Wakefield acceleration in planetary atmospheres: a possible source of MeV electrons. The collisionless case.

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Abstract

Intense electromagnetic pulses interacting with a plasma can create a wake of plasma oscillations. Electrons trapped in such oscillations can be accelerated under certain conditions to very high energies. We study the optimal conditions for the wakefield acceleration to produce MeV electrons in planetary plasmas under collisionless conditions. We apply the results to the Earth atmosphere identifying possible sources of waves and plasmas where the conditions could be fulfilled in order to produce MeV electrons. Wakefield acceleration might play a role in the triggering of Terrestrial Gamma ray Flashes (TGF's).

Keywords: Wakefield acceleration, MeV electrons, TGF's

1. Introduction

Electromagnetic pulses interacting with a plasma can create a wake of plasma oscillations through the action of the nonlinear ponderomotive force [1]. Electrons trapped in the wake can be accelerated to high energies. In the laboratory very high-power electromagnetic radiation from lasers is used to accelerate electrons to high energies in a short distance. However, in atmospheric plasmas the distance range goes from metres to kilometres, and the conditions for the

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wakefield acceleration takes place are different as the plasma characteristics are different. We will focus in this work on the conditions to have a collisionless behaviour when considering the interaction of an electromagnetic wave with a plasma.

The acceleration of the electrons in atmospheres has been the subject of attention since the discovered of Terrestrial Gamma Flashes (TGF's). TGF's were first discovered by accident in 1994 [2] and only limited experimental data is available [3]. One of the goals of the European Space Agency (ESA) mission ASIM [4] will be to provide experimental data on TGF's. Any theory about the TGF generation should be able to successfully explain the observed duration, energy spectra, and photon fluence [5]. There is some agreement that the emission is produced via Bremsstrahlung [6] when high energy electrons collide with nuclei in the air releasing energy. The satisfactory theory for TGF's needs to explain the origin of the high-energy electrons [7] which can trigger the ignition of the gamma flashes [8]. At present there are two realistic TGF's mechanism. One is called the Relativistic Runaway Electron Avalanche (RREA) feedback mechanism, and for a detailed description we point to the reference [9]. The other is the leader/streamer theory [10], in which seed electrons are produced in the streamer head and accelerated in the stepped leader electric field (there is the current pulse version of that [11]).

The wakefield phenomena could also supply the initial seed of energetic electrons, coming from the interaction of a electromagnetic pulse with an existing plasma previously created. Although the idea of production of high energy electrons by an electromagnetic pulse due to a lightning return stroke was explored previously [12], the novelty of our proposal is the interaction of the pulse with a plasma already present in the atmosphere and created for example by another discharge or any electromagnetic activity.

In order to explore the idea, we assume that the acceleration takes place within the plasma under collisionless conditions, and thus the friction force [13] due to collisions with neutral molecules, which is the main braking force in other theories [14] is not considered. That is valid provided that some conditions for the pulse and plasma are fulfilled. We establish those conditions in the present study. Whether the inclusion of electron-neutral collisions is necessary, the conditions should be modified. That would be the subject of future work.

The outline of the paper is the following. First we summarize the optimal conditions under which an electromagnetic wave can create a wakefield in a plasma, so acceleration of electrons to MeV energies is possible. The acceleration mechanism is demonstrated through computer simulations and we establish a criteria for this to work. We use a unipolar and bipolar form of the electromagnetic pulse as found in lightning for example [15]. We present a scenario for the production of the plasma and the electromagnetic pulse in planetary atmospheres, and in particular for the Earth atmosphere we include some discussion. We make some rough estimations for the plasma conditions in negative step leaders. Finally we summarise the main results and end with some conclusions.

2. Optimal conditions for wakefield acceleration

When an electromagnetic wave encounters a plasma it can create a wake inside the plasma through the action of a nonlinear ponderomotive force [1]. The following is a set of optimal conditions (derived from a single-particle approximation discussed in detail in the Appendix A), required to accelerate electrons to MeV energies.

First we need a plasma region with the following characteristics,

- The numbers of particles in the Debye sphere must be large, $N_D \gg 1$. For electrons, $N_D = 4\pi n_e \lambda_{De}^3/3$, where n_e is the electron density and λ_{De} the electron Debye length. An useful expression of the Debye length under equilibrium conditions is given by $\lambda_{De} = 69\sqrt{T/n_e}\,(\mathrm{m})$, where T is the equilibrium temperature of the electrons in Kelvin and n_e is expressed in m^{-3} [16], so we have

$$N_D \approx 1.38 \times 10^6\, T^{3/2}/n_e^{1/2} \gg 1. \quad (1)$$

- The Debye length must be much smaller than the characteristic size of the plasma region,

$$\lambda_{De} \ll L. \tag{2}$$

  We recall that the condition to have a plasma is λ_{De} being small than the characteristic dimension of the system.

