FRICTIONAL INTERFACE CRACK-TIP SINGULAR STRESS FIELD IN ANISOTROPIC COMPOSITES

HUA-PENG CHEN*

* School of Engineering, the University of Greenwich,
Chatham Maritime, Kent ME4 4TB, UK
e-mail: h.chen@greenwich.ac.uk, www2.gre.ac.uk

Key words: Stress Singularity, Frictional Interface, Crack Tip, Composites Laminates.

Abstract. This study presents the asymptotic displacement and stress fields at the crack tip of frictional interface where slip can occur along the interface between two anisotropic composite laminates. The results show that real values corresponding to the order of stress singularities may exit at the crack tip of the frictional interface between two anisotropic layers. The order of stress singularity largely depends on the coefficient of friction within the interface and the material properties of anisotropic composite laminates such as fibre orientations.

1 INTRODUCTION

The investigations of damage mechanisms and failure criteria of composite materials, which may be caused by interfacial failure or crack growth due to stress concentration, are critical for the design and assessment of composite laminates [1]. These investigations require the knowledge of the deformation and stress fields in the vicinity of interfacial crack tip, when numerical methods such as finite element methods are adopted in singular stress analyses [2, 3]. In classical fracture mechanics, the open model was developed by assuming that the crack surfaces are traction free and the interface between two dissimilar materials is perfectly bonded [4]. The use of this model leads to the stress singularity of inverse square root, the introduction of stress intensity factors, and the prediction of crack growth. This model however has inherent nature of displacement and stress oscillations near the singular points, which could be physically inadmissible.

In order to resolve the difficulties associated with the oscillatory deformation and stress field, a contact model was then introduced, by assuming that a frictional contact exists near the tip of an interface crack [5-8]. This assumption is based on experimental observations, i.e. a limited frictional contact zone may exist near the tip of interface cracks in dissimilar bimaterials [9]. Due to the presence of frictional contact, only real values of the order of stress singularities associated with fracture modes may exist, so that stable displacement and stress fields can be achieved [10,11].

The stress singularities near the singular points with a perfectly bonded interface between two anisotropic elastic materials have been widely investigated [12-15]. However, these studies mainly focused on singular stresses near the singular points at perfectly bonded interfaces of composite materials. On the other hand, most existing studies on stress singularities of contact problems with a frictional interface are limited to isotropic materials. There is very limited research undertaken for the deformation and stress field near the crack tip of anisotropic composite materials with a frictional interface, such as cases for stress singularities in fibre reinforced composite laminates.

In this study, the displacement and stress fields near the singular point are obtained on the basis of Ting's theory and Stroh's formulation for anisotropic materials [16,17]. The characteristic equations are developed from the given external and interfacial boundary conditions of the problems with a frictional interface. Therefore, the order of stress singularities is obtained by solving for the roots of the constructed characteristic equations. The obtained results show that only real values associated with the order of stress singularities may exist at interfacial crack tip in anisotropic composites with a frictional interface. This agrees with the conclusion for the stress singularities of isotropic bimaterials with a frictional interface. The research further investigates the influence of the friction coefficient of the slip interface and the material properties such as fibre ply-angles of composite laminates on stress singularities. The analytical angular distributions of displacements and stress components near the singular points are presented for the anisotropic composite materials with a frictional interface, which is useful for finite element analysis.

2 ASYMPTOTIC FIELDS NEAR CRACK TIP

Assume that the displacements u_i, the strains ε_{ij} and the stresses σ_{ij} are independent of longitudinal coordinate x_3 when the composite is sufficiently long, as shown in Fig. 1. The asymptotic displacements near a singular point in anisotropic materials is given by Ting [16] and Chen *et al*. [11] as

$$u_i(\theta) = r^{\delta+1}\sum_{\alpha=1}^{3}[a_{i\alpha}\eta_\alpha^{\delta+1}(\theta)q_\alpha + \overline{a}_{i\alpha}\overline{\eta}_\alpha^{\delta+1}(\theta)\hat{q}_\alpha]/(\delta+1) \tag{1}$$

