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# A COMBINED FINITE-DISCRETE ELEMENT MODEL FOR REINFORCED CONCRETE UNDER SEISMIC LOAD

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**Abstract.** In this work a numerical model for analysis of reinforced concrete structures under seismic load is presented. The model uses the combined finite-discrete element method; thus taking into account the discontinuous nature of the reinforced concrete at the failure stages. The application of the combined finite-discrete element method includes a number of deformable discrete elements that interact with each other, fracture, fragmentation and disjoint during the seismic load. To these a robust model for reinforcement bars has been added. Interaction solutions between bars and concrete have also been developed and implemented into the open source Y2D combined finite-discrete element code. This way it is possible to describe initiation of the cracks, crack propagation and fracture which are important mechanisms in the analysis of reinforced concrete structures under seismic load. Through numerical examples these have been demonstrated and tested using reinforced concrete structure under an experimentally recorded earthquake accelelogram.

## 1 INTRODUCTION

The development of the numerical models for simulation of the response of reinforced concrete structures under seismic load, taking into consideration non-linear behaviour of the concrete and reinforcement, makes possible the results of high accuracy almost equivalent to the results obtained by expensive laboratory experiments. One of the main causes of concrete non-linear behaviour is cracking. A reliable model for simulation of the opening and closing the cracks is especially important in the structures under seismic load because a significant part of seismic energy is lost in cracking. The most of the models described in literature is based on finite element method where the cracking is described with smeared crack models or with embedded models where the cracks are modelled as discontinuity within the elements by enriching interpolation function. In contrast to the finite elements, in discrete models cracks are simulated as discontinuities of displacement between two elements.

A numerical model for analysis of reinforced concrete structures under seismic load developed in this work is based on combined finite-discrete element method [1].

Transition from continua to discontinua in the combined finite discrete element method occurs through fracture and fragmentation processes. A typical combined finite discrete element method based simulation may start with a few discrete elements and finished with very large number of discrete elements. Fracture occurs through alteration, damage, yielding or failure of microstructural elements of the material.

There has been a number of fracture models proposed in the context of both discrete element methods and combined finite discrete element method. Some of the models are based on a global approach applied to each individual body, while others used a local smeared crack approach or local single-crack approach. In this work a model for plane crack initiation and crack propagation in concrete is used [2]. The model combines standard finite element formulation for the hardening part of the constitutive law with the single-crack model for the softening part of stress-strain curve. Finite elements are used to model behaviour of the material up to the ultimate tensile strength while a discrete crack model is used for modelling of the crack opening and separation along edges of finite elements.

In this work an embedded model of reinforcing bars [3,4] is implemented in Y2D combined finite-discrete element code. The concrete structure is discretized on triangular finite elements, while the reinforcing bars are modelled with linear one-dimensional elements which can be placed in arbitrary position inside the concrete finite elements. The main assumption of the model is that the concrete tensile strength will be exceeded before the yield stress will achieved in steel. The behaviour of the structure is linear-elastic up to the opening of the crack. After that joint elements in concrete as well as in reinforcing bar are occurred. The concrete and reinforcing bars are analyzed separately, but they are connected by the relation between the size of the concrete crack and strain of the reinforcing bar [5,6]. Cyclic behaviour of the steel during the seismic load is modelled with improved Kato's model [7].

The formulation which has allowed the introducing of the seismic load modelled by an earthquake accelelogram is developed in this model.

## 2 MODELLING OF THE REINFORCED CONCRETE STRUCTURE

In this work an embedded model of reinforcing bars [3,4] is implemented in Y2D code based on combined finite-discrete element method. The concrete structure is discretized on triangular finite elements, while the reinforcing bars are modelled with linear one-dimensional elements which can be placed in arbitrary position inside the concrete finite elements. The model of the reinforced concrete structure with the embedded reinforcing bar is shown in Figure 1.

In the first step the reinforcement is modelled as bar AB (Figure 1.). Intersection between the sides of concrete finite elements and reinforcing bars gives the reinforcement finite elements. The behaviour of the structure is linear-elastic up to the opening of the crack. In that phase the triangular concrete element and line element of reinforcing bar behave as one body. The deformation of the triangular element influence to the deformation of the reinforcing bars. When the crack in concrete appears, joint element in concrete as well as joint element in reinforcing bars is occurred.

