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AERODYNAMIC DAMPING OF MEMBRANES IN STILL AIR

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Summary. The aerodynamic mechanisms modifying the dynamic response of membrane structures oscillating in still air are discussed. Emphasis is placed on aerodynamic damping and its connection to vortex shedding from the membrane edges. A simplified vortex-particle method for the prediction of aerodynamic damping and added mass for two-dimensional configurations is implemented and tested. It appears to be valuable for preliminary assessments of the corresponding parameters.

1 INTRODUCTION

Under atmospheric conditions, no matter if windy or not, oscillations of membrane structures are necessarily accompanied by air movements. With increasing lightness of the structures and oscillation amplitudes, these movements may result in considerable forces that strongly modify the dynamic response of the structure. An unfavourable consequence of this interaction may be the effective load increase due to the inertial forces caused by the added air mass that moves with the membrane. This load-increasing effect is on the other hand counteracted by the transfer of vibration energy from the oscillating structure to the surrounding air taking the form of an additional aerodynamic damping. Certainly, the most accurate way to cover these effects numerically or experimentally is to perform full FSI (fluid-structure-interaction) simulations or to do wind tunnel tests with exactly scaled aero-elastic models. However, both techniques are often far too expensive or even technically infeasible. This is especially true with regard to the majority of projects that require quick solutions and do not budget for highly advanced research efforts. Therefore, it is necessary to formulate and assess simplified procedures to model the complexities of real fluid structure interactions without disregarding too much of the underlying physics.

A first step towards this direction is the restriction to a limited number of modes and the introduction of linearized relations between deformation and deformation-induced forces. Such an approach is frequently applied in bridge aerodynamics where the stability characteristics of bridge sections at a specified reduced frequency are derived from the aerodynamic derivatives. The aerodynamic derivatives are measures quantifying the change rate of aerodynamic forces (lift, drag and torsional moment coefficients) due to modal displacements, velocities and accelerations of the section and can be obtained from section wind tunnel tests or numerical (CFD) computations carried out in two or three dimensions. Details about the theory can be found in numerous publications and textbooks (e.g. [1] and references therein).

Despite the evident differences between membrane structures and bridges, a similar approach can be used for membranes. If only one mode of vibration is considered and restriction is made to the still-air case, which is usually not of interest in bridge aerodynamics, the set of aerodynamic derivatives effectively collapses to just two parameters describing the air influence on the dynamical characteristics: added mass and aerodynamic damping.

The term “added mass” refers to the effect of air that is accelerated and decelerated with every oscillation cycle and therefore leaves an additional pressure footprint on the membrane in phase with the motions. Understanding and modelling of added mass effects have advanced relatively far. An extensive review is provided in AlSofi *et al.* [2]. For the still-air situation, the added mass effect is well represented when the flow induced by the modal movements is regarded as irrotational except at the boundaries where a time-varying thin vortex-sheet is assumed. A corresponding analytical solution had been suggested by Minami [3] and was confirmed experimentally [4]. Due to the predominantly irrotational nature of the flow induced by the membrane movements, the problem of added-mass computation is well-suited for boundary element methods which do not require any spatial discretization of the entire flow domain but just a discretization of the surfaces [5]. The scaling of the added mass in still air is only related to the geometry of the setup; the added mass ratio is independent of the oscillation frequency. For the range of amplitudes of practical interest with $a_0/L<<1$, where a_0 is the maximum oscillation amplitude and L is a representative length scale of the structure, the added mass is also amplitude-independent.

The potential flow approach that proves to be useful for the modelling of added mass, however, implies that there is no energy flux from the oscillating structure into the fluid where it would be finally dissipated. In reality, viscous friction at the boundaries and separation of the vortex sheet at the membrane edges are unavoidable and will be perceived by the membrane as aerodynamic damping. It is very important, however, to distinguish between the still-air case (which we are focussing on) and a situation with oncoming wind where negative damping cannot be excluded.

An overview over the current state of research on aerodynamic damping of membranes is provided by AlSofi *et al.* [2] together with a presentation of results obtained from FSI simulations. Moreover, a concept is proposed on how to combine wind-tunnel time series from rigid model tests and simplified FSI computations in order to obtain realistic results. As the computed results show, the dependencies of aerodynamic damping on the considered parameters, especially on the ratio a_0/L between oscillation amplitude and length scale of the structure, are more intricate than for added mass and not fully understood yet.