Now the coupling conditions for the electromagnetic wave and the plasma,

- The frequency of electron and ions collisions must be smaller than the electromagnetic pulse angular frequency,

$$\nu_{ei} \ll \omega_0. \tag{3}$$

- The damping rate also must be negligible,

$$\nu = \frac{\omega_{pe}^2}{\omega_0^2}\nu_{ei} \ll \omega_{pe}, \tag{4}$$

  where $\omega_{pe} = e\sqrt{n_e/(m_e\epsilon_0)}$. The physical meaning of this condition comes as ν represents the rate of energy lost from the electromagnetic wave, i.e, $\nu(E_0^2\epsilon_0)/2$. It must be balanced by the rate at which the oscillatory energy of the electrons $n_e e^2 E_0^2/2m_e$ is dissipated by electron-ion scattering with frequency ν_{ei}. If we assume a wave package with group velocity v_g, the energy of the wave will decay within a characteristic length of $l = v_g/\nu$, and we should expect that to be bigger or at least the order of the Debye length, $\lambda_{De} \leq l$.

- For the acceleration of electrons to MeV energies, we need the further condition derived from our simulations,

$$\tilde{\omega} = \frac{m_e c\omega_0}{eE_0} < 1. \tag{5}$$

  We will discuss further this condition in the next section.

3. The acceleration of electrons in atmospheric plasmas

In this section we will study how electrons in a plasma are accelerated by an electromagnetic pulse propagating through the plasma under the collisionless conditions. We will first set the equations to describe the electron dynamics and then compare their predictions with some particle in cell code simulations to verify the results. We will use in particular two kind of pulses, a unipolar and a bipolar one, as they are a good approximation for the pulses that one can find for example in the electromagnetic activity of the Earth atmosphere [17, 18].

Let us consider an electromagnetic pulse which is propagating in the x-direction inside a plasma,

$$\mathbf{E}(\mathbf{r},t) = \int dk A(k) e^{i(kx-\omega t)} \hat{\mathbf{e}}_y + \text{c.c.}\,, \tag{6}$$

where $A(-k) = A(k)^*$ and $A(k)$ for $k > 0$ is nonzero only in the vicinity of a central wave number k_0. From the relation (A.14) we take for the pulse a lead frequency $\omega_0 = (w_{pe}^2 + c^2 k_0^2)^{1/2}$. Expanding around k_0 yields the following expressions for the electromagnetic field,

$$\begin{aligned} \mathbf{E}(\mathbf{r},t) &= E_0\, f(x - v_g t) \cos[(\omega_0 - v_g k_0)t]\, \hat{\mathbf{e}}_y, \\ \mathbf{B}(\mathbf{r},t) &= \frac{v_g}{c^2} E_0\, f(x - v_g t) \cos[(\omega_0 - v_g k_0)t]\, \hat{\mathbf{e}}_z, \end{aligned} \tag{7}$$

where

$$v_g = \left(\frac{\partial \omega}{\partial k}\right)_{k=k_0} = \frac{c^2 k_0}{\omega_0} = c\,\varepsilon^{1/2} \tag{8}$$

is the propagation speed inside the plasma, ε is the plasma dielectric function (A.11), and $f(x) = E_y(\mathbf{r},0)/E_0$ gives the shape of the pulse, E_0 being the amplitude of the electric field.

As discussed in the previous section, the condition $\omega_0 \geq \omega_{pe}$ is required for the propagation of the electromagnetic pulse inside the plasma. In the following we will consider $\omega_0 \geq 10\,\omega_{pe}$. For $\omega_0 = 10\,\omega_{pe}$, v_g differs from the speed of light in vacuum in less than 1%. For larger values of ω_0, the pulse propagation speed is even closer to c. The electromagnetic pulse will start interacting with the electrons at $t = 0$. The interaction will end at a time of the order of $2\pi/\omega_0$. Since $\omega_0 - v_g k_0 = \omega_{pe}^2/\omega_0$, the cosine term in (7), $\cos[(\omega_0 - v_g k_0)t] = \cos[(\omega_{pe}/\omega_0)^2 \omega_0 t] \approx 1$ can be safely neglected throughout the process. We will also assume that the damping rate (A.15) of the pulse is negligible. Those assumptions will impose some constrains that we will discuss at the end of this section. With these approximations, the electromagnetic field can be written as

$$\mathbf{E}(\mathbf{r},t) = E_0\, f(x-ct)\hat{\mathbf{e}}_y, \tag{9}$$
$$\mathbf{B}(\mathbf{r},t) = \frac{E_0}{c} f(x-ct)\hat{\mathbf{e}}_z. \tag{10}$$