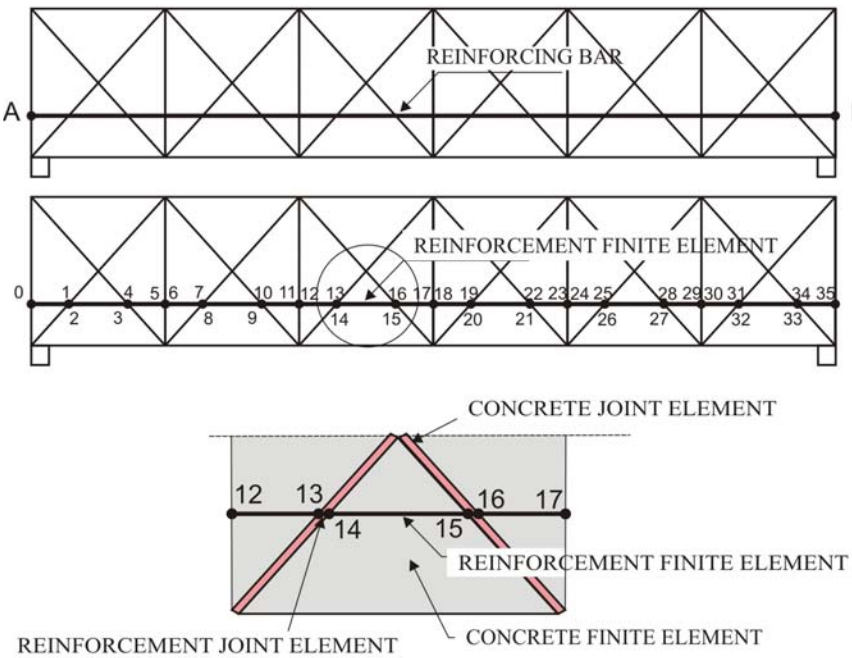



**Figure 1**: Discretization of reinforced concrete structure

## 3 NON-LINEAR MATERIAL MODEL IN JOINT ELEMENTS

### 3.1 Basic crack model

A model of discrete crack in reinforced concrete element with joint elements of concrete and reinforcing bar described in previous section is shown in Figure 2.

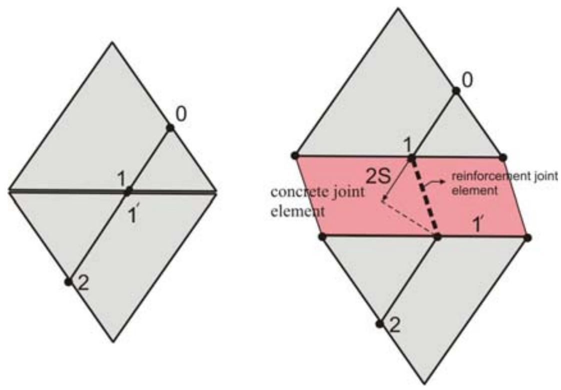



**Figure 2**: Discrete crack: crack initiation and crack development in concrete

### 3.2 Concrete model in joint element

In this work the combined finite-discrete element method uses the concrete model [2] which is based at crack initiation and crack propagation of mode I loaded cracks. The model is developed on the basis of experimental stress-strain curves for concrete in tension [8].

The area under the stress-strain curve consists of two parts (Figure 3). Part ″A″ is used for modelling of the concrete behaviour up to the crack opening while part ″B″ represents strain softening after the tensile strength is exceeded. The assumption of the discrete crack model is that the cracks coincide with the finite element edges. The total number of nodes for each of the finite element meshes is doubled and the continuity between elements is realized through the penalty method. Separation of the edges induces a bonding stress, which is a function of the size separation δ (Figure 3.).

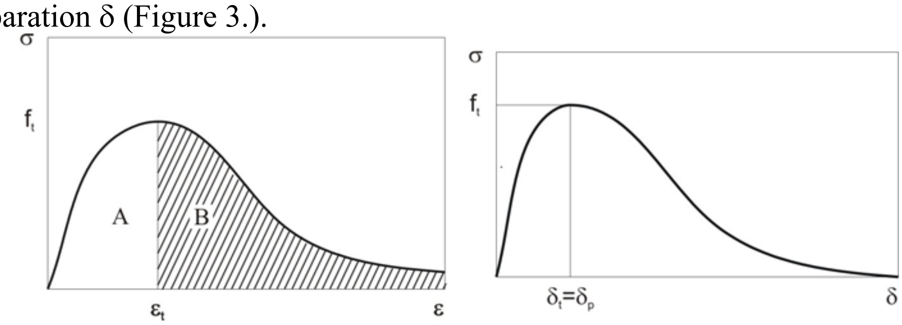



**Figure 3**: Strain softening stress-strain and stress-displacement curves

No separation of the adjacent elements occurs before the tensile strength is reached, i.e. $\delta_t = 0$. The edges of two adjacent elements are held together by normal and shear springs (Figure 4.) Procedure of the separation of the elements and complete relationship for the normal and shear bonding stress are given in [2].

