

# NUMERICAL MODELLING OF A RC BEAM-COLUMN CONNECTION SUBJECTED TO CYCLIC LOADING BY COUPLING DIFFERENT NON LINEAR MATERIAL MODELS

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**Key words:** Reinforced Concrete, Beam-Column Connection, Non-Linear Behaviour.

**Abstract.** The aim of this work is to study the effects of steel reinforcement array inside the RC beam-column connection as it is subjected to cyclic loading, through the numerical simulation of the joint adopting different stirrup's arrays: in order to simulate a real case of study, two specimens extracted from an experimental test of 12 RC beam-column connections reported in literature are modelled in the FEAP code. About numerical simulations based on the Finite Element Method (FEM), the non linear response of the RC beam-column connection is evaluated taking into account the non linear thermodynamic behaviour of each component: for concrete, it is used a damage model; for steel reinforcement, a classical plasticity model is adopted; for steel-concrete bonding, a plasticity-damage model is applied. At the end, the experimental structural response is compared to the numerical results, as well as the distribution of shear stresses and damage inside the concrete core of the beam-column connection, which are analyzed for a low and high confinement.

## 1 INTRODUCTION

Nowadays, Reinforced Concrete (RC) is one of the most important hybrid materials used in the construction industry and consequently its efficient behaviour depends on different aspects basically related to structural design and constructive techniques. Both of them must fulfil with local regulatory requirements for structural security which, given the complex nature of RC, adopt a lot of technical simplifications in design rules and construction codes in order to reduce the effects of uncertainties. In the case of structural design, the accomplishment is done by specifying the geometrical dimensions of the structural element as well as the quantification and location of the respective steel reinforcement. Theoretically, the respect of these specifications must assure the loading capacity of the structural element, or in other words, a good transference of internal efforts and stress between concrete and steel bars. However, in some occasions a blind application of these design specifications complicates unnecessarily the construction work of the structural elements, in particular the beam-column connections, which are at the same time, the key-points for the structural stability of the whole

system. In the other hand, the unreasoned removing of steel bars might reduce dramatically the resistance of the joint, particularly in the event of an earthquake.

Being the beam-column connection the main point of transmission of forces between horizontal elements (beams) and vertical elements (columns), it should provide enough stiffness to the structural system and because of this, there is a high concentration of stresses that potentially might produce any damage in concrete and/or plastic deformations on steel bars. That is the reason why beam-column connection is one of the most risky points of failure in RC structures.

By the way, modern design is strongly dependent of the numerical method adopted for structural analysis –typically a standard finite element code- and the better prediction of the real response (efforts and displacements) is directly derived from the computational capabilities of the code. Modelling of any mechanical problem should include not only the definition of a set of load combinations, but also the selection of a proper finite element associated to efficient material models as well as a good representation of the real boundary conditions. Perhaps due to the complexity of a complete modelling, the local study of any RC connection is practically disregarded by structural engineers, while steel reinforcement array is basically proposed from practical recommendations extracted from limited experimental tests. In consequence, the quantity of steel reinforcement inside the connection might be overestimated or simply poor distributed.

The aim of this work is to study the effects of steel reinforcement array inside the RC beam-column connection as it is subjected to cyclic loading, through the numerical simulation of the joint adopting different stirrup's arrays and quantities. About numerical simulations based on the Finite Element Method (FEM), we evaluated the non linear response of the RC beam-column connection taking into account the non linear thermodynamic behaviour of each component: for concrete, we adopted the concrete damage model proposed by Mazars [1]; for steel reinforcement, we used a classical plasticity model with Von Mises criterion; for steel-concrete bonding, an elastic-plastic-damage model [2,10] is applied. In order to calibrate the modelling, we adopted as an experimental reference the results reported by [3] for a RC beam-column connection.

### **1.1 Basic concepts about beam-column connections**

According to [4], the beam-column connections may be classified following two criteria:

- by the geometrical configuration of reinforcement,
- by the local behaviour of the whole joint.

In the first case, there are internal joints (if the steel bars of beams pass across the joint) and external joints (when the steel bars of beams are anchored into the joint). For the second case, there are elastic joints (if plastic articulations appear into the structural element –beam or column-) and inelastic joints (if any non linearity appears into the connection). The mechanisms of failure of the beam-column connection identified by different authors [5, 6, 7, 8, 9] are the following:

- Beam reinforcement anchorage is not enough inside the joint and the bar slips,
- Shear forces developed into the joint activate the inelastic response of the core of concrete.
- A poor transference of shear forces may produce a failure plan between the joint and the beam, or between the joint and the column.

In general, the most accepted criteria of failure out of the connection is the SC-WB (Strong Column – Weak Beam), which means that any plastic articulation should appear on the beam instead of on the column.