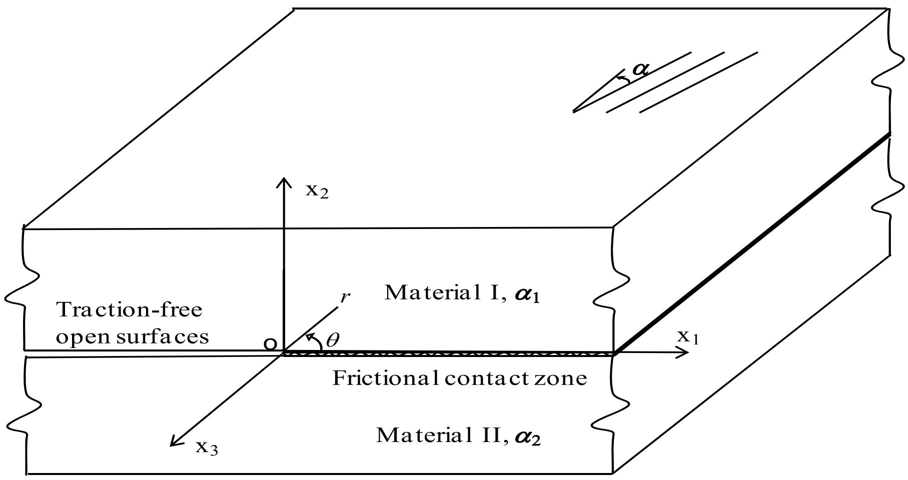

Figure 1: Traction-free open angle-ply laminated composite layers with frictional contact at the interface

where δ is a constant to be determined; variable $\eta = \cos\theta + p\sin\theta$; an over bar denotes the complex conjugate; q_α and $\hat{q}_\alpha$ are arbitrary constants; and the coefficients p_k and $\{a_k\}$ (k=1, 2, 3) are the eigenvalues and eigenvectors of the equation associated with the equilibrium equation [16], expressed here as

$$[\mathbf{Q} + p(\mathbf{R} + \mathbf{R}^T) + p^2\mathbf{T}]\boldsymbol{a} = \mathbf{0} \tag{2}$$

in which **Q**, **R** and **T** are 3×3 matrices associated with elasticity constants of the anisotropic elastic material. The stress components σ_{i1} and σ_{i2} on the plane of x_1 and x_2 are

$$\sigma_{i1}(\theta) = -r^\delta \sum_{\alpha=1}^{3} [b_{i\alpha} p_\alpha \eta_\alpha^\delta(\theta) q_\alpha + \bar{b}_{i\alpha} \bar{p}_\alpha \bar{\eta}_\alpha^\delta(\theta) \hat{q}_\alpha] \tag{3.1}$$

$$\sigma_{i2}(\theta) = r^\delta \sum_{\alpha=1}^{3} [b_{i\alpha} \eta_\alpha^\delta(\theta) q_\alpha + \bar{b}_{i\alpha} \bar{\eta}_\alpha^\delta(\theta) \hat{q}_\alpha] \tag{3.2}$$

where vectors $\boldsymbol{b}_\alpha$ are related to material constants, calculated from $\boldsymbol{b}_\alpha = (\mathbf{R}^T + p_\alpha \mathbf{T})\boldsymbol{a}_\alpha$. From Eq. (3), it can be seen that the stresses become singular and are of the order r^δ when r approaches zero if the real part of δ, Re(δ), is negative. The surface traction t_i on the polar plane θ near the singular point is calculated from Eq. (3) as

$$t_i(\theta) = -\sigma_{i1}\sin\theta + \sigma_{i2}\cos\theta = r^\delta \sum_{\alpha=1}^{3} [b_{i\alpha} \eta_\alpha^{\delta+1}(\theta) q_\alpha + \bar{b}_{i\alpha} \bar{\eta}_\alpha^{\delta+1}(\theta) \hat{q}_\alpha] \tag{4}$$

To determine the angular distributions of asymptotic fields, the displacements u_i in Cartesian coordinates (x_1, x_2, x_3) are transferred in polar coordinates (r, θ, z) as

$$u_\theta = -u_1 \sin\theta + u_2 \cos\theta \tag{5.1}$$

$$u_r = u_1 \cos\theta + u_2 \sin\theta \quad ; \quad u_z = u_3 \tag{5.2}$$

and the normal and tangential stress components on the plane θ in the polar coordinates are

$$\sigma_{\theta\theta} = \sigma_{11}\sin^2\theta + \sigma_{22}\cos^2\theta - \sigma_{21}\sin 2\theta \tag{6.1}$$

$$\sigma_{r\theta} = \tfrac{1}{2}(\sigma_{22} - \sigma_{11})\sin 2\theta + \sigma_{21}\cos 2\theta \quad ; \quad \sigma_{z\theta} = \sigma_{32}\cos\theta - \sigma_{31}\sin\theta \tag{6.2}$$

In order to obtain real displacement and stress fields near the singular point, two real matrices **S** and **L**, associated with the non-singular complex coefficient matrices **A**=$\{a_1 \ \ a_2 \ \ a_3\}$ and **B**=$\{b_1 \ \ b_2 \ \ b_3\}$, are defined in [16] as

$$\mathbf{S} = j(2\mathbf{A}\mathbf{B}^T - \mathbf{I}) \quad , \quad \mathbf{L} = -2j\mathbf{B}\mathbf{B}^T \tag{7.1}$$

$$\mathbf{S} = j(2\mathbf{A}\mathbf{B}^T - \mathbf{I}) \quad , \quad \mathbf{L} = -2j\mathbf{B}\mathbf{B}^T \tag{7.2}$$