Since the pulse frequency is larger than the plasma frequency, and the plasma frequency bigger than the collision frequency, the dynamics of the electrons will be collisionless while interacting with the pulse, so can be modelled by [19]

$$\frac{d\mathbf{v}}{dt} = -\frac{e}{m_e}\left(1-\frac{v^2}{c^2}\right)^{1/2}\left[\mathbf{E}+\mathbf{v}\times\mathbf{B}-\frac{\mathbf{v}(\mathbf{v}\cdot\mathbf{E})}{c^2}\right]. \tag{11}$$

We define dimensionless variables $\tilde{t} = teE_0/(m_e c)$ and $\tilde{\mathbf{r}} = \mathbf{r}eE_0/(m_e c^2)$, so that the scaled velocity is just $\tilde{\mathbf{v}} = \mathbf{v}/c$. Using (9) and (10), the equation of motion (11) becomes

$$\begin{aligned}\frac{d\tilde{v}_x}{d\tilde{t}} &= -\sqrt{1-\tilde{v}^2}(\tilde{v}_y - \tilde{v}_x\tilde{v}_y)\,\tilde{f}[\tilde{\omega}(\tilde{x}-\tilde{t})],\\ \frac{d\tilde{v}_y}{d\tilde{t}} &= -\sqrt{1-\tilde{v}^2}(1-\tilde{v}_y^2-\tilde{v}_x)\,\tilde{f}[\tilde{\omega}(\tilde{x}-\tilde{t})],\\ \frac{d\tilde{v}_z}{d\tilde{t}} &= -\sqrt{1-\tilde{v}^2}(-\tilde{v}_z\tilde{v}_y)\,\tilde{f}[\tilde{\omega}(\tilde{x}-\tilde{t})],\end{aligned} \tag{12}$$

where $\tilde{f}(k_0 x) = f(x)$ takes into account the assumed form of the wave-packet, and

$$\tilde{\omega} = \frac{m_e c\,\omega_0}{eE_0}. \tag{13}$$

We are interested in pulses which are able to accelerate electrons to kinetic energies $K = m_e c^2[(1-\tilde{v}^2)^{-1/2}-1]$ in the order of several MeV after the interaction time, which is $\tilde{T} \approx 2\pi/\tilde{\omega}$. Therefore, in this model the interaction with the pulse is controlled by the dimensionless parameter $\tilde{\omega}$. Pulses with different amplitudes and frequencies but same ratio ω_0/E_0 (and shape) produce the same acceleration.

Figure 1 shows the relativistic kinetic energy obtained for a free electron initially at rest at the origin accelerated by a pulse with the shape of the form

$$\tilde{f}(\tilde{\omega}x) = \cos[\tilde{\omega}(x-\tilde{T}/4)][H(x+\tilde{T}/2)-H(x)], \tag{14}$$

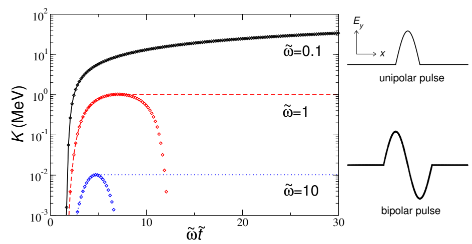

Figure 1: Kinetic energy vs time of an electron accelerated by a unipolar (dotted lines) and a bipolar (crosses) EM pulse, for several $\tilde{\omega}$. The pulses, depicted in the right side, are assumed to be translation invariant in the directions perpendicular to the direction of propagation (along the x-axis).

which is unipolar (see right of Fig. 1), and also for the bipolar shape

$$\tilde{f}(\tilde{\omega}x) = \cos[\tilde{\omega}(x - \tilde{T}/4)][H(x+\tilde{T}) - H(x)], \tag{15}$$

where $H(x)$ is the Heaviside step function. The results were obtained by solving numerically (12). The simulations show that for values $\tilde{\omega} \lesssim 0.1$, regardless whether the shape is unipolar or bipolar, the kinetic energy keeps growing after five periods T_0, which is an indication that the electron is actually trapped by the pulse, soon achieving very high energies. For $\tilde{\omega} \approx 1$, the electromagnetic pulse is not so strong as to carry away the electron. It is still able to produce accelerations to energies in the MeV scale, though for the bipolar pulse the electron acceleration is later reversed by the own electromagnetic packet. For larger values of the reduced frequency, $\tilde{\omega} \gtrsim 10$, the impulse produced by the pulse is well below the MeV range.