## 2 ESSENTIAL COMPONENTS FOR THE NON-LINEAR MODELLING

### 2.1 The experimental test of reference

The experimental work carried out by Alameddine and Ehsani [3] consisted in reproducing the structural response of an external beam-column joint subjected to cyclic loading in order to verify the recommendations of the ACI-ASCE-352 code. The tests were classified in three groups of four specimens (see figure 1-a), each group with a specific concrete high resistance. Each specimen was designated by two letters and a number, indicating the level of the maximal joint shear stress (first letter), the level of confinement induced by the number of stirrups (second letter) and the value of the compressive strength. For example, the LH11 denomination designates a specimen with a Low shear stress, High confinement level, and a compressive strength of 11 ksi. In other words, these three variables were observed and studied:

- a) The compression strength of concrete (55.8 MPa (8 ksi), 73.8 MPa(11 ksi) and 93.8 MPa ( 14ksi) respectively);
- b) The maximal value of the shear stress into the connection, with a minimal value of 7.6 MPa (1100 psi) and a maximum of 9.7 MPa (1400 psi); and
- c) The contribution of the stirrups by improving the confinement of the core of concrete (see Table 1 for stirrup characteristics).

**Table 1:** Reinforcement on the transversal section of specimen's elements

Specimen	LL	LH	HL	HH
$A_{s1c}$	2 # 8, 1 # 7	2 # 8, 1 # 7	3 # 8	3 # 8
$A_{s2c}$	2 # 7	2 # 7	2 # 8	2 # 8
$A_{s3c}$	2 # 8, 1 # 7	2 # 8, 1 # 7	3 # 8	3 # 8
$A_{s1b}$	4 # 8	4 # 8	4 # 9	4 # 9
$A_{s2b}$	4 # 8	4 # 8	4 # 9	4 # 9
Number of stirrups	4	6	4	6
$\rho_t$	1.2	1.8	1.2	1.8
$h_s/d_{b,col}$	20	20	20	20
Development length $l_{dh}$ (inches) required for $f'_c=8,000$ (psi) (Recommendations 1985)	8.9	8.9	10.0	10.0
Development length $l_{dh}$ (inches)	10.5	10.5	10.5	10.5

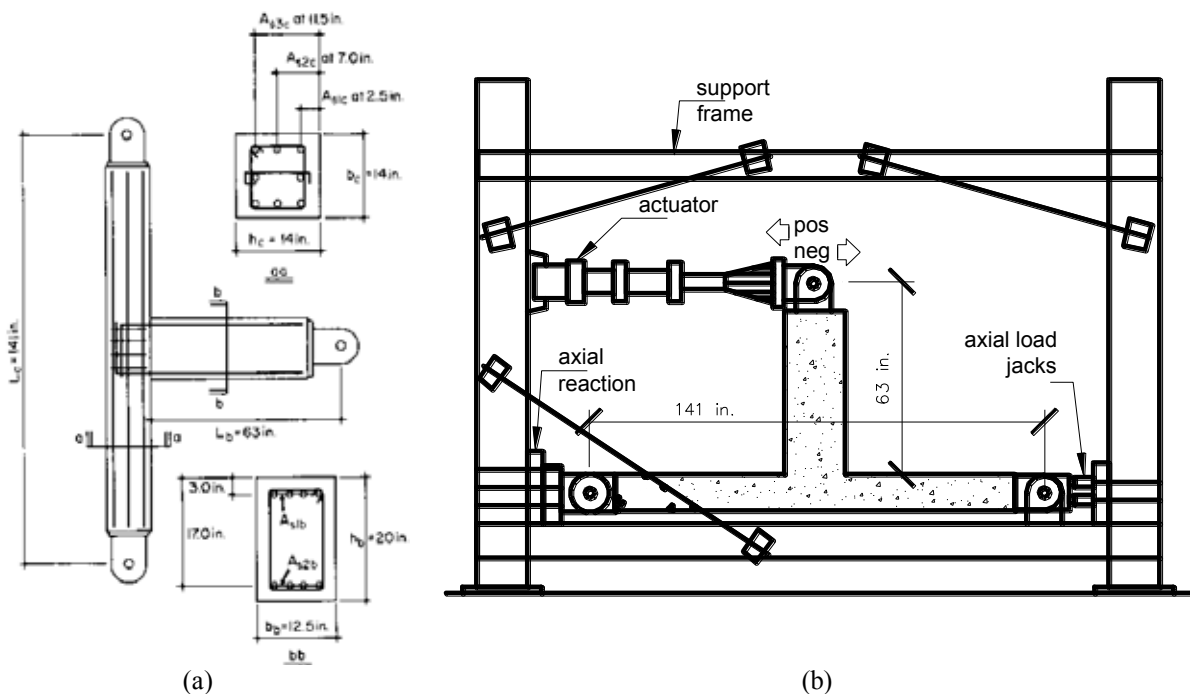
Notes:

$1 \text{ psi} = 6.89 \text{ kPa}$ ;  $1 \text{ inch} = 25.4 \text{ mm}$ ; L: Low; H: High;  
the first letter indicates the level of shear stress; the  
second letter indicates the level of confinement.