The scope of this paper is not to offer a ready-to-use method to quantify aerodynamic damping but rather to explore the underlying mechanism and to check for the appropriateness of vortex methods for its modelling. As outlined above, the added mass effect in still air can be relatively well captured by potential flow theory. Therefore, our initial assumption is that the flow field around an oscillating membrane is in fact well-represented by potential flow theory but becomes perturbed by vorticity released into the flow domain at the membrane edges. The presence of a large irrotational region and some smaller areas of highly concentrated vorticity suggests the use of vortex particle (or vortex filament) methods. In order to concentrate on the very essentials of the underlying mechanisms we restrict ourselves to two dimensional geometries (1d membrane within a 2d flow domain) and only consider the still air case.

This paper is organized as follows: First (chapter 2), the generalized equation of motion for a 1d-membran surrounded by a two-dimensional fluid is stated and the generalized damping and mass parameters are introduced. This is followed by an illustration of a possible mechanism leading to aerodynamic damping (chapter 3) and a brief introduction to the vortex-particle method (chapter 4). In chapter 5, the method is applied in order to compute the added mass and aerodynamic damping for an example documented in [2] and finally (chapter 6), a short outlook is given.

2 EQUATION OF MOTION

Starting point for our analysis is a 1d-membrane with a constant mass distribution surrounded by an infinite two-dimensional flow domain. We consider only the first mode of vibration which we assume to be known and not to be modified by added-air impacts.

The generalized equation of motion for the first mode equals the equation of motion for a 1-DOF oscillator:

$$m_{gen}\ddot{q} + c_{gen}\dot{q} + k_{gen}q = F_{gen}(t) \tag{1}$$

where m_{gen}, c_{gen}, k_{gen} are the generalized mass, damping and stiffness, respectively, and F_{gen} is the generalized external force.

For the sake of simplicity, we consider the case of a free oscillation after an initial modal displacement. It is assumed that external forces acting on the membrane depend exclusively on its current motion velocity and acceleration:

$$F_{gen}(t) = F_{gen,mot}(t) = f(\dot{q}, \ddot{q}) \tag{2}$$

As outlined above, we consider only the still-air case. Strictly speaking, this would imply that fluid motions that were induced during previous oscillation cycles are still moving around in the vicinity of the membrane and may influence the surface pressure at later instants. However, the influence of this flow field evolution is neglected in equation (2) - possible consequences will be addressed in chapter 5.

Linearization of equation (2) in the range of small amplitudes may be expressed as:

$$F_{gen,mot}(q, \dot{q}, \ddot{q}) \approx \frac{dF_{gen}}{d\dot{q}}\dot{q} + \frac{dF_{gen}}{d\ddot{q}}\ddot{q}\,. \tag{3}$$

Inserting (3) into the equation of motion (1) results in the equation of motion including the added quantities:

$$\left(m_{gen} + m_{gen,air}\right)\ddot{q} + \left(c_{gen} + c_{gen,air}\right)\dot{q} + k_{gen} q = 0 \tag{4}$$

where the added mass and added damping are:

$$c_{gen,air} = -\frac{dF_{gen}}{d\dot{q}} \tag{5}$$

$$m_{gen.air} = -\frac{dF_{gen}}{d\ddot{q}} \tag{6}$$

Thus, the added mass and damping parameters of a specific mode (here the lowest mode of vibration) describe the dependence of the additional generalized air forces on the velocity and acceleration of the modal displacement. Assuming the validity of equation (2), it should be possible to determine these parameters from the surface pressure timeseries obtained from experimental or numerical tests with membrane sections undergoing forced motions. Considering, for example, a sinusoidal motion $q(t) = q_0 \cdot \sin(2\pi f \cdot t)$, the added mass can be derived from the sinusoidal part of the pressure reaction while the aerodynamic damping is related to the cosine component.

Thus, modelling the added quantities is basically equivalent to modelling the flow and pressure patterns resulting from the air displacements around an oscillating structure. These flow patterns shall be briefly discussed in the following chapter.