These results were numerically verified using the Finite Difference Time Domain Particle in Cell software VORPAL [20]. Figure 2 shows the kinetic energy profile after a time interval of 1.44×10^{-7} s, for a plasma of density $n_0 = 10^{10}\,\mathrm{m}^{-3}$ with a unipolar pulse defined by the boundary condition $\mathbf{E}(0, y, z, t) = E_0 \sin(\omega_0 t)\, H(\pi/\omega_0 - t)\, \hat{\mathbf{e}}_y$, where H is the Heaviside step function. For $E_0 = 10^5$ V/m and $\tilde{\omega} = 1.071$, as predicted, the pulse is able to produce electrons just in the MeV scale, whereas the pulse with $E_0 = 10^6$ V/m yields $\tilde{\omega} = 0.1071$, able to accelerate free electrons to very large energies. Note that for the case of the

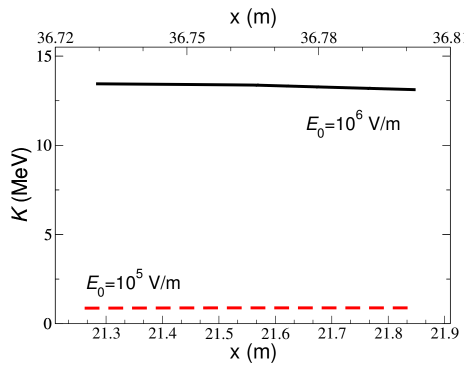

Figure 2: Energy of electrons in a plasma of density $n_0 = 10^{10}\,\mathrm{m}^{-3}$ after an acceleration time interval of 1.44×10^{-7} s due to an electromagnetic unipolar pulse of angular frequency $\omega_0 = 2\pi \times 10^7$ rad/s. The continuous line corresponds to a pulse of amplitude $E_0 = 10^6$ V/m and its abscissa is the upper one. The dashed line correspond to $E_0 = 10^5$ V/m, being its abscissa the lower one. The plasma frequency is $\omega_{pe} = 5.6 \times 10^6$ rad/s.

higher electric field amplitude, the acceleration occurs at larger distances.

In Fig. 3, we have plotted the threshold conditions for which the wakefield is able to accelerate electrons to MeV energies, based on the requirements imposed by the condition (5). A yellow a band around 50-60 MHz, which corresponds to the typical emitted frequencies in negative leaders, has also been plotted in Fig. 3 for reference. A range of pulse intensities where $\tilde{\omega}$ is less than unity can be observed, so in principle the acceleration to MeV energies is possible under these conditions. It can be observed that as the peak amplitude increases from the lower value 5×10^5 V/m, the parameter $\tilde{\omega}$ decreases, so the acceleration is more effective.

Still, the rest of conditions (1)–(4) must be fulfilled. For instance, choosing —as in the above simulations— $\omega_{pe} = \omega_0/10$, the condition (4) is satisfied provided that the electron-ion collision rate is not larger than a magnitude of 100. This imposes some restrictions for the plasma in which the EM pulse propagates. The plasma density is related to the plasma frequency so for the $50-60$ MHz band, the plasma density should be of the order 10^{10} m^{-3}. If we assume a plasma temperature of a few eV, and the mean atomic number of the

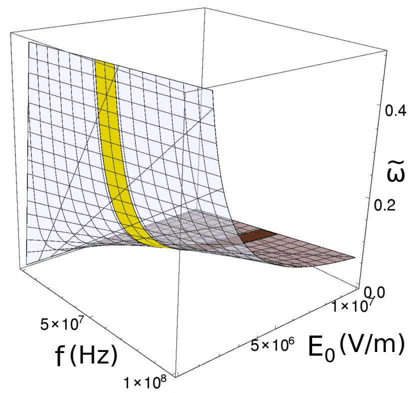

Figure 3: The phase diagram for wakefield conditions able to accelerate electrons to MeV energies. The yellow a band around 50-60 MHz indicates the typical emitted frequencies in negative leaders.

air $Z = 7.2$ (Earth atmosphere), it can be easily checked that the conditions (1) and (2) are satisfied. With those values, the characteristic plasma length λ_{De} is tens of centimeters, and ν_{ei} is less than 1 rad/s, which is much less than ω_0, in agreement with (3).

