. Particular material behaviours are defined in specific spring topologies placing positional constraints on variable

(user-defined) networks of particle-springs. A hierarchy is established where certain springs serve to provide tensile and bending stiffness (in- and out-of-plane forces) while other particle-springs define meshes representative of a physical geometry. In this topological construct, the relationships within the condition of the textile hybrid are relative and not explicitly expressive of select material descriptions.

The approximation of the mechanical behaviour for bending stiffness is captured in particular topological arrangements of springs and the springs' properties of stiffness. Three primary numerical methods have been established in Computer Graphics: cross-over, vertex position and vertex normal [8]. The vertex normal method has been expanded upon in structural engineering simulating three degrees of freedom at each particle (node) in order to calculate the behaviour of bending-active beam elements [9].

#### **4.1 Interaction**

Spring-based methods enable explorations of textile hybrids to advance complexity and specification in their topological relationships via relative descriptions of force characteristics. A certain freedom from specific mechanical descriptions allows for a minimization of the pre-planning effort prior to beginning the form-finding process. Most importantly, housed within a functional modelling environment, relationships can be actively manipulated in topology and behavior, while the time-based solver continually runs. Where the complexity of the textile hybrid belies intuition, iterative feedback through the computational environment elicits knowledge in particular topological and behavioural manipulations. In a programmable modelling environment, such specific manipulations can be encapsulated and embedded in order to advance the potentials for geometric performance. Such was accomplished with a modelling environment programmed in Processing (Java) by the second author.

## **5 CASE STUDIES**

### **5.1 Umbrella Marrakech**

At the Institute of Building Structures and Structural Design, a new type of membrane structure was developed and realised in collaboration with HFT Stuttgart. The project is based on a student workshop that developed shading solutions for an outdoor plaza space at an architecture school in Marrakech, Morocco. The design proposal of a funnel-shaped membrane roof was further developed by the first author with the aim of minimizing anchoring forces to the surrounding buildings. The introduction of a bending-active supporting structure for the free edges of the membrane proved to be a very efficient solution. After a successful test setup in Stuttgart, which took place in June 2011, the structure was mounted by students from Stuttgart and Morocco in March 2012.

The structure features six elastically bent glass fibre rods with a length of approx. 7.5m. The rods push out three additional corner points on both free edges of the structure. The funnel-shaped membrane has a span of approx. 11m x 11m and an eaves height of 5.5m resulting in a membrane surface of approx. 110m<sup>2</sup> (Fig. 2).

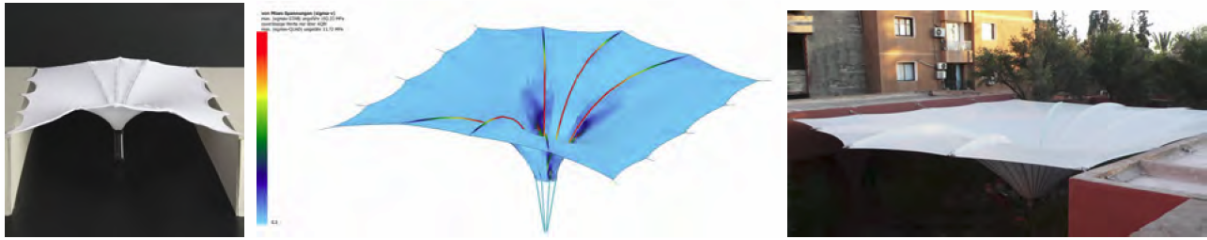


Figure 2: Physical model, FEM simulation and realized structure of the Marrakech umbrella

For the relatively simple geometry of the umbrella, the simultaneous form-finding approach is used (Fig. 3). Here, controlling the stability of the beam during form-finding is a particular challenge, since the stabilizing effect the membrane has on the beam is only activated in the post form-finding configurations. This necessitates a temporary restraining of the beam perpendicular to the bending plane. This is particularly the case because the utilised FEM Software Sofistik® uses the transient stiffness method where the large deformations which occur during the form-finding are enabled by a temporary reduction of the elastic stiffness in the membrane and edge-cable elements [4]. Floating coupling elements are used to control the off-set distance between the mechanically pre-stressed membrane elements and elastically deformed beam elements.