3 A QUALITATIVE PICTURE OF AERODYNAMIC DAMPING

As outlined in the introduction, added mass effects are well represented assuming a vortex sheet at the current position of the membrane and an otherwise irrotational flow field. Figure 1 shows an image taken from Minami [3] which illustrates the flow pattern outside the ends of a vibrating membrane. The corresponding potential flow solution exhibits singularities at the membrane ends so that the tangential flow velocities as well as the surface suction locally grow to infinity. In the instant of maximum velocity, i.e. when $\dot{q} = \max$ and q=0, the pressure distributions on the upper and lower surface of the membrane would cancel out and not produce any net force leading to damping.

Under real conditions, the vortex sheet will neither remain infinitely thin nor attached to the membrane surfaces. Due to viscosity, it will diffuse from the boundaries into the flow domain. In the high Reynolds number limit, the flow will immediately separate at the membrane edges and the separated vortex sheet will roll up into a spiral vortex (Figure 2). This spiral vortex grows and its further dynamics depends on the subsequent motions of the membrane as will be shown in chapter 5.

Despite possible Reynolds-number dependencies of the flow dynamics caused by a moving membrane, it appears reasonable to assume the Reynolds number to be high enough so that viscosity has no further influence except that it enforces the formation of the vortex sheet and its separation at the edges. In this case, the viscous vorticity diffusion from the membrane surface into the flow domain along the boundaries does not need to be considered and all vorticity entering the flow domain may be assumed to be released from the edges. Thus, the flow in the domain around the oscillating membrane may be regarded as an initially irrotational, temporally varying flow field which is continuously contaminated by vorticity released from the membrane edges. The corresponding vortical disturbances result in pressure perturbations causing the damping of the oscillation.

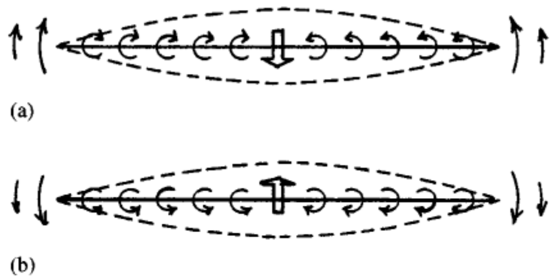


Figure 1: Schematic patterns of flow around the outside ends of the vibrating membrane (image taken from [3])

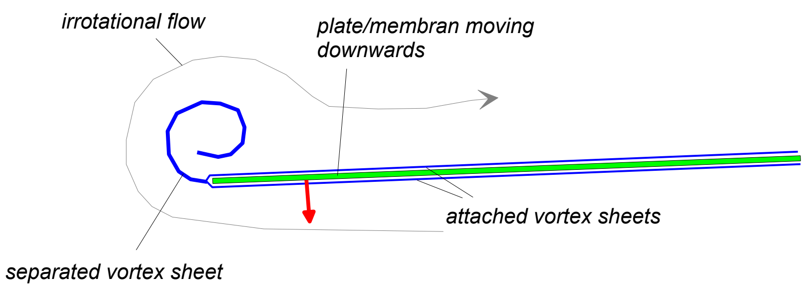


Figure 2: Formation of the spiral vortex immediately after the onset of downward motion of the object (symbolized by the red arrow)

4 VORTEX-PARTICLE METHOD

The basics of Vortex Particle Methods (VPM) are extensively described in numerous articles and textbooks, e.g. Walther, Larsen [6]. Vortex particle methods make use of the fact that the vorticity ω is conserved along the trajectories of the flow as long as viscous diffusion of vorticity is not directly accounted for:

$$\frac{D\omega}{Dt} = 0 \tag{7}$$

The vorticity field at a particular instant is represented by N vortex particles with circulation Γ_i (i=1…N) and position x_i:

$$\omega(\mathbf{x}) = \sum_{i=1 \dots N} \zeta_\varepsilon(\mathbf{x_i} - \mathbf{x})\Gamma_i \tag{8}$$

The function ζ_ε is a smooth approximate to the Dirac function (see, e.g., [7]) which is applied in order to avoid the infinite velocity that would be caused by a singular vortex if $(\mathbf{x_i} - \mathbf{x}) \to 0$.