4. A proposed scenario for MeV electron production

We have shown in previous section how an electromagnetic pulse, interacting with an already created plasma, is able to accelerate electrons to high MeV energies provided some conditions are fulfilled. In a laboratory, the electromagnetic pulse is provided by a laser source, and the plasma is created ionizating a gas at certain pressure, either using microwaves or any other technique. In a planetary atmosphere, the pulse must be created by electromagnetic activity, such as lightning, leader strokes, etc. The plasma also must be present, and the again some electromagnetic activity is needed for the creation of the plasma.

Candidates for the creation of the pulse are for example lightning discharges cloud to cloud, or cloud to earth [17, 18]. When the pulse is created, it propa-

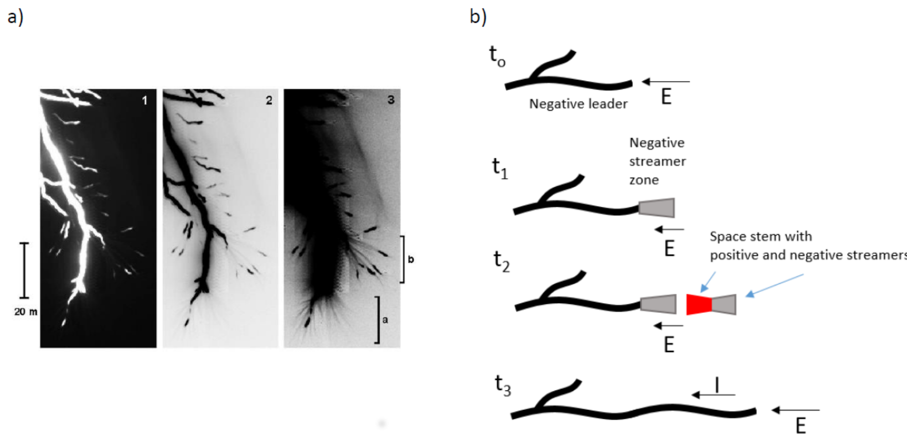

Figure 4: Video images and sketch of the stepping process of negative leaders. a) Standard video frame and inverted images of a negative downward leader (adapted from [25]). b) Sketch of the stepping process of a negative leader. At time $t = t_0$, very intense electric fields are found at the leader tip, which will be quickly screened by the formation of the streamer zone at $t = t_1$. At the instant t_2, a space stem is formed with a bidirectional development that will finish with the encounter of the negative and positive streamers, leading a high current impulse at t_3.

gates at basically the speed of light through the planetary atmosphere. If that pulse finds in its way a plasma, then the wakefield mechanism might be fired.

Intense and pulsating electric fields can be found in lightning leaders (Fig. 4a). So in principle, leaders could be a good candidate to support wakefield mechanism. Also we have another source of electromagnetic pulses. The recent observations of lightning initiation mechanisms [18] have shown that the formation of narrow bipolar events by means of volumetric positive streamers in the cloud yields the most intense radiation produced by lightning from VHF to lower frequencies. In such case, the occurrence of a TGF would be related to the first moments of lightning. This phenomenon is indeed consistent with the analysis of the TGF parent lightning by means of the Lightning Mapping Array occurred over Nashville [32]. Here the TGF was observed to occur at the initiation of a compact intra-cloud flash with an intense VHF power emission

[32]. This case shows the existence of intense VHF emissions

About the origin of the plasma, we can speculate that is another discharge, or even the same discharge that creates the pulse. For example, for EM pulses of order 10^5 V/m emitted in the $50-60$ MHz band which is the typical band of negative leaders, the plasma density that the pulse encounters should be of the order 10^{10} m^{-3} in order to accelerate electrons to MeV energies. For a characteristic temperature of a plasma created in lightning (30000 K), the characteristic plasma length λ_{De} should be the order of tens of centimeters. The plasma densities required for these conditions has been found in numerical simulations for sprite streamers in the Earth atmosphere [21] and might be present in other transient events out of equilibrium at lower temperatures. Note that sprite halos occur in the Earth's upper atmosphere at altitudes of ≈ 80 km so the encounter with the halo of a pulse created at low altitudes able to trigger the wakefield acceleration would be unlikely.