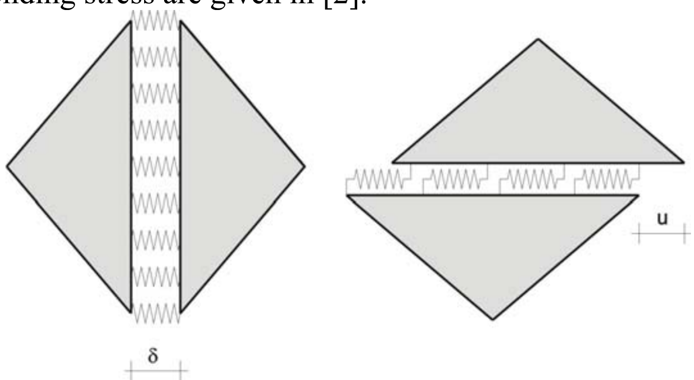



**Figure 4:** Normal and shear springs between the finite elements

### 3.3 Steel material model in reinforcement joint element

The main assumption of the model is that the concrete tensile strength will be exceeded before the yield stress will achieved in steel. The behaviour of the structure is linear-elastic up to the opening of the crack. After that joint elements in concrete as well as in reinforcing bar are occurred. The concrete and reinforcing bars are analyzed separately, but they are connected by the relationship between the size of the concrete crack and strain of the reinforcing bar.

In this work a model of the relationship between the concrete crack size and strain of the reinforcing bar developed by Shima [5] and Shin [6] is applied. The model is based on experimental strain-slip curves and represents well approximation of the behaviour of reinforcing bar with the expressed plastic strain caused by cyclic loading.

The steel strain-slip relation before the yielding of reinforcing bar is given by expressions:

$$s = \varepsilon_s (6 + 3500\varepsilon_s) \tag{1}$$

$$s = \left(\frac{S}{D}\right) \cdot K_{fc}, \quad K_{fc} = \left(\frac{f_c'}{20}\right)^{2/3} \tag{2}$$

where $s = s(\varepsilon_s)$ is normalized steel slip, D is bar diameter and $f_c'$ is concrete strength.

Normalized slip in the post-yield range is given by expression:

$$s = s_{pl} + s_e . \tag{3}$$

where $s_e$ is slip in the elastic region and $s_{pl}$ is slip in the yield region. A strain distribution along the reinforcing bar in the post-yield region is shown in Figure 5 where $\varepsilon_{se}$ is a strain at the yield boundary point on the elastic region and $\varepsilon_{sp}$ is a strain at the yield boundary point on the yield region.