where superscript T represents the transpose of a matrix; **I** is an identity matrix; and j indicates the standard imaginary unit with the property of $j^2 = -1$. From Eq. (7), the real and imaginary parts of matrices $\mathbf{A}\mathbf{B}^{-1}$ and $\overline{\mathbf{A}\mathbf{B}}^{-1}$ are then obtained from

$$\mathbf{AB}^{-1} = -\mathbf{SL}^{-1} - j\mathbf{L}^{-1} \quad , \quad \overline{\mathbf{AB}}^{-1} = -\mathbf{SL}^{-1} + j\mathbf{L}^{-1} \tag{8.1}$$

$$\mathbf{AB}^{-1} = -\mathbf{SL}^{-1} - j\mathbf{L}^{-1} \quad , \quad \overline{\mathbf{AB}}^{-1} = -\mathbf{SL}^{-1} + j\mathbf{L}^{-1} \tag{8.2}$$

It can be showed that $\mathbf{SL}^{-1}$ is anti-symmetric matrix with $\mathbf{SL}^{-1} = -(\mathbf{SL}^{-1})^T$ and diagonal elements of zero and $\mathbf{L}^{-1}$ is symmetric and positive definite matrix with $\mathbf{L}^{-1} = (\mathbf{L}^{-1})^T$.

3 ORDER OF STRESS SINGULARITY

For the frictional contact problem shown in Fig. 1, the frictional interface is located on the plane $\theta = \theta_f = 0$, the continuity conditions of the stresses for material I and material II on the plane are given by

$$\boldsymbol{t}^I(0) - \boldsymbol{t}^{II}(0) = \boldsymbol{0} \tag{9}$$

The interface boundary conditions governed by the Coulomb's law of friction within the frictional interface $\theta_f = 0$ are expressed here as

$$u_2^I(0) - u_2^{II}(0) = 0 \tag{10.1}$$

$$\sigma_{12}^I(0) + f\cos\varphi\sigma_{22}^I(0) = 0 \tag{10.2}$$

$$\sigma_{32}^I(0) + f\sin\varphi\sigma_{22}^I(0) = 0 \text{ where } \sigma_{22}^I(0) \le 0 \tag{10.3}$$

where the normal stress of material I within the frictional interface should remain compressive, and the coefficient of friction f is assumed to be constant within the slip interface in all directions. Angle φ is the direction of the tangential resultant stress with respect to positive axis x_1, determined by

$$\tan\varphi = \frac{\sigma_{32}^I(0)}{\sigma_{12}^I(0)} = \frac{\Delta u_3(0)}{\Delta u_1(0)} = \frac{u_3^I(0) - u_3^{II}(0)}{u_1^I(0) - u_1^{II}(0)} \tag{11}$$

Application of the formulation for surface traction in Eq. (4) to the traction free boundary conditions for material I at $\theta = +\pi$ and material II at $\theta = -\pi$ leads to

$$e^{j\pi\delta}\mathbf{B}^I\boldsymbol{q}^I + e^{-j\pi\delta}\overline{\mathbf{B}}^I\hat{\mathbf{E}}^I\hat{\boldsymbol{q}}^I = \boldsymbol{0} \tag{12.1}$$

$$e^{-j\pi\delta}\mathbf{B}^{II}\boldsymbol{q}^{II} + e^{j\pi\delta}\overline{\mathbf{B}}^{II}\hat{\boldsymbol{q}}^{II} = \boldsymbol{0} \tag{12.2}$$

Similarly, the stress continuity conditions within the frictional interface in Eq. (9) gives

$$\mathbf{B}^I\boldsymbol{q}^I + \overline{\mathbf{B}}^I\hat{\boldsymbol{q}}^I - \mathbf{B}^{II}\boldsymbol{q}^{II} - \overline{\mathbf{B}}^{II}\hat{\boldsymbol{q}}^{II} = \boldsymbol{0} \tag{13}$$

Applying the asymptotic fields for stress components in Eq. (3) and the displacements in Eq. (1) into Eq. (10) for the frictional contact boundary conditions leads to

$$\begin{bmatrix}0 & 1 & 0\end{bmatrix}[\mathbf{A}^I\boldsymbol{q}^I + \overline{\mathbf{A}}^I\hat{\boldsymbol{q}}^I - \mathbf{A}^{II}\boldsymbol{q}^{II} - \overline{\mathbf{A}}^{II}\hat{\boldsymbol{q}}^{II}] = 0 \tag{14.1}$$

$$\begin{bmatrix}1 & f\cos\varphi & 0\end{bmatrix}[\mathbf{B}^I\boldsymbol{q}^I + \overline{\mathbf{B}}^I\hat{\boldsymbol{q}}^I] = 0 \tag{14.2}$$

$$\begin{bmatrix}0 & f\sin\varphi & 1\end{bmatrix}[\mathbf{B}^I\boldsymbol{q}^I + \overline{\mathbf{B}}^I\hat{\boldsymbol{q}}^I] = 0 \tag{14.3}$$