Leaders themselves also can be the source of several plasma regimes, from hot plasma channels to coronas, frequently found simultaneously in them. Early observations [22] highlighted the stepped propagation of downward negative leaders to ground. Streamers normally start at high-frequency rates from leader tips. The streamer zone at the leader tip (t_1 in Fig. 4b) provides the charge that will surround—cover— the hot leader channel when the leader advances [23]. In front of the streamer zone of a negative leader, a bidirectional streamer zone —with positive and negative directed streamer fronts— is established, originating from the so–called space stem (t_2 in Fig. 4b). The positive end of the space stem moves towards the leader tip that eventually becomes a positive leader end. When both meet (t_3 in Fig. 1b), the charges are neutralized, leaving uncompensated positive charge at the other stem end. At this moment, the space leader is negatively charged, acquiring the potential of the leader. This process is accompanied by a fast current pulse and a certain amount of energy release, including radiation [23]. In recent years, by means of high-speed video imaging, it has been possible to resolve the leader step processes occurring in natural lightning. The multi-instrumental observation of a negative dart-stepped leader [24] revealed that both fast dE/dt pulses and X-ray emissions are simultaneously emitted by the negative stepped leader. Using a limited bandwidth antenna, dE/dt pulses of about 10 $\text{kVm}^{-1}\mu\text{s}^{-1}$ peak-to-peak at 80 m (horizontal distance), with a pulse width of 50 ns, were associated with leader steps. Later [25], the detailed step evolution of a negative stepped leader and a dart leader was reported by means of high-speed video. In the negative speed leader tip, weakly filamentary radial structures were identified (Fig. 1a). In the same area, strongly luminous segments occurred with the same linear orientation. These features are consistent with laboratory observations of leaders, where the weakly luminous filaments correspond to corona streamers and the intense luminous segments to space leaders. Comparing to laboratory sparks, when the luminous segments (space leaders) attach with the main leader channel, a current surges able to generate the measured pulse of optical emissions and microwave radiation. In the laboratory, X-ray emissions are found to be associated with the encounter of positive and negative streamer zones in long sparks. In the case of negative sparks, the encounters were produced by space bidirectional streamers similar to the space leaders at negative leader tips of lightning. Moreover, peaks of microwave radiation, in addition to the X-ray emissions have been recently detected in long laboratory sparks [26].

So let us examine the question whether a plasma of size larger than 10 cm and electron density less than 10^{10} m^3 can be found in the vicinity of a stepped leader. If the plasma is fully ionized, then the air densities are too large at altitudes of 10 to 15 km. So in that case, to achieve a lower the plasma density the air should not be fully ionized, and the electron-neutral collisions need to be included in our analysis. There is however another possibility to be explored. We can consider that the expansion of the plasma channel or region makes the density decreases. The other possibility is the depletion of charge in the discharge channel due to current transport. The pulse could then encounters a region of very low density plasma and the wakefield under collisionless conditions would work. These possibility remains to be studied in details as well as the inclusion of collisions with neutrals.

5. Conclusions

In this paper we have studied the conditions for the creation of MeV electrons in atmospherics plasmas. An intense electromagnetic pulse interacting with the plasma can create a wake on the plasma. Electrons trapped in such oscillations can be accelerated under certain conditions to high energies. We have shown that those electrons could reach energies in the MeV range, thus being able to ignite gamma bursts. The pulse can be generated by electromagnetic activity in the atmosphere. The novelty of our proposal is the interaction of the pulse with a plasma already present in the planetary atmosphere and created for example by the same or another discharge or any electromagnetic activity. We have checked the conditions using numerical simulations.

In particular we have identified in the Earth atmosphere the negative lightning stepped leaders, where intense and pulsating electric fields are found as a candidate to observe the phenomena, although it does not exclude other scenarios. Under the conditions studied in this work, the wakefield mechanism could trigger the ignition of TGF's.

We stress that if those conditions are fulfilled, the pulse can propagate inside the plasma without damping and electrons will be accelerated dragged by the pulse. However, the conditions imply that the pulse propagates inside the plasma under collisionless regime. In the case that the collisions could not be neglected, still the amplification of the pulse can be achieved via the coupling of the electromagnetic wave and the electron plasma by an ion density fluctuation [33]. In this case, conditions (3) and (4) would not hold and condition (5) should be modified. A full study of the poderomotive force acting on the plasma is then needed and will the the subject of future work.

Even in the case that the inclusion of collisions with neutrals might prevent to achieve MeV energies, the role for wakefield acceleration on a more moderate scale could be extremely important. Acceleration of an electron to MeV may occur in several steps. The presence of a DC electric field would enhance the process. The acceleration provided by wakefield mechanism would be enough to get energies in keV range, after which the DC applied field could take the electrons to MeV energies.