Based on a cyclic controlled displacement test (see figure 1-b), the initial displacement in the free edge of the beam was of  $\pm \frac{1}{2}$  inches (13 mm), being increased in  $\pm \frac{1}{2}$  inches (13 mm) in each cycle of loading; during the test a small axial load was applied in the top of the column.

At the end of the test, they reached distortions up to 7% corresponding to a maximum displacement of  $4 \frac{1}{2}$  inches (114 mm), concluding that:

- Elevated shear stresses reduce significantly the load capacity of the connection.
- The value of the ultimate shear stress recommended by the ACI-ASCE-352 for the joint was lower than the value observed in the experimental tests for high resistance concrete.
- The increment of transversal reinforcement reduces the deterioration into the connection, avoiding the failure of the reinforcement anchorage.



**Figure 1:** Description of the experimental test: (a) dimensions of the specimen, in inches; (b) Mounting of the experimental test

## 2.2 Brief description of the non linear models adopted for different material behaviours

One of the main ideas of this research was to adopt and combine different non linear models based on a thermodynamic formulation, in order to include in the structural response the effects of the different dissipative phenomena associated to each inelastic material behaviour.

*Concrete behaviour: the non linear damage model of Mazars.*

The model of Mazars [1] was specially conceived for the particular behaviour of concrete, which is different in compression compared to traction. As any other model of damage, this model is based on the calculation of an effective stress (equation 1) which is function of two scalar damage variables,  $D_t$  and  $D_c$  -traction and compression damage respectively- (equations 2,3). Nevertheless, instead of building the surface of failure in the space of stresses, this one is built in the space of strains, needing the calculation of an equivalent strain (equation 4).

$$\boldsymbol{\sigma} = (\mathbf{1} - \mathbf{D})\mathbf{E}^e : \boldsymbol{\varepsilon}^e \quad (1)$$

$$\mathbf{D} = \alpha_t \mathbf{D}_t + \alpha_c \mathbf{D}_c \quad (2)$$

$$D_i(\tilde{\boldsymbol{\varepsilon}}) = \mathbf{1} - \frac{(\mathbf{1} - A_i)\boldsymbol{\varepsilon}_{do}}{\tilde{\boldsymbol{\varepsilon}}} - \frac{A_i}{\exp[B_i(\tilde{\boldsymbol{\varepsilon}} - \boldsymbol{\varepsilon}_{do})]} \quad (i = t, c) \quad (3)$$

$$\tilde{\boldsymbol{\varepsilon}} = \sqrt{\sum_i (\boldsymbol{\varepsilon}_i^+)^2} \quad \boldsymbol{\varepsilon}_i^+ = \max(\mathbf{0}, \boldsymbol{\varepsilon}_i) \quad (4)$$

In the last equations,  $\alpha_t, \alpha_c, A_i, B_i, \boldsymbol{\varepsilon}_{do}$  are model parameters which can be determined from experimental tests; for this work, their values are presented in Table 2.

**Table 2:** Material parameters for the damage model of Mazars

$A_c$	1.446
$B_c$	1570
$\boldsymbol{\varepsilon}_{do}$	7.428E-05
$A_t$	0.97
$B_t$	8000
f <sub>c</sub> (PSI)	81
Confinement index	1.06
f <sub>t</sub> (PSI)	407.49

*Steel behaviour: a classical non linear plasticity model with hardening.*

For the steel bars, a classical elasto-plastic model based on Von Mises Criterion was chosen, which includes isotropic hardening.

*Bond behaviour: a non linear plasticity-damage model based on a non-width finite element.*

An elastic-plastic-damage model for bonding [10] was adopted in the formulation due to its various advantages: a) thermodynamics formulation, written in stress-strain terms; b) capacity of coupling between: “cracking and frictional sliding”, and “tangential and normal stresses”; c) able to take account of confinement influence and lateral pressure; d) great stability for monotonic and cyclic loading. This model was already used in the prediction of the structural response of tie tests [2], and its robustness is supported by a 2D non-width interface element, which is fully described in [11].