From Eq. (12) and Eq. (13), the arbitrary constant vectors are obtained as

$$\hat{\boldsymbol{q}}^I = -e^{j2\pi\delta}\overline{\mathbf{B}}^{I^{-1}}\mathbf{B}^I\boldsymbol{q}^I \quad ; \quad \boldsymbol{q}^{II} = -e^{j2\pi\delta}\mathbf{B}^{II^{-1}}\mathbf{B}^I\boldsymbol{q}^I \quad ; \quad \hat{\boldsymbol{q}}^{II} = \overline{\mathbf{B}}^{II^{-1}}\mathbf{B}^I\boldsymbol{q}^I \tag{15}$$

By using Eq. (15), the characteristic equations in Eq. (14) can be rewritten as

$$[0 \quad 1 \quad 0][\boldsymbol{\Omega} - e^{j2\pi\delta}\overline{\boldsymbol{\Omega}}]\mathbf{B}^I\boldsymbol{q}^I = 0 \tag{16.1}$$

$$(1-e^{j2\pi\delta})[1 \quad f\cos\varphi \quad 0]\mathbf{B}^I\boldsymbol{q}^I = 0 \tag{16.2}$$

$$(1-e^{j2\pi\delta})[0 \quad f\sin\varphi \quad 1]\mathbf{B}^I\boldsymbol{q}^I = 0 \tag{16.3}$$

where coefficient matrix $\boldsymbol{\Omega}$ related to material constants is defined, by using Eq. (8), as

$$\boldsymbol{\Omega} = \boldsymbol{\Omega}^r + j\boldsymbol{\Omega}^i = \mathbf{A}^I\mathbf{B}^{I^{-1}} - \overline{\mathbf{A}}^{II}\overline{\mathbf{B}}^{II^{-1}} = -(\mathbf{S}^I\mathbf{L}^{I^{-1}} - \mathbf{S}^{II}\mathbf{L}^{II^{-1}}) - j(\mathbf{L}^{I^{-1}} + \mathbf{L}^{II^{-1}}) \tag{17}$$

in which the real part $\boldsymbol{\Omega}^r = -(\mathbf{S}^I\mathbf{L}^{I^{-1}} - \mathbf{S}^{II}\mathbf{L}^{II^{-1}})$ is anti-symmetric matrix with properties of $\omega^r_{\alpha\beta} = -\omega^r_{\beta\alpha}$ and $\omega^r_{\alpha\alpha} = 0$, and the imaginary part $\boldsymbol{\Omega}^i = -(\mathbf{L}^{I^{-1}} + \mathbf{L}^{II^{-1}})$ is symmetric matrix with properties of $\omega^i_{\alpha\beta} = \omega^i_{\beta\alpha}$.

It is found that if δ is a root, so is $\delta+n$ where n is an integer. For stress singularities with $-1 < \mathrm{Re}(\delta) < 0$, the root δ of $1-e^{j2\pi\delta} = 0$ can be ignored. By considering Eqs. (16.2) and (16.3), where the corresponding coefficient vectors are linearly independent, we have

$$\mathbf{B}^I\boldsymbol{q}^I = \hat{k}\{-f\cos\varphi \quad 1 \quad -f\sin\varphi\}^T \tag{18}$$

where $\hat{k}$ is an arbitrary constant and may be a complex number. Substituting Eq. (18) into Eq. (16.1) produces the characteristic equation for obtaining the nontrivial solution for the arbitrary constants to solve for the root δ, expressed here as

$$e^{j2\pi\delta} = \frac{\omega_{22} - f\omega_{21}\cos\varphi - f\omega_{23}\sin\varphi}{\overline{\omega}_{22} - f\overline{\omega}_{21}\cos\varphi - f\overline{\omega}_{23}\sin\varphi} \tag{19}$$

The root δ of Eq. (19), bounded by $-1<\mathrm{Re}(\delta)<0$, may characterise the order of stress singularities at the tip of frictional interface between the anisotropic composite laminates, expressed explicitly after considering the properties of matrices $\boldsymbol{\Omega}^r$ and $\boldsymbol{\Omega}^i$ in Eq. (17) as

$$\mathrm{ctg}\pi\delta = \frac{-f(\omega^r_{21}\cos\varphi + \omega^r_{23}\sin\varphi)}{(\omega^i_{22} - f\omega^i_{21}\cos\varphi - f\omega^i_{23}\sin\varphi)} \tag{20}$$

Consequently, the order of stress singularity is determined from the characteristic equation Eq. (20) of the crack at the frictional interface between two anisotropic composite materials.