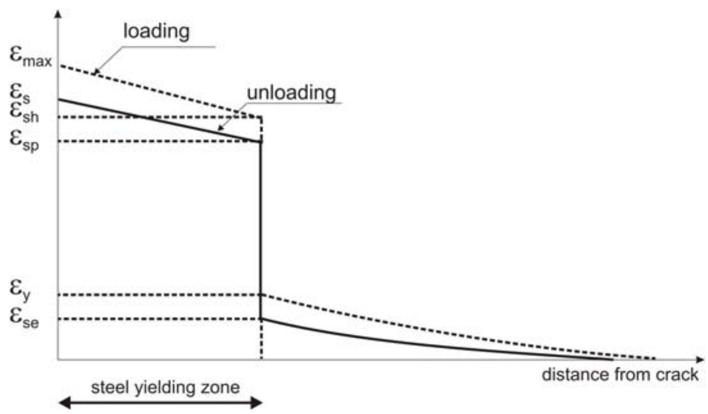


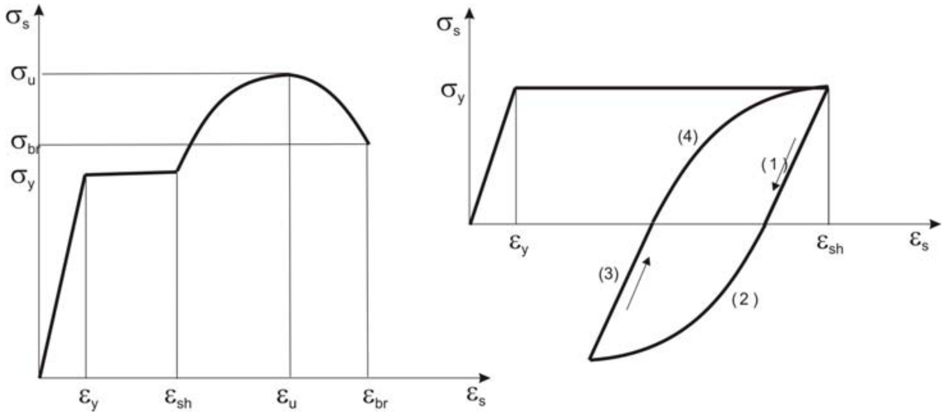

**Figure 5:** Strain distribution along the reinforcing bar in the post-yield range

If the deformation of concrete is ignored, the steel slip can be determined by integration of steel strain along the reinforcement axis. Normalized plastic steel slip in the yield region is given by an expression:

$$s_{pl} = \frac{(1+\beta)\varepsilon_s + \varepsilon_{sh} - \beta\varepsilon_{\max}}{\varepsilon_{\max} + \varepsilon_{sh}} \cdot (s_{\max} - s_y^*) \tag{4}$$

where $\beta = \sigma_{max} / \sigma_y$ represents the gradient of the line shown in Figure 5., $\sigma_{max}$ is the maximum stress in reinforcing bar under tensile loads, $s_{max}$ is a function of $\varepsilon_{max}$ and $s_y^* = \varepsilon_s (2 + 3500\varepsilon_s)$.

In this paper non-linear material model for steel is based on experimental stress-strain curve and it is shown on Figure 6a. Cyclic behaviour of the steel during the seismic load is modelled with improved Kato's model [7] (Figure 6.b).




a) Stress-strain relation for steel b) Cyclic behaviour of steel

**Figure 6**: Material model for steel

# 4 EXAMPLES

## 4.1 Reinforced concrete beam

Reinforced concrete beam (Figure 7.) subjected to self weight is analyzed in order to verified implemented model of the reinforcing bars in static conditions. The span of the beam is 6.0 (m). Analysis is performed with two discretization presented in Figure 7. The midspan deflection, the stress in the concrete and the stress in the reinforcing bar in the marked point are analyzed. The results obtained by numerical model are compared with the analytical results in the Table 1.

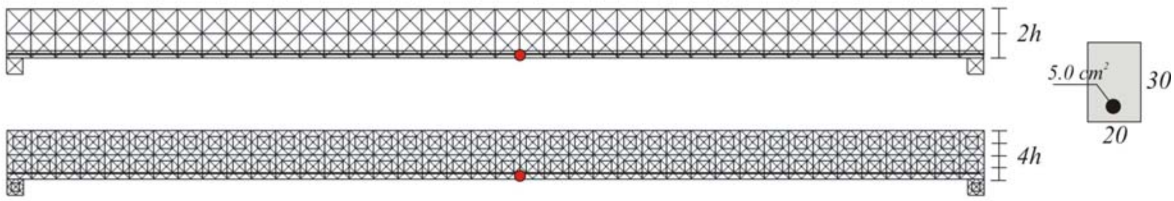



**Figure 7**: Reinforced concrete beam

**Table** 1: Comparison of numerical solutions with analytical ones

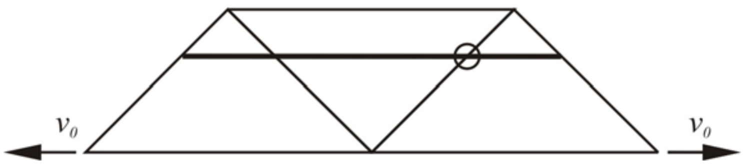
| | Analytical | Numerical - 2h | Numerical - 4h |
|---|---|---|---|
| Midspan deflection (mm) | 1.351 | 1.15 | 1.350 |
| Concrete stress (N/m$^2$) | $1.65\cdot10^6$ | $1.40\cdot10^6$ | $1.65\cdot10^6$ |
| Stress in reinforcing bar (N/m$^2$) | $11.60\cdot10^6$ | $9.89\cdot10^6$ | $11.56\cdot10^6$ |

Presented results show that the numerical solutions is very close to the analytical ones for the discretization 4h.