### 3 NUMERICAL ANALYSIS OF THE BEAM-COLUMN CONNECTION

#### 3.1 Hypothesis, limitations and strategy of the proposed modelling

In order to make our simulations, we have chosen the finite element code FEAP v.7.4 [12], an open-source code with license in which is possible to implement user material models and user finite elements. The models of Mazars and bonding were specifically integrated into the code, but the last one is only available for a 2D formulation. For this reason and due to the limitations in memory capacity, we have decided to model in a 2D-space the beam-column connection, which may provide acceptable results –in comparison with 3D models- if some simplifications are made. For example, in a real RC structural element, the steel reinforcement forms a cage embedded into the concrete, inducing a particular concentration of stresses in the concrete around each bar; however, taking into account that bending is acting only in one plane, and assuming that the most important shear stresses might be developed in the same plane, it is possible to “homogenize” the steel reinforcement in layers for a 2D simulation. In the case of the stirrups, only the branches parallel to the bending plane are taken into account, modelled with one truss element whose transversal section corresponds to the total area of the stirrups. Because the concrete cannot develop large rotations, any possible geometrical non linearity was not considered into the model.

In which concerns to the adopted strategy, we followed the next steps:

- a) Selection of the experimental reference
- b) Definition of the cases to simulate:
  - only longitudinal steel without stirrups;
  - with minimal quantity of stirrups;
  - with the quantity of stirrups indicated in experimental test,
- c) Cases with bond material model (still in progress):
  - Bonding included only in the flexural steel
  - Bonding included in all of the reinforcement
- d) Comparison of results

#### 3.2 Construction of the model

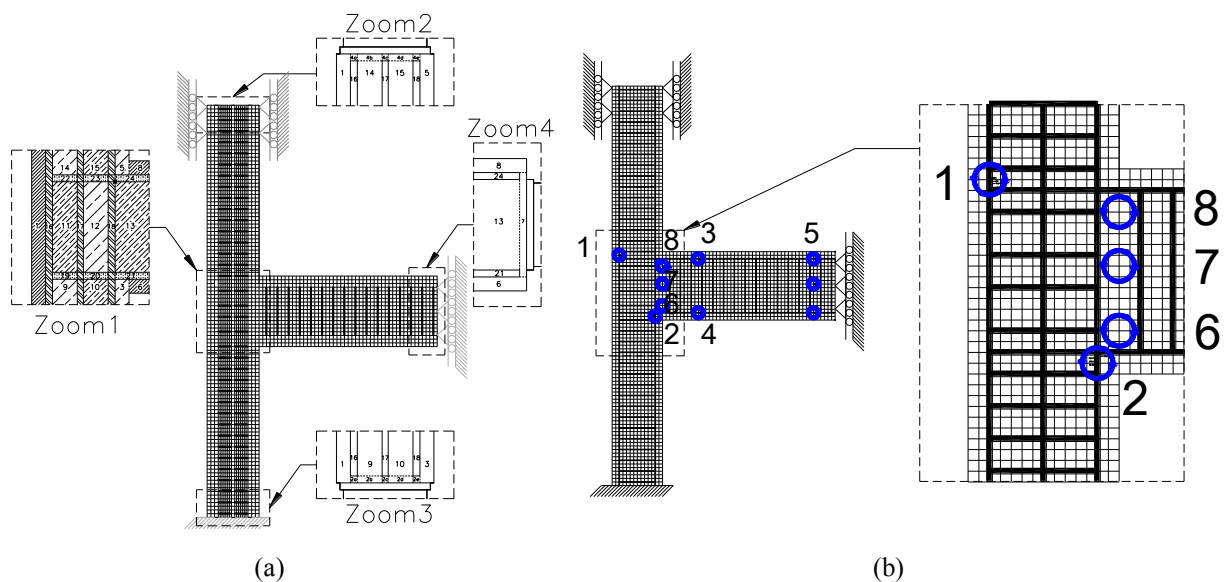
According to the proposed strategy, among the 12 corner-reinforced concrete beam-column subassemblies reported in the experimental reference, we selected the LL11 and LH11 specimens, having both of them the same geometrical and material properties, except for the number of stirrups inside the core of concrete (four stirrups for “Low confinement”, and six stirrups for “High confinement”).