Since the circulation of each vortex particle is conserved, the dynamics of the flow field depends on the velocity on each one of these particles induced by the other particles. The velocity field is linked to the vorticity field via the Poisson equation

$$\Delta\psi = -\omega \tag{9}$$

where ψ is the stream function so that:

$$\mathbf{u} = \begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} \psi_z \\ -\psi_x \end{bmatrix} \tag{10}$$

An explicit time scheme is applied in order to compute the evolution of the flow field. The membrane is discretized using boundary elements assigned with a specific circulation which are calculated for each time step in dependence of the current flow field and the current membrane position such that there is no vortex-induced flow through the membrane surface and such that the overall circulation of all elements and particles is zero. Usually, vortex-particle methods seek to approximate the effect of vorticity diffusion by a random walk which is superimposed to the particle convection [6]. This is omitted in our method because, as outlined above, the effect of vorticity release at the membrane edge is regarded as dominant and the random walk would quickly populate the vicinity of the membrane surfaces with vortex particles and thus increase the computational effort. However, an a-priori definition of the separation points is necessary.

Given a flow field with N vortex elements, the computational effort increases with N^2 if all particle interactions are accounted for. This would render the method very inefficient for many practical cases. There are several numerical recipes to overcome this drawback. Nevertheless, it turned out that a fairly low number of particles already delivers valuable results for the simple test case documented in the following chapter. At this moment, no further attention has been paid to the question of accelerating the numerical procedure.

5 EVALUATION OF A SELECTED CASE

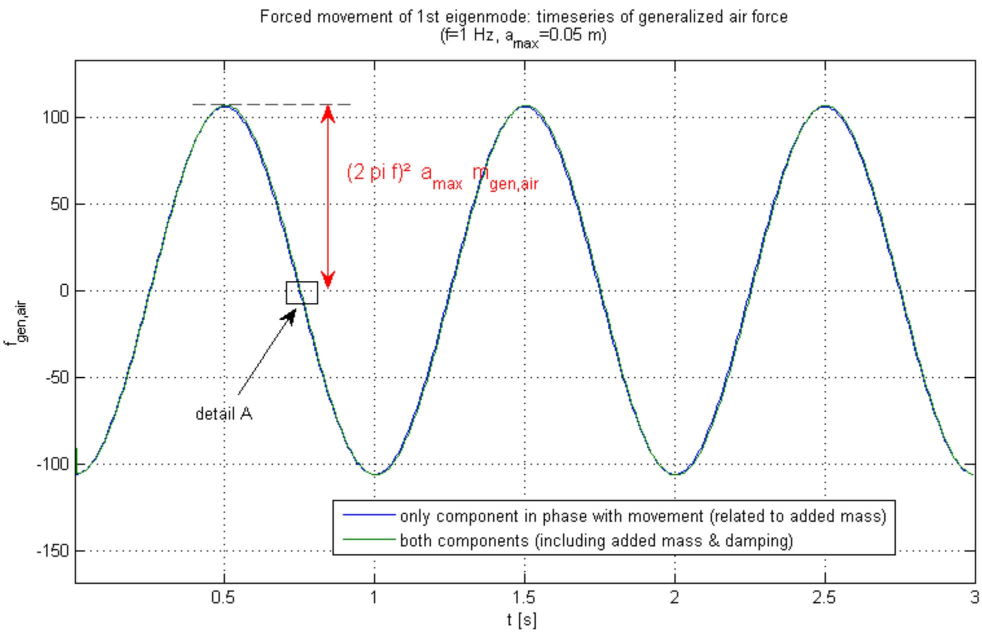
FSI simulations of differently configured one-dimensional and two-dimensional membranes are documented in [2]. We will focus on the test case of a membrane spanning 10 meters with a uniform mass distribution of 2 kg/m². The 1d case represents an infinitely wide membrane while the membrane of the 2d case is 20 meters wide and geometrically fixed at the lateral edges. The evaluation of aerodynamic damping has been done for the two-dimensional case at different initial amplitudes so that the results are not strictly comparable to our 1d run. However, an inspection of the graphically represented displacements for the 1d membrane suggests that the differences in aerodynamic damping between the 1d and the 2d case should be around 20% so the results should be comparable.