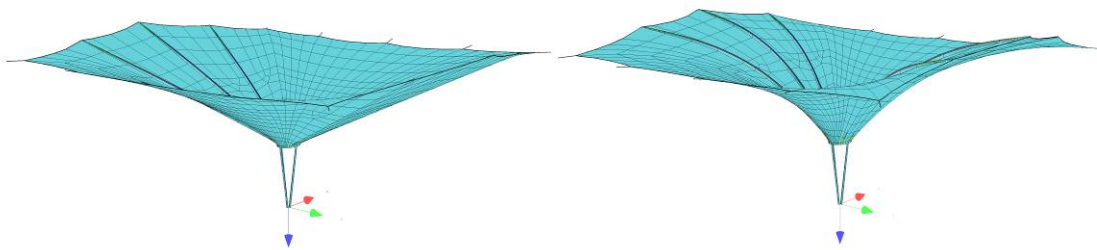


Figure 3: Simultaneous form-finding of bending- and form-active members in FEM

The influence of the elastic beam's mechanical properties on the form-finding result becomes particularly visible in this structure where the cantilevering condition of the beam offers little geometrical constraint on the equilibrium shape. Figure 4 shows a comparison of different beam stiffness ratios  $n$  in the FEM simulation results from simultaneously form-finding the form- and bending-active elements of the structure.

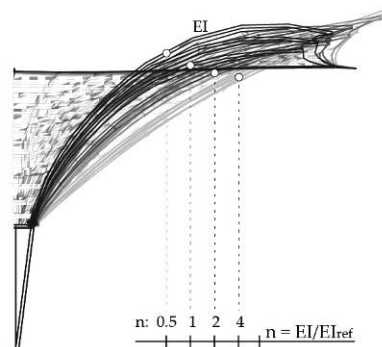


Figure 4: Influence of the beam element stiffness on the form-finding result

## 5.2 M1 Project

The Textile Hybrid M1 at La Tour de l'Architecte showcases the research on hybrid form- and bending-active structure systems. The scientific goal of the project was the exploration of formal and functional possibilities in highly integrated equilibrium systems of bending-active elements and multi-dimensional form-active membranes. The resulting multi-layered membrane surfaces allows not only for structural integration but also serves a functional integration by differentiating the geometry and orientation of the membrane surfaces. The site selected for the design is a historical and structurally sensitive tower in Monthoiron, France. The tower is based on a design by Leonardo Da Vinci from the 16th century, which brought the owners to the idea of making the tower usable for exhibitions. On the basis of a spatial program, a textile hybrid system is developed where short-cutting of forces produces a minimization of the loading on the tower. In the context of this project, the M1 is developed as a representative pavilion.

The scientific goal of this project was a formal and functional exploration of textile hybrid systems through the establishment of an iterative design process, which passes through various modelling environments and finishes with a realised structure (Fig 5).

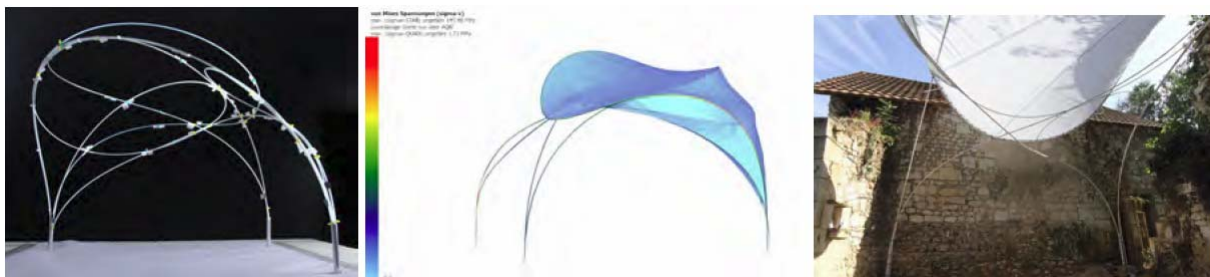


Fig. 5: physical- and FE model as well as erection of the primary form- and bending-active system