The basic model was constructed in a 2D space based on a plane strain formulation, using QUAD4 elements (4-node quadrangular element with 4 integration points) for the concrete body and TRUSS2 elements (2-node bar element) for the steel reinforcement. Initially, the reinforcement was modelled with QUAD4 elements as well, but due to their minimal dimensions, there were some numerical problems by a non-realistic excessive concentration

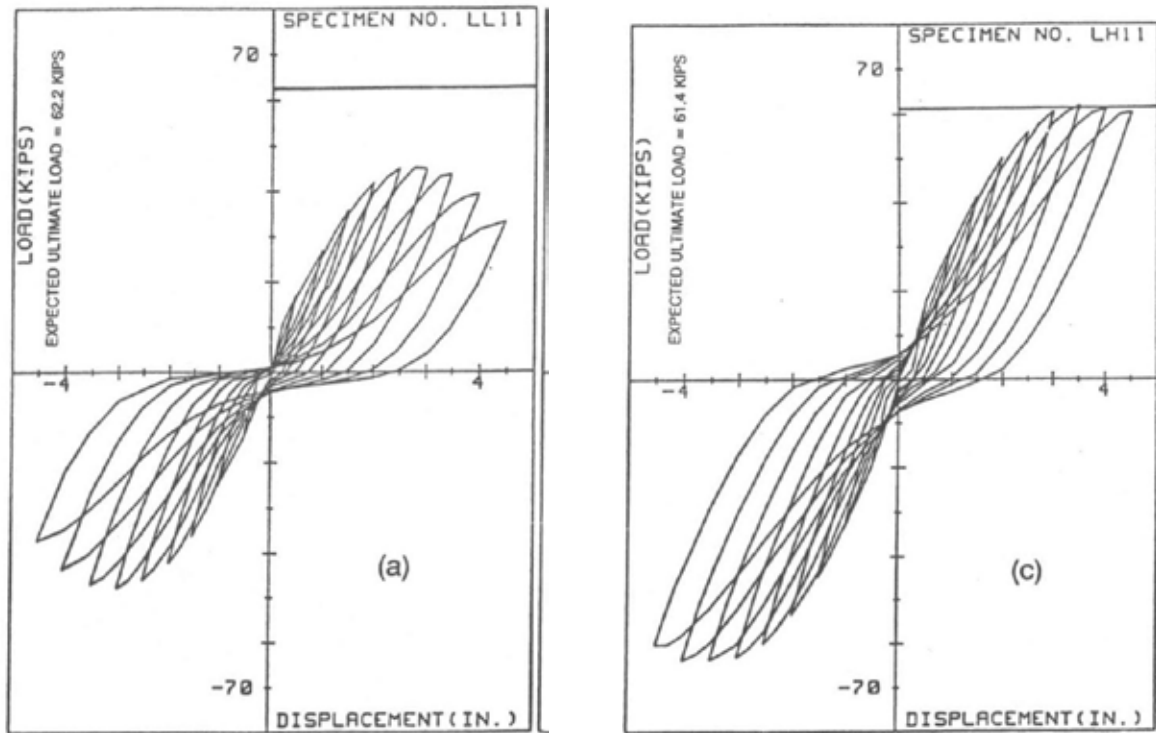
of stresses around the union between longitudinal steel and the stirrups. In which concerns to the bonding, two non-width interface elements were initially placed between the concrete QUAD4 element and the steel QUAD4 element –one on each side of the steel-, but due to the numerical instabilities mentioned previously, they were replaced by a unique interface element linking the steel rebar to the concrete elements: this numerical solution is still in progress.

About boundary conditions, bottom face of the column is fully-restrained, while the top face was constrained only in the transversal direction because a constant axial load was applied and distributed at the same face. By the way, the free edge of the beam is restrained in the axial direction, with a cyclic displacement imposed in its transversal direction (see figure 2-a).

In the experimental test, at least eight displacement transducers were positioned in order to follow the evolution of displacements over the concrete face of the joint (see figure 2-b). In the same way, we followed the numerical evolution of these points, in order to construct the corresponding load-displacement response.



**Figure 2:** Meshing of the beam-column connection: (a) boundary conditions and reinforcement array; (b) points of observation of the stress-strain relationship according to the experimental tests.

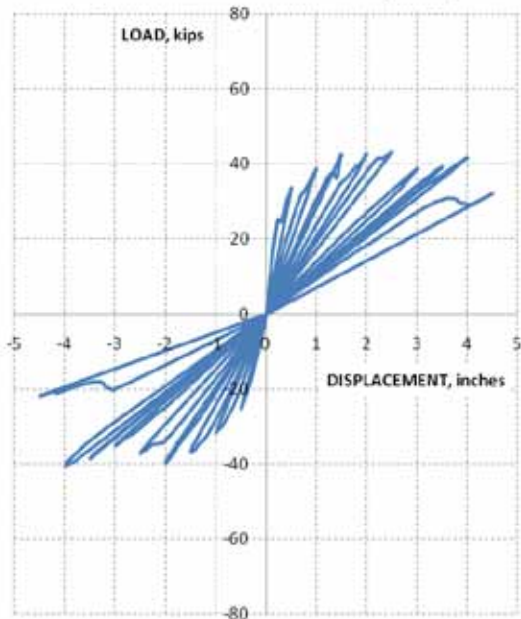


(a)