The vortex method was applied to a membrane undergoing a forced oscillation at different amplitudes (a_0/L=0.005, 0.015, 0.02 and 0.03 in analogy to [2] and the added mass and aerodynamic damping parameters were obtained in the way described in the second chapter. An example for a timeseries of the generalized air force acting on the membrane induced by the forced oscillation is given in Figure 3. In the upper image it is sketched how to obtain the generalized added mass while the detail in the lower image illustrates the phase shift between motion and force which is linked to the damping. The time step size of the vortex particle method has been reduced until the phase shift was found to converge to a fixed value. As suggested above, the air pressures acting on the moving membrane are not purely dependent on its current velocities and accelerations as implied in equation (2) but are somewhat distorted by the remaining vortical structures from previous motions. This leads to differences in the phase shift between the subsequent cycles. The aerodynamic damping decrement was therefore derived from the first oscillation cycle only.

The obtained results for the reduced frequencies f_a (due to the added mass effect) and the aerodynamic damping ratios for the different investigated amplitudes are stated in Table 1. They can be interpreted as follows:

- The reduced frequency due to added mass corresponds almost exactly to the value obtained in [2] for the 1d-case. The deviation between the value for the 1d configuration and the 2d configuration is due to the different geometry. However, as the results in [2] and other work suggest, the added mass effect is almost independent on the amplitude (at least if a_0/L<<1) and there are only slight variations of the reduced frequency with increasing amplitude.
- The aerodynamic damping is about 30% below the value of AlSofi *et al.* [2] at the lowest investigated amplitude (a_0/L=0.005) but grows stronger with increasing amplitude reaching a value of about 20% above the FSI value for a_0/L=0.03. At the moment, it is unclear in how far these differences stem from the different dimensionality of the domains. However, from an engineering point of view, the obtained aerodynamic damping decrements are within the same range so that the vortex particle method could readily serve as a quick tool for a rough prediction of aerodynamic damping for simple configurations.

Apart from the usability of the obtained results, the time-dependent vorticity distributions obtained from the VPM give some insight into the basic flow patterns induced by the oscillation of the membrane. A series of flow field snapshots at time steps of quarter cycles is displayed in Figure 4. The vorticity distributions around the single vortex particles were somewhat smoothed in order to allow for a better graphical display. The oscillation timeseries which starts with an initial displacement of the membrane and a definition of the plot position



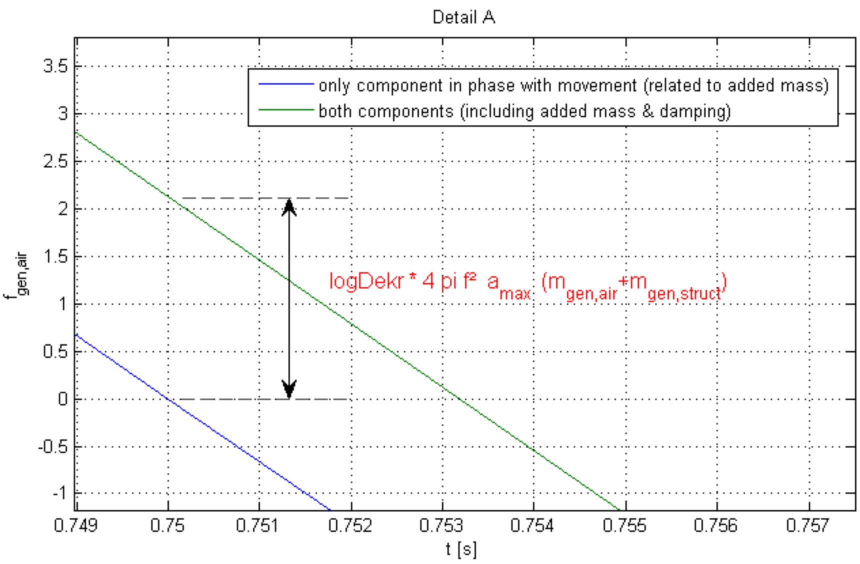


Figure 3 : Time series of the reaction force obtained from a forced-oscillation simulation of the 1d-membrane example and geometrical interpretation of the target parameters $m_{gen,air}$ and δ_{air}. As shown in the detail (lower image), the phase of the overall generalized force (green line) is somewhat shifted from the membrane motion.