The M1 structure is comprised of 110 meters of GFRP rods, 45m<sup>2</sup> of membrane material covering an area of approx. 20m<sup>2</sup> and anchored to the ground with only three foundations resting against the existing stone structures which neighbour the tower. In total, the textile hybrid structure weighs approximately 60 kilograms (excluding foundations), with clear spans ranging from 6 to 8 meters. Fig. 8 shows the finished structure, elaborating upon the hybrid nature of the system, where the organisation of bending-active beams and tensile surfaces creates moments of long span arches, overlapping grid-shell conditions, and doubly-curved pure tensile surfaces, both at a macro- and meso-scale.

For generative studies, the spring-based modelling environment as described above is utilised alongside exhaustive physical form-finding experiments. The computational modelling allows for complex topologies to be developed and altered, quickly registering feedback from the prototypical physical studies (Fig 6). In particular, this approach was utilised for the form-finding of the secondary textile hybrid system, a series of differentiated cells providing additional structure to the primary envelope and variation to the illumination qualities of the space.



Fig. 6: Spring-based computational modelling of internal cell logic. Cell assembly in final structure.

As both a design avenue and method for material specification, FEM is utilised. The parameters of the complex equilibrium system are explored to determine the exact geometry and evaluate the structural viability. The complexity of the M1 geometry necessitates a separation of the form-finding steps and applying the successive approach as introduced above. After the form-finding of the beam elements, the membrane mesh is generated on the given boundary conditions of the edge beams. Because of the general elasticity of the structure, the membrane pre-stress largely deflected some of the beams and therefore has a significant influence on the overall geometry. By means of automatic mesh generation, the membrane surfaces are added and a final form-finding of the fully coupled textile hybrid is undertaken (see Fig. 7). This form-found structural analysis model allows verification of the geometrical shape including its residual stress, as well as analysing the deformations and stress levels under external wind loads. Furthermore, the form-found membrane surfaces are processed directly by the textile module of the software for patterning (Fig 1).

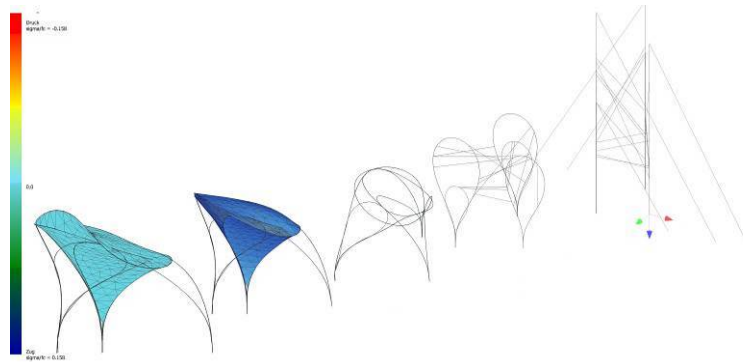


Fig. 7: Successive form-finding of the M1 textile hybrid using FEM

Thus, all three design models, the physical and both generative, and specific simulation techniques informed each other in this iterative design process. The realised structure was fully based on the digital information computed through these modelling environments. The fitting harmonious pre-stress of the patterned membrane surfaces proved the validity of this design and simulation process (Fig. 8).



Fig 8: The textile hybrid M1: Internal view of multilayer membrane system with integrated cells.  
External view in the context of the tower ruin.

## 5 CONCLUSIONS

The two case study project discussed in this paper showed that a full exploration of the functional and formal potentials of textile hybrid structures necessitates the use and close interaction of various modelling environments.

The Marrakech Umbrella relied on FEM simulation alone and thereby proved the ability of computing complex equilibrium systems with modern commercial FEM software. However this project was not able to explore further new structural and formal possibilities of textile hybrid systems. This aspect was taken up by the M1 project in which a design methodology was established that enabled a cross integration of information from various other modelling environments. On-going research is focusing on refining this process to establish a closer interaction of the physics-based particle –spring computational modelling environment with the FEM simulation.

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