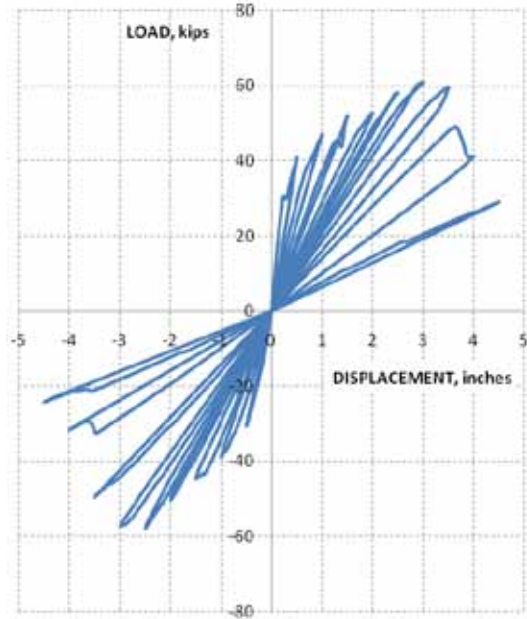
(b)

LOAD vs. DISPLACEMENT (LL11)

LOAD vs. DISPLACEMENT (LH11)



(c)



(d)

**Figure 3:** Load-displacement structural response of the beam-column connection: (a) experimental curve for specimen LL11; (b) experimental curve for specimen LH11; (c) numerical curve for specimen LL11; (d) numerical curve for specimen LH11.



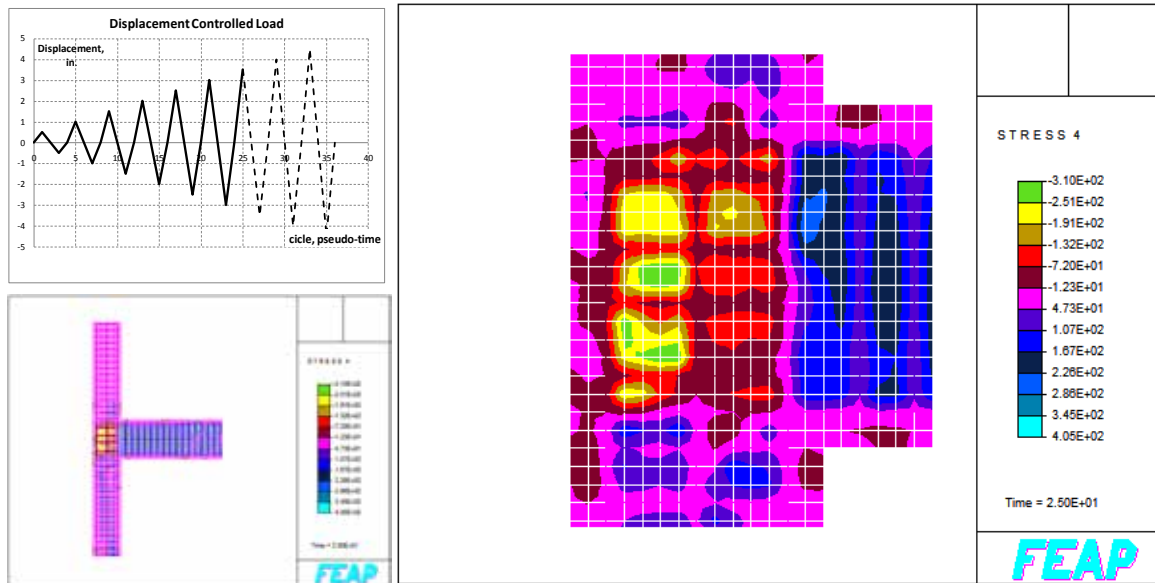


Figure 4: Shear stress distribution on specimen LL11 (low confinement) for a displacement of 3.5 inches.