Acknowledgment

This work has been partially supported by the Spanish Ministerio de Economía y Competitividad, under project ESP2015-69909-C5-4-R.

Appendix A.

Let us recall how an electromagnetic wave interacts with a plasma and how a plasma modifies the propagation of the electromagnetic waves [33]. The starting point to describe the evolution of a plasma is the Vlasov equation, which up to first order in the expansion parameter $1/N_D$, reads

$$\frac{\partial f_j}{\partial t} + \mathbf{v} \cdot \frac{\partial f_j}{\partial \mathbf{r}} + \frac{q_j}{m_j} \left(\mathbf{E} + \mathbf{v}_j \times \mathbf{B} \right) \cdot \frac{\partial f_j}{\partial \mathbf{r}} = \sum_k \left(\frac{\partial f_{jk}}{\partial t} \right)_C , \tag{A.1}$$

being $f_j(\mathbf{r}, \mathbf{v}, t)$ the distribution function of the j species. The right hand side of (A.1) represents the collisional terms. The parameter N_D is the number of particles in the Debye sphere. For electrons, $N_D = 4\pi n_e \lambda_{De}^3/3$, where n_e is the electron density and λ_{De} the electron Debye length. An useful expression of the Debye length under equilibrium conditions is given by $\lambda_{De} = 69\sqrt{T/n_e}\,(\mathrm{m})$, where T is the equilibrium temperature of the electrons in Kelvin and n_e is expressed in m^{-3} [16], so we have the condition

$$N_D \approx 1.38 \times 10^6\, T^{3/2}/n_e^{1/2} \gg 1, \tag{A.2}$$

for (A.1) to be valid. Note as pointed in the main text, that the condition to have a plasma is $\lambda_{De} \ll L$, being L the characteristic dimension of the system.

From (A.1) it is a standard procedure to calculate the first moment equations. We will assume that collisions do not change the number of species, so in the averaging we will take

$$\int d\mathbf{v} \sum_k \left(\frac{\partial f_{jk}}{\partial t} \right)_C = 0.$$

Hence, the change of the momentum of the j species becomes

$$n_j \frac{\partial \mathbf{u}_j}{\partial t} + n_j \mathbf{u}_j \cdot \frac{\partial \mathbf{u}_j}{\partial \mathbf{r}} = \frac{q_j}{m_j} n_j \left(\mathbf{E} + \mathbf{u}_j \times \mathbf{B}\right) - \frac{1}{m_j} \frac{\partial p_j}{\partial \mathbf{r}} - \sum_{k \neq j} \left(\frac{\partial}{\partial t} n_j \mathbf{u}_j \right)_k , \quad \text{(A.3)}$$

where n_j is the density and $\mathbf{u}_j$ the velocity of the j species and p_j the pressure.

Let us consider a plasma composed of ions with positive charged $+eZ$ and electrons. We will assumed that ions form a fixed background with density $n_i = n_e/Z$, where n_e is the corresponding electron density. Then we only need to treat the dynamics of the electron fluid. We can investigate the damping of an electromagnetic wave of the form $\mathbf{E}(\mathbf{r}) \exp(-i\omega t)$ considering the linearised plasma response. Writing

$$\left(\frac{\partial}{\partial t} n_e \mathbf{u}_e \right)_i = \nu_{ei} n_e \mathbf{u}_e,$$

where ν_{ei} is the collisional frequency of the scattering of electrons by ions, from (A.3) to first order we get

$$\frac{\partial \mathbf{u}_e}{\partial t} = -\frac{e}{m_e} \mathbf{E}(\mathbf{r})\, e^{-i\omega t} - \nu_{ei} \mathbf{u}_e. \quad \text{(A.4)}$$

The solution of (A.4) results

$$\mathbf{u}_e(\mathbf{r}, t) = \frac{-ie}{m_e(\omega + i\nu_{ei})} \mathbf{E}(\mathbf{r}) e^{-i\omega t}. \quad \text{(A.5)}$$

The plasma conductivity σ can be calculated from the current density of the plasma $\mathbf{j} = -en_e \mathbf{u}_e$. Using (A.5)

$$\mathbf{j} = i\varepsilon_0 \frac{\omega_{pe}^2}{\omega + i\nu_{ei}} \mathbf{E}(\mathbf{r}) e^{-i\omega t} = \sigma\, \mathbf{E}(\mathbf{r}, t), \quad \text{(A.6)}$$

where the plasma frequency is defined as

$$\omega_{pe}^2 = n_e e^2/(\varepsilon_0 m_e), \quad \text{(A.7)}$$

and the plasma conductivity σ is a complex quantity,

$$\sigma = i\varepsilon_0 \frac{\omega_{pe}^2}{\omega + i\nu_{ei}}. \quad \text{(A.8)}$$