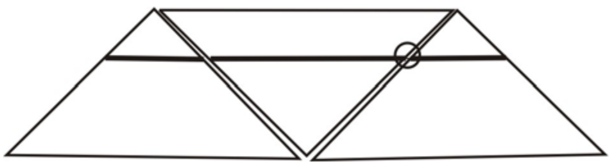
## 4.2 Analysis of steel behaviour in joint element of reinforcing bar

A simple example of reinforced concrete structure consists of 3 triangular finite elements with a reinforcing bar is analyzed. The length of triangular finite element side is 1.4 (m) and the cross-section area of reinforcing bar is 0.05 (m$^2$). Compressive concrete strength is 30 (MPa) while the characteristics of the steel are $\sigma_y$=275·10$^6$(N/m$^2$), $\sigma_u$=430·10$^6$(N/m$^2$), $\varepsilon_{sh}$=0.005, $\varepsilon_u$=0.01. The structure is subjected to initial velocity of 80 (m/s). The applied time interval is $\Delta t$=0.3·10$^{-5}$s.

Figure 8 shows initial stage of the structure and stage of the structure after the cracking of reinforcing bar. Stress-strain relation in joint element of reinforcing bar, which is presented in Figure 9, shows cyclic behaviour of the steel.




a) Initial stage



b) Stage after the cracking of reinforcing bar

**Figure 8**: Reinforced-concrete element subjected to initial velocity

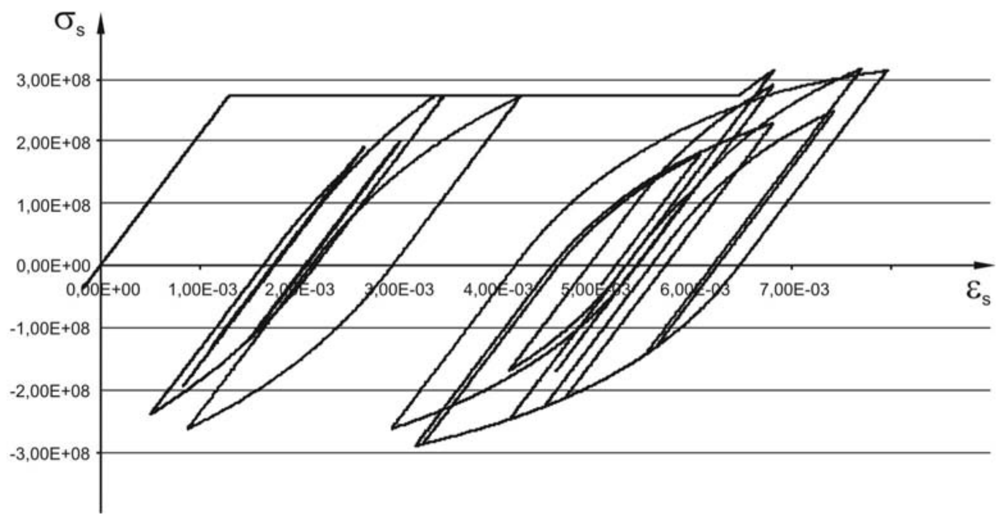



**Figure 9**: Stress-strain relation in reinforcing bar

## 5 CONCLUSIONS

- This paper presents a new numerical model for analysis of reinforced concrete structures under seismic load based on combined finite-discrete element method. The model combines standard finite element formulation for the hardening part of the constitutive law with the single-crack model for the softening part of stress-strain curve.
- An embedded model of reinforcing bars is implemented. The concrete structure is discretized on triangular finite elements, while the reinforcing bars are modelled with linear one-dimensional elements which can be placed in arbitrary position inside the concrete finite elements.
- The main assumption of the model is that the concrete tensile strength will be exceeded before the yield stress will achieved in the steel. The behaviour of the structure is linear-elastic up to the opening of the crack. After that joint elements in concrete as well as in reinforcing bar are occurred. Interaction solutions between bars and concrete have also been developed and implemented into the open source Y2D combined finite-discrete element code. Cyclic behaviour of the steel during the seismic load is modelled with improved Kato's model.

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