are sketched in Figure 5. Image 1 (in Figure 4) shows the vorticity shortly after the onset of motion which is concentrated in the vicinity of the membrane edge. When the membrane passes its rest position (image 2), the spiral vortex composed of negative vorticity from the lower surface of the membrane has clearly developed. The growing spiral vortex induces counteracting positive vorticity which is shed from the upper surface of the membrane. This is already clearly visible after half a cycle (image 3), i.e. when the membrane changes its direction of motion. Now, the background potential flow turns around and transports the initial spiral vortex downwards around the edge, where it separates and develops into a dipole vortex together with the counteracting vorticity field resulting from the vortex sheet that separates from the upper surface (image 4). It appears that the dipole moves away due to its self-induced velocity and finally detaches from the membrane. As images 5 and 6 show, the process is repeated in a similar way after one entire oscillation cycle has past. However, the detached dipole vortices are still left in the domain and it is unclear in how far they influence the further development of the flow field or become decorrelated or dissipated after some time.

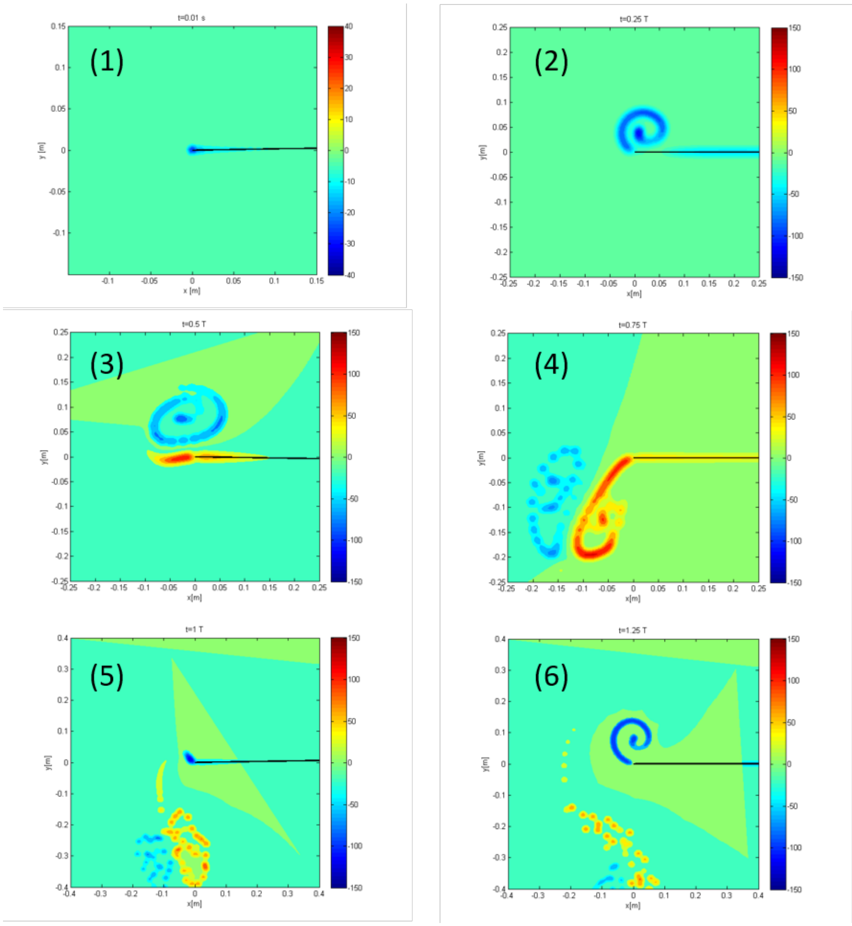


Figure 4: Snapshots of the vorticity field at time t=0.01T (1), t=0.25 T (2), t=0.50 T (3), t=0.75 T (4), t=1.00 T (5), t=1.25 T (6) obtained from a forced-oscillation simulation of the 1d-membrane example (a_0/L=0.005). T is the entire oscillation period. The visible frame represents a small area in the vicinity of the left edge (see Figure 5). The membrane and the time series of its prescribed motion are displayed in Fig. 5.