From Maxwell's equations for harmonic fields,

$$\begin{aligned} \nabla \times \mathbf{E}(\mathbf{r}) &= i\omega \mathbf{B}(\mathbf{r}), \\ \nabla \times \mathbf{B}(\mathbf{r}) &= \mu_0 \sigma \mathbf{E} - i\frac{\omega}{c^2} \mathbf{E}(\mathbf{r}). \end{aligned} \quad \text{(A.9)}$$

The second equation in (A.9) can be written as

$$\nabla \times \mathbf{B}(\mathbf{r}) = -i\frac{\omega}{c^2}\varepsilon\,\mathbf{E}(\mathbf{r}), \tag{A.10}$$

where

$$\varepsilon = 1 - \frac{\omega_{pe}^2}{\omega(\omega + i\nu_{ei})}, \tag{A.11}$$

is the dielectric function of the plasma. Taking the curl in equations (A.9) and (A.10), one gets the wave equations

$$\begin{aligned} \nabla^2\mathbf{E}(\mathbf{r}) &- \nabla\left(\nabla\cdot\mathbf{E}(\mathbf{r})\right) + \frac{\omega^2}{c^2}\varepsilon\,\mathbf{E}(\mathbf{r}) = 0, \\ \nabla^2\mathbf{B}(\mathbf{r}) &+ \frac{1}{\varepsilon}\nabla\varepsilon\times\left(\nabla\times\mathbf{B}(\mathbf{r})\right) + \frac{\omega^2}{c^2}\varepsilon\,\mathbf{B}(\mathbf{r}) = 0, \end{aligned} \tag{A.12}$$

that give the spatial behaviour of the electric and magnetic fields in the plasma.

In the case of a neutral plasma with an uniform density $\nabla\varepsilon = 0$ and $\nabla\cdot\mathbf{E} = 0$, for harmonic electromagnetic waves $\mathbf{E}(\mathbf{r}) \sim \exp(i\mathbf{k}\cdot\mathbf{r})$, the equations (A.12) yield $\omega^2\varepsilon = c^2k^2$. So using (A.11)

$$\omega^2 = \omega_{pe}^2\left(1 - i\frac{\nu_{ei}}{\omega}\right) + c^2k^2, \tag{A.13}$$

in which it is assumed that $\nu_{ei} \ll \omega$. Equation (A.13) means that electromagnetic waves are damped. Writing $\omega = \omega_r - i\nu/2$, where ν is the damping rate, equation (A.13) becomes

$$\begin{aligned} \omega_r &= \sqrt{\omega_{pe}^2 + k^2c^2}, \\ \nu &= \frac{\omega_{pe}^2}{\omega_r^2}\,\nu_{ei}. \end{aligned} \tag{A.14}$$

The damping rate can be computed from the zero-order distribution function. This allows to get an expression for the collision frequency, namely [33, 16]

$$\nu_{ei} = \frac{1}{3(2\pi)^{3/2}}\frac{Z\omega_{pe}^4}{n_e v_e^3}\ln\Lambda = 3.61\times10^{-6}Z\ln\Lambda\frac{n_e}{T^{3/2}}, \tag{A.15}$$

where we use $v_e = \sqrt{K_BT/m_e}$ for the thermal velocity of electrons. The factor Λ is the ratio of the maximum and minimum impact parameters. The maximum impact parameter r_{max} is given by the Debye length λ_{De}, as the Coulomb potential is shielded out over that distance. The minimum impact parameter is given by the classical distance of closest approach $r_{min} = Ze^2/4\pi\varepsilon_0 m_e v_e^2$, which averaging over all the particle velocities and assuming a Maxwellian distribution yields $\bar{r}_{min} = Ze^2/(12\pi\varepsilon_0 K_B T)$, so

$$\Lambda = \frac{\lambda_{De}}{\bar{r}_{min}} = \frac{12\pi n_e \lambda_{De}^3}{Z}. \tag{A.16}$$

For typical values of plasmas, $\ln\Lambda \approx 10$. We must notice that (A.15) is only an approximation where the zero order electron distribution is taken Maxwellian. For non equilibrium processes, the collisional damping rate could be less. For example in the case of a super-Gaussian distribution, the collisional damping is reduced by a factor of 2.

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