4 FIELDS AT CRACK TIP

From Eq. (18), the corresponding arbitrary constant vectors in Eq. (15) can be found, and the real arbitrary constant k is introduced to have the relationship $\hat{k} = k(1-e^{-j2\pi\delta})$ in order to obtain real displacement and stress fields around the tip of the frictional interface. By using the obtained arbitrary constant vectors, Eq. (11) becomes

$$\tan\varphi = \frac{[0 \quad 0 \quad 1][\mathbf{\Omega} - e^{j2\pi\delta}\overline{\mathbf{\Omega}}]\{-f\cos\varphi \quad 1 \quad -f\sin\varphi\}^T}{[1 \quad 0 \quad 0][\mathbf{\Omega} - e^{j2\pi\delta}\overline{\mathbf{\Omega}}]\{-f\cos\varphi \quad 1 \quad -f\sin\varphi\}^T} \tag{21}$$

Substituting the characteristic equation in Eq. (20) into Eq. (21) and considering the properties of matrices $\mathbf{\Omega}^r$ and $\mathbf{\Omega}^i$ in Eq. (17) give the uncoupled equation to solve for the angle φ as

$$\begin{aligned}&[\omega_{22}^i - f\omega_{21}^i\cos\varphi - f\omega_{23}^i\sin\varphi][(\omega_{12}^r - f\omega_{13}^r\sin\varphi)\sin\varphi - (\omega_{32}^r - f\omega_{31}^r\cos\varphi)\cos\varphi]\\&\quad - f[\omega_{21}^r\cos\varphi + \omega_{23}^r\sin\varphi][(\omega_{12}^i - f\omega_{11}^i\cos\varphi - f\omega_{13}^i\sin\varphi)\sin\varphi\\&\quad - (\omega_{32}^i - f\omega_{31}^i\cos\varphi - f\omega_{33}^i\sin\varphi)\cos\varphi] = 0\end{aligned} \tag{22}$$

After angle φ is determined from Eq. (22), the value of δ is obtained from Eq. (20) without requiring the iterative procedure. From the obtained arbitrary constant vectors, the stress components σ_{i2}^I in Eq. (3.2) at the frictional interface at $\theta = 0$ are expressed as

$$\{\sigma_{i2}^I(0)\}^T = kr^\delta 4\sin^2\pi\delta\{-f\cos\varphi \quad 1 \quad -f\sin\varphi\}^T \tag{23}$$

Furthermore, the displacements in Eq. (1) for material I u_i^I and for material II u_i^{II} at the frictional interface at $\theta = 0$ are given, respectively, as

$$\{u_i^I(0)\}^T = kr^{\delta+1}\frac{4}{\delta+1}\sin\pi\delta[-\sin\pi\delta\mathbf{S}^I\mathbf{L}^{I^{-1}} + \cos\pi\delta\mathbf{L}^{I^{-1}}]\{-f\cos\varphi \quad 1 \quad -f\sin\varphi\}^T \tag{24. 1}$$

$$\{u_i^{II}(0)\}^T = kr^{\delta+1}\frac{4}{\delta+1}\sin\pi\delta[-\sin\pi\delta\mathbf{S}^{II}\mathbf{L}^{II^{-1}} - \cos\pi\delta\mathbf{L}^{II^{-1}}]\{-f\cos\varphi \quad 1 \quad -f\sin\varphi\}^T \tag{24. 2}$$

Similarly, the displacements of material I u_i^I at $\theta = +\pi$ and of material II u_i^{II} at $\theta = -\pi$ are obtained from

$$\{u_i^I(+\pi)\}^T = -kr^{\delta+1}\frac{4}{\delta+1}\sin\pi\delta\,\mathbf{L}^{I^{-1}}\{-f\cos\varphi \quad 1 \quad -f\sin\varphi\}^T \tag{25. 1}$$

$$\{u_i^{II}(-\pi)\}^T = k\,r^{\delta+1}\frac{4}{\delta+1}\sin\pi\delta\mathbf{L}^{II^{-1}}\{-f\cos\varphi \quad 1 \quad -f\sin\varphi\}^T \tag{25. 2}$$

Consequently, from the properties of matrices $\mathbf{\Omega}^r$ and $\mathbf{\Omega}^i$, the shifts within the frictional interface in x_1 and x_3 directions are calculated from

$$\Delta u_1(0) = kr^{\delta+1}\frac{4}{(\delta+1)}\sin\pi\delta\{\sin\pi\delta(\omega_{12}^r - f\omega_{13}^r\sin\varphi) - \cos\pi\delta(\omega_{12}^i - f\omega_{11}^i\cos\varphi - f\omega_{13}^i\sin\varphi)\} \tag{26.1}$$

$$\Delta u_3(0) = kr^{\delta+1}\frac{4}{(\delta+1)}\sin\pi\delta\{\sin\pi\delta(\omega_{32}^r - f\omega_{31}^r\cos\varphi) - \cos\pi\delta(\omega_{32}^i - f\omega_{31}^i\cos\varphi - f\omega_{33}^i\sin\varphi)\} \tag{26.2}$$