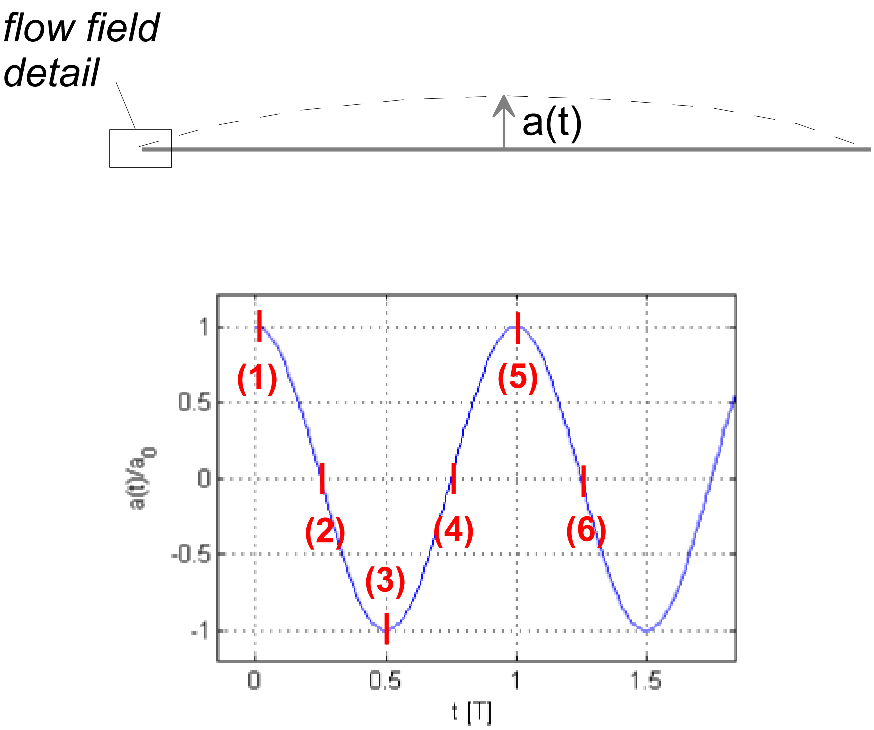


Figure 5: Sketch of the test case (1d-membrane with sinusoidal eigenmode) with indication of the detail frame shown in Figure 4 (upper image) and display of the membrane motion time series including indication of the snapshot times of images 1 to 6 in Figure 4 (lower image).

Table 1: Reduced frequencies due to added mass and aerodynamic damping decrements for the test case at different amplitudes a_0. The values obtained in [2] are displayed for comparison. Index "1d" refers to the output of the 1d-membrane tests (2d fluid domain) and index "2d" to the 2d-membrane tests (3d fluid domain). All values are based on the vacuum eigenfrequency f_s stated in the second column.

a_0/L	f_s [Hz]	Free oscillation test using FSI (from AlSofi et al., 2015)			Forced oscillation test using vortex particle method	
		$f_{a1,1d}$ [Hz]	$f_{a1,2d}$ [Hz	$\delta_{a1,2d}$	$f_{a1,1d}$ [Hz]	$\delta_{a1,1d}$
0.005	0.820	0.322	0.372	0.054	0.325	0.041
0.015	1.022	-	0.453	0.080	0.404	0.089
0.020	1.170	-	0.511	0.089	0.462	0.107
0.030	1.510	-	0.645	0.118	0.594	0.137

6 CONCLUSION AND OUTLOOK

A first attempt was made in order to explain and model the aerodynamic damping of membranes in still air in terms of vortex dynamics. This approach may be regarded as an extension to already existing potential flow models for the determination of added air mass. The obtained results and insights are of preliminary character and further testing and verifying are necessary. An essential fact is that vorticity is concentrated in the vicinity of the membrane edges for realistic vibration amplitudes while large parts of the flow domain remain irrotational. This definitely suggests the use of vortex methods which may be capable of capturing the flow field dynamics with a relatively little number of vortex particles and do not require extensive numerical efforts. We have restricted ourselves to the two-dimensional case, where vortices are modelled as particles. This does not reflect real-life conditions in the engineering of membrane structures. However, an extension of the model to three-dimensions (vortex filament method) might be promising.

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