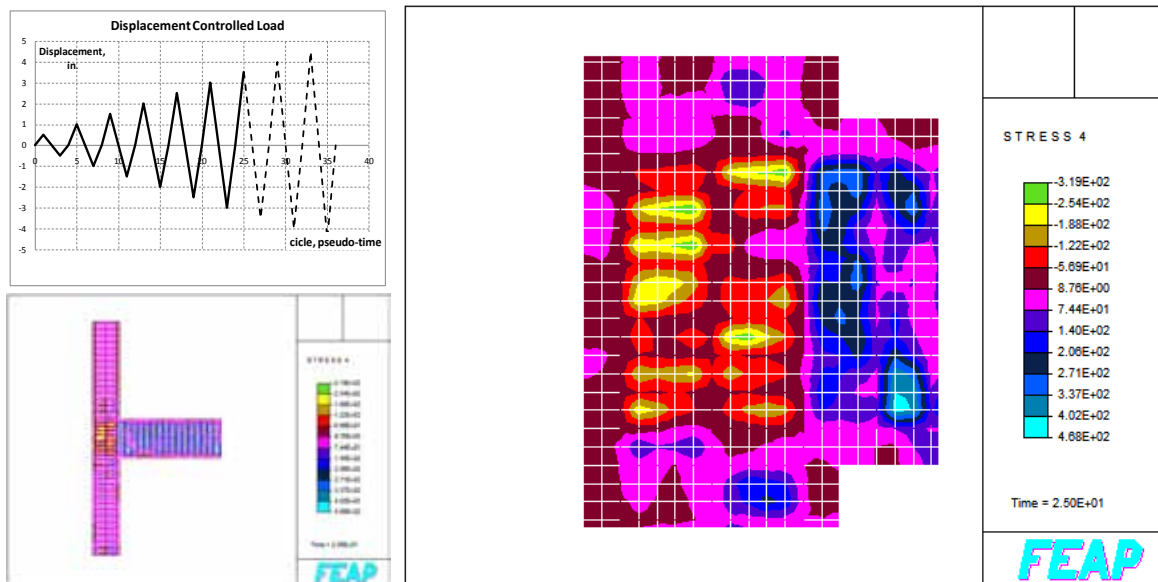


Figure 5: Shear stress distribution on specimen LH11 (high confinement) for a displacement of 3.5 inches.

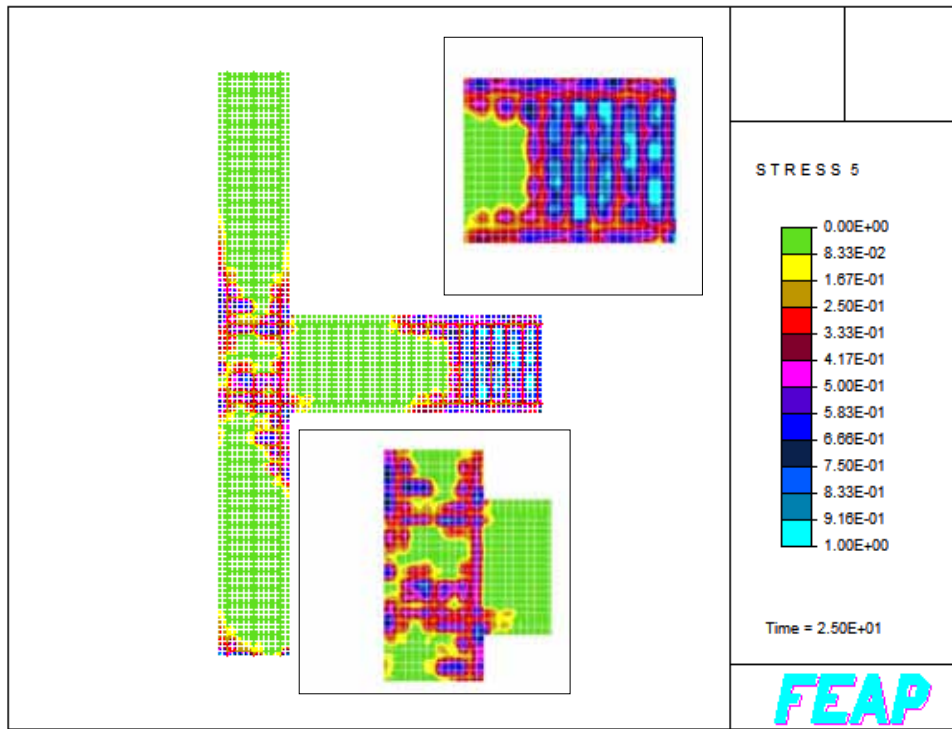


Figure 6: Damage distribution on specimen LL11 (low confinement) for a displacement of 3.5 inches.