From Eq. (25) and Eq. (17), the gap in the front of the frictional interface tip between material I at $\theta = +\pi$ and material II at $\theta = -\pi$ is expressed as

$$\Delta u_2(\pm\pi) = u_2^I(+\pi) - u_2^{II}(-\pi) = kr^{\delta+1}\frac{4}{(\delta+1)}\sin\pi\delta\,(\omega_{22}^i - f\cos\varphi\omega_{21}^i - f\sin\varphi\omega_{23}^i) \tag{27}$$

The boundary conditions for the frictional contact problems described in Eq. (10) require compressive normal stress within the frictional interface and positive gap in the front of its end point to keep separation propagating forward, as discussed in [5,10]. From Eq. (23), the compressive normal stress within the frictional interface $\sigma_{22}^{I}(0)=kr^{\delta}4\sin^{2}\pi\delta$ requires the arbitrary constant $k\le 0$ to remain the normal contact. Since $-1<\mathrm{Re}(\delta)<0$, the requirement of positive gap in Eq. (27) leads to

$$w_{22}^{i}\ge f(\cos\varphi w_{21}^{i}+\sin\varphi w_{23}^{i}) \tag{28}$$

We found that stress singularities can appear at the tip of frictional interface between two anisotropic layers if the material constants satisfy the requirement given in Eq. (28). This is different from the conclusion given in [5] that no stress singularities can appear in the frictional contact problems for isotropic materials.

5 NUMERICAL RESULTS

For fibre reinforced composite laminates with a frictional interface shown in Fig. 1, the engineering constants for the orthotropic material are assumed to be E_L=137.90GPa, E_T=E_Z=14.48GPa, μ_{LT}=μ_{LZ}=μ_{TZ}=4.98GPa, and υ_{LT}=υ_{LZ}=υ_{TZ}=0.21, where E is Young's modulus and the subscripts *L, T* and *Z* refer to the fibre, transverse and thickness directions of an individual layer, respectively. Fig. 2 shows the results for the order of stress singularities as a function of the fibre orientation of material I α_1 varying from 0° to 90° with α_2 fixed to α_2=0° and the coefficient of friction *f* in the contact zone ranging from 0 to 3. From the results it is found that the stress singularity becomes weaker as the coefficient of friction *f* increases. The values of the order of stress singularities are significantly affected by the fibre orientation α_1 when the coefficient of friction becomes high, e.g. *f*=3. In the case of frictionless interface *f*=0, the order of singularities remains the same value of -0.5 for any fibre orientation angle.

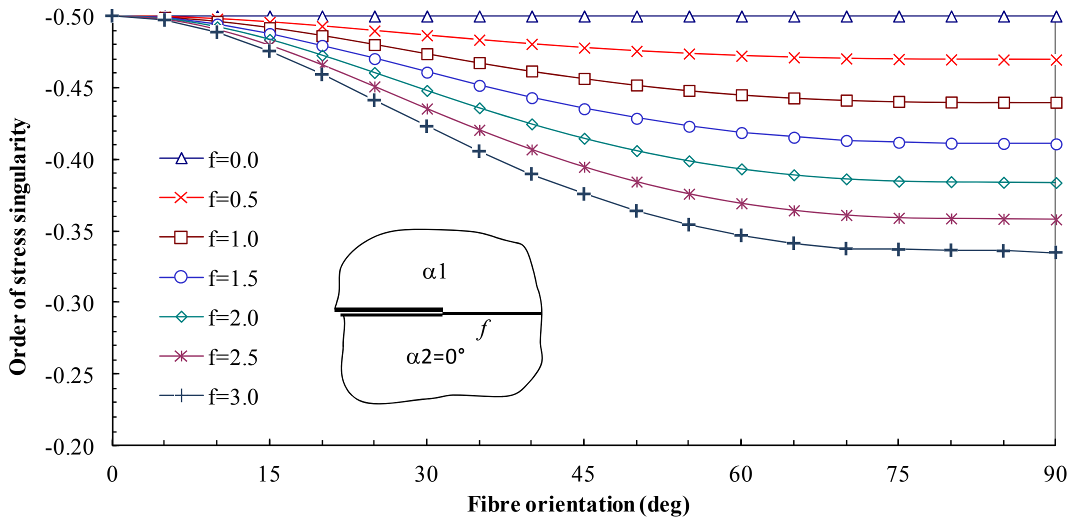

Figure 2: Root δ as a function of the fibre orientation of material I α_1 and the coefficient of friction *f*.

The normalised angular distribution functions of the displacements u_θ, u_r, u_z and the stress components $\sigma_{\theta\theta}$, $\sigma_{r\theta}$, $\sigma_{z\theta}$ in the crack tip of frictional interface between two anisotropic laminates are presented in Figs. 3 and 4, respectively. The fibre orientations for two composite materials α_1=45° and α_2=−45° and the friction coefficient of the slip interface f=0.5 are considered in this case. The order of stress singularity δ=−0.477952 and the angle φ=-4.186° to axis x_1 are accurately obtained for the contact problem by solving the coupled nonlinear equations only after a few iterations with the relative error for δ less than 10^{-6}. From the results in Fig. 3, it can be seen that the normal displacement u_θ is continuous at the frictional interface at θ=0°, but not its slope, to maintain the normal contact at the frictional interface between two composite wedges. The displacements u_r and u_z however are not continuous at the frictional interface, so that the relative slips occur in the negative direction of coordinate r (x_1) and the positive direction of coordinate z (x_3) between two materials, respectively. The displacement distribution functions vary smoothly with the angle θ within the regions of individual materials around the crack tip, and show that large deformation may occur on the free surfaces. The results in Fig. 4 demonstrate that the stress components $\sigma_{\theta\theta}$, $\sigma_{r\theta}$, $\sigma_{z\theta}$ on the plane of coordinate θ are continuous at the frictional interface between two materials at $\theta = 0$ because it is assumed that the normal contact is maintained at the frictional interface. The normal stress $\sigma_{\theta\theta}$ gives a negative value at the frictional interface θ=0° to maintain the normal contact. The values of stress components gradually approach zero near the external open surfaces, vanishing at θ=180° and θ=−180° to satisfy the traction-free conditions on the surfaces of composite laminates .

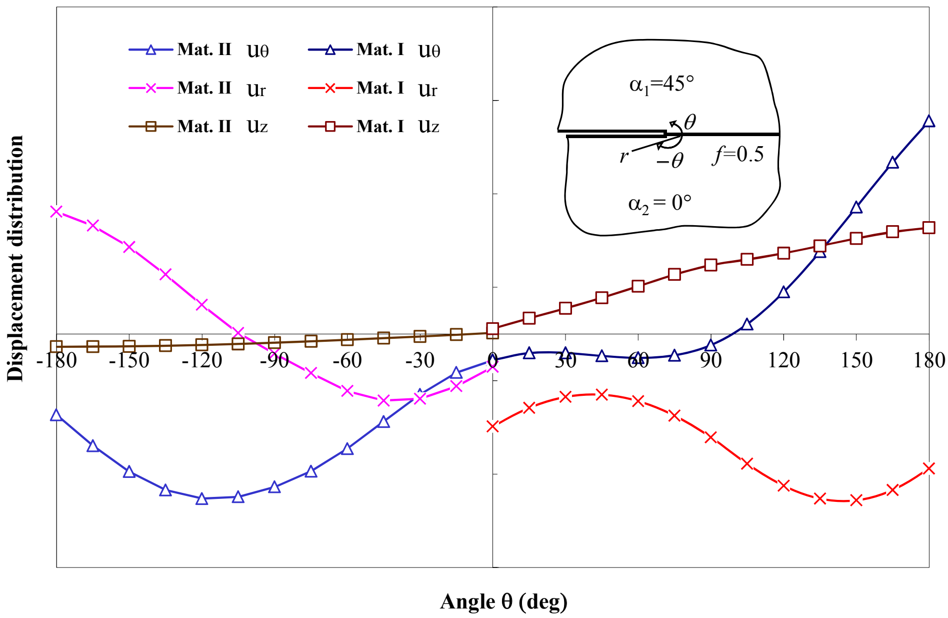

Figure 3: Normalised angular distribution functions of the displacements u_θ, u_r, u_z near the corner of the free-free fibre reinforced composite laminates (+45°/0°) with frictional interface of f = 0.5.