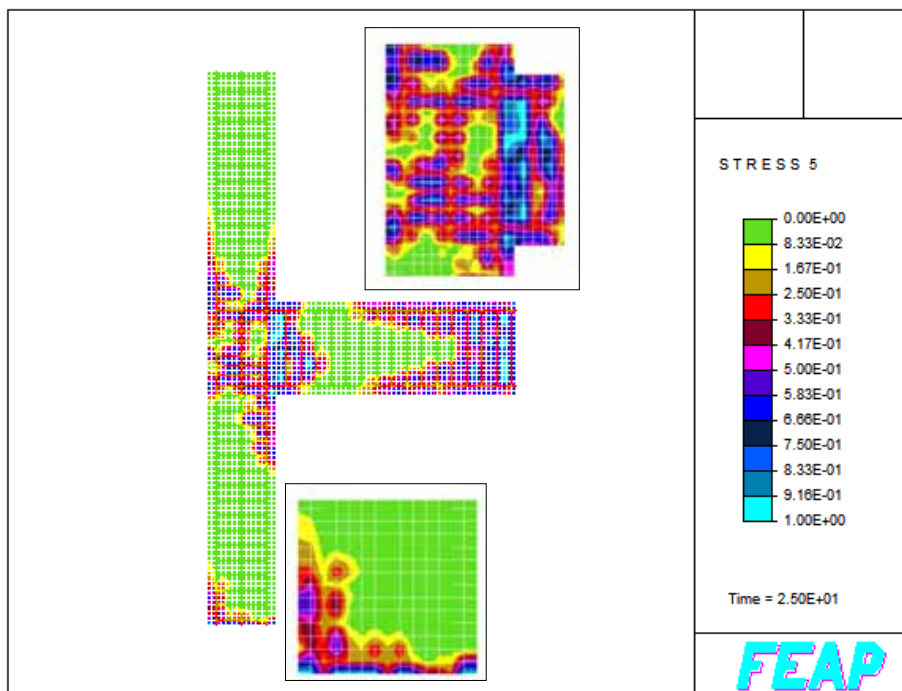


Figure 7: Damage distribution on specimen LH11 (high confinement) for a displacement of 3.5 inches.

#### 4 DISCUSSION OF RESULTS

In the next paragraphs, we will discuss the numerical results of the modelling coupling the nonlinear behaviour of steel and concrete. It should be mentioned that they do not include the bonding cases, which are still in progress. So on, first at all we compare the structural response of the two specimens, both experimental and numerical, in figure (3). Figures (3-a) and (3-c) correspond to the LL11 specimen, in which the maximal load capacity was reached between 40 and 45 kips for a displacement near to two inches. For the LH11 specimen, figures (3-b) and (3-d) show a maximal load capacity near to 60 kips, very close to three inches of displacement. By comparing experimental curves with numerical results, it can be appreciated that some key-values are very similar (maximal load capacity associated to the lateral displacement), but their shapes are far away from any similitude. In numerical curves, all the unloading branches go directly to the origin, without any accumulated permanent displacement as it is observed in the experiments. Typically, the origin of these permanent displacements is associated to the crack friction on concrete. For cyclic loads, the damage model of Mazars includes only the slope variation of the elastic unloading, since cracks on concrete are closed as soon as there is a reversibility of loading, assuming no friction on cracks. Because of this, it is not possible to reproduce numerically any dissipative boucle or permanent deformation. This was already explained by Ragueneau et al. [13], who presented a modified version of Mazars model which includes these effects.

Figures (4) and (5) show the distribution of shear stresses inside the specimens. In both of them, the concentration of stresses is determined by the disposition of the stirrups, being greater the affected area when the reinforcement is lower inside the core. In fact, when no stirrups are placed inside the core, the damage is reached almost immediately, even if the longitudinal bars of the column and beams pass through the joint. Other relevant points observed in numerical simulations are the following: (a) in both cases, the highest value of shear stress was reached on the beam, and not in the column or in the connection; (b) when the number of stirrups is increased inside the core, the principal damage is placed out of the core, exactly in the plane of connectivity between the beam and the core of the connection (see figure 7); and (c) if the constant axial load on the column is not included in the modelling, the resistance of the beam-column connection decreases substantially (according to [14]).

In general, all the simulations stopped as soon as a non convergence condition was reached. Sometimes this problem was solved by reducing the time step, in particular in the picks of the displacement when unloading started. From a physical point of view, this non convergence corresponds to the instant when a set of concrete elements reaches a high level of damage. Figures (6) and (7) show the level and distribution of damage in concrete for both specimens respectively. Apparently, damage is higher in LH11 specimen, but in reality is better distributed along the stirrups, although the numerical value seems to be elevated. The implementation of bond elements must reduce this effect on the concrete body, as it was demonstrated in [2], due to the redistribution of stresses induced by bonding, which allows a small slip between steel bars and concrete, avoiding a false premature degradation of concrete as it is observed in these simulations.

## 5 CONCLUSIONS

In this work, we have focused in modelling the non linear structural response of a beam-column connection subjected to cyclic loading. Thus, we have: (a) modelled different cases of beam-column connection and reproduced their experimental structural response, including different material models for steel and concrete (elasto-plastic and damage models respectively); (b) corroborated the influence of the stirrups in the resistance of the connection; and (c) analyzed the damage distribution inside the core of the connection.

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