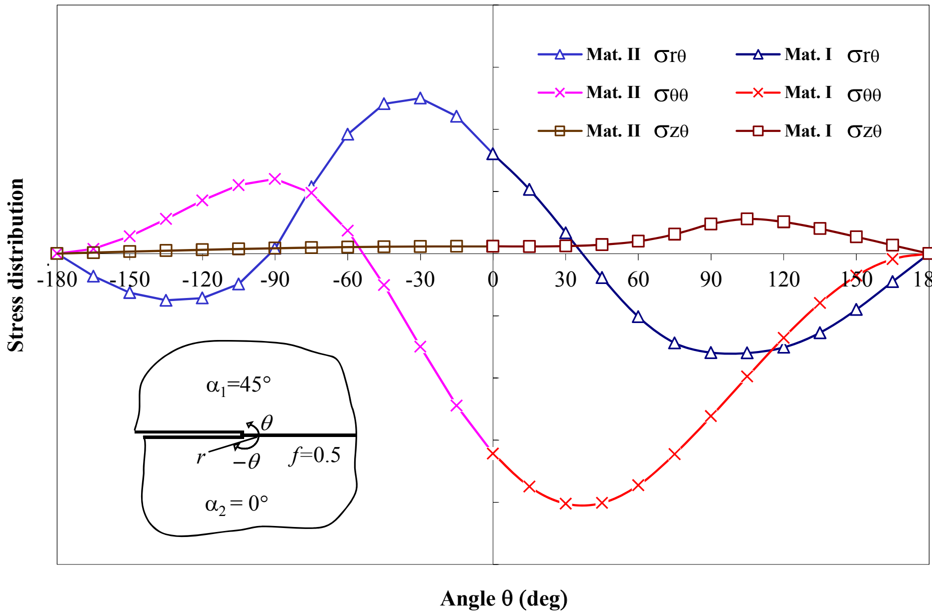

Figure 4: Normalised angular distribution functions of the stress components $\sigma_{\theta\theta}$, $\sigma_{r\theta}$, $\sigma_{z\theta}$ near the corner of the free-free fibre reinforced composite laminates (+45°/0°) with frictional interface of $f = 0.5$.

6 CONCLUSIONS

The obtained results show that stress singularities can appear at the tip of the frictional interface if their material constants satisfy the requirement propose. Only real values associated with the order of stress singularities may exist for the contact problems with a frictional interface in anisotropic composite laminates. The fibre orientation of composite materials has stronger influence on the stress singularity when the friction coefficient is higher. The effect of the friction coefficient at the slip interface on the stress singularities increases when stronger mismatch between two materials is present. Large deformation may occur on the traction-free surfaces near the crack tip, and the peak value of the normal stress may exist at the fictional interface for the case investigated.

REFERENCES

[1] Chen, H.P. Stress singularities in anisotropic multi-material wedges and junctions. *International Journal of Solids and Structures* (1998) **35**:1057-1073.

[2] Ayatollahi, M.R. and Nejati, M. An over-deterministic method for calculation of coefficients of crack tip asymptotic field from finite element analysis. *Fatigue & Fracture of Engineering Materials & Structures* (2011) **34**(3): 159–176.

[3] Huynh, D.B.P. and Belytschko, T. The extended finite element method for fracture in composite materials. *International Journal for Numerical Methods in Engineering* (2009) **77**(2):214–239.

[4] Williams, M.L. The stresses around a fault or crack in dissimilar media. *Bulletin of the Seismological Society of America* (1959) **49**:199-204.
[5] Dundurs, J. and Comninou, M. Some consequences of the inequality conditions in contact and crack problems. *Journal of Elasticity* (1979) **9**:71-82.
[6] Wijeyewickrema, A.C. and Keer, L.M. Matrix cracking in a fibre-reinforced composite with slip at the fibre-matrix interface. *International Journal of Solids and Structures* (1993) **30**:91-113.
[7] Poonsawat, P., Wijeyewickrema, A,C, and Karasudhi, P. Singular stress fields of angle-ply and monoclinic bimaterial wedges. *International Journal of Solids and Structures* (2001) **38**(1):91-113.
[8] Fuenmayor, F.J., Giner, E. and Tur, M. Extraction of the generalized stress intensity factor in gross sliding complete contacts using a path-independent integral. *Fatigue & Fracture of Engineering Materials & Structures* (2005) **28**(12):1071–1085.
[9] Miniatt, E., Waas, A. and Anderson, W. An experimental study of stress singularities at a sharp corner in a contact problem. *Experimental Mechanics* (1990) **30**:281-285.
[10] Audoly, B. Asymptotic study of the interfacial crack with friction. *Journal of the Mechanics and Physics of Solids* (2000) **48**:1851-1864.
[11] Chen, H.P., Guo, Z. and Zhou, X. Stress singularities of contact problems with a frictional interface in anisotropic bimaterials, *Fatigue and Fracture of Engineering Materials and Structures* (2012) **35**(8):718–731.
[12] Ting, T.C.T. Explicit solution and invariance of the singularities at an interface crack in anisotropic composites. *International Journal of Solids and Structures* (1986) **22**:965-983.
[13] Profant, T., Ševeček, O., Kotoul, M. and Vysloužil, T. Dislocation tri-material solution in the analysis of bridged crack in anisotropic bimaterial half-space. *International Journal of Fracture* (2007) **147**:199-217.
[14] Ni, L. and Nemat-Nassera ,S. Interface cracks in anisotropic dissimilar materials: An analytic solution. *Journal of the Mechanics and Physics of Solids* (1991) **39**:113-144.
[15] Gao, H., Abbudi, M. and Barnett, D.M. Interfacial crack-tip field in anisotropic elastic solids. *Journal of the Mechanics and Physics of Solids* (1992) **40**:393-416.
[16] Ting, T.C.T. *Anisotropic Elasticity: Theory and Applications*. Oxford University Press, Oxford, UK (1996).
[17] Stroh, A.N. Steady state problems in anisotropic elasticity. *Journal of Mathematical Physics* (1962) **41**:77-103.