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Fuzzy Heterogeneous Neurons for Imprecise Classification Problems

Julio J. Valdés Lluís A. Belanche René Alquézar

Secció d'Intel·ligència Artificial.
Dept. de Llenguatges i Sistemes Informàtics.
Universitat Politècnica de Catalunya.
c/Jordi Girona Salgado 1-3
08034 Barcelona, Spain.
{valdes, belanche, alquezar}@lsi.upc.es

Abstract

In the classical neuron model, inputs are continuous real-valued quantities. However, in many important domains from the real world, objects are described by a mixture of continuous and discrete variables, usually containing missing information and uncertainty. In this paper, a general class of neuron models accepting heterogeneous inputs in the form of mixtures of continuous (crisp and/or fuzzy) and discrete quantities admitting missing data is presented. From these, several particular models can be derived as instances and different neural architectures constructed with them. Such models deal in a natural way with problems for which information is imprecise or even missing. Their possibilities in classification and diagnostic problems are here illustrated by experiments with data from a real world domain in the field of environmental studies. These experiments show that such neurons can both learn and classify complex data very effectively in the presence of uncertain information.

Keywords: Heterogeneous Neural Networks; Fuzzy Logic; Genetic Algorithms; Uncertainty and Missing Data.

1 Introduction

The classical neuron model –where inputs are continuous real-valued quantities, and net input is computed as the scalar product of the input and the weight vector– was extended by the notion of an heterogeneous neuron introduced in [11]. Model instances of this concept provide neurons accepting mixtures of real and discrete quantities (i.e. numerical and qualitative information) possibly also containing missing data. A second important feature of this class of models is that their internal stimulation is based on a *similarity* or *proximity relation* [1] between the input and the weight tuples. In particular, a family of models considering neuron outputs as a composition of a similarity function with a sigmoid-like squashing function was shown to be a reasonable brick for constructing layered network architectures mixing heterogeneous with classical neurons. These networks were shown to be capable to learn from non-trivial data sets with an effectiveness comparable, and sometimes better, than that of classical methods. They exhibited a remarkable robustness when information degrades due to the increasing presence of missing data. As was stated in [11] when discussing the general framework for heterogeneous neurons, future studies should consider more general models constructed by taking into account mappings among wider classes of sets as domains and images (i.e inputs and output). The purpose of this paper is to consider extensions in which fuzzy sets may occur as part of the input. This will introduce more flexibility by accepting training processes using imprecise data, both in the input *and* the weights. This way, heterogeneous neurons of this kind may accept a mixture of real, qualitative, and *fuzzy* quantities, possibly with missing information. In what follows, a neuron of this type is presented and its learning capabilities are illustrated by a real world example: an environmental study –using geophysical data processing– aimed at detecting the presence of underground caves.

2 The Heterogeneous Neuron Model Revisited

An heterogeneous neuron was defined in [11] as a mapping $h : \hat{\mathcal{H}}^n \rightarrow \mathcal{R}_{out} \subseteq \mathbb{R}$, satisfying $h(\phi) = 0$ (ϕ is the empty set). Here $\mathbb{R}$ denotes the reals and $\hat{\mathcal{H}}^n$ is a cartesian product of an arbitrary number of *source sets*. Source sets may be extended reals $\hat{\mathcal{R}} = \mathbb{R} \cup \{\mathcal{X}\}$, and/or finite sets of the form $\hat{\mathcal{O}}_i = \mathcal{O}_i \cup \{\mathcal{X}\}$, $\hat{\mathcal{M}}_i = \mathcal{M}_i \cup \{\mathcal{X}\}$. Each of the $\mathcal{O}_i$ has a full order relation, while the $\mathcal{M}_i$ have not. The special symbol $\mathcal{X}$ denotes the unknown element (missing information) and it behaves as an incomparable element w.r.t. any ordering relation. According to this definition, neuron inputs are possibly empty arbitrary tuples , composed by n elements among which there might be reals, ordinals, nominals and missing data.

A particular class of heterogeneous neurons can be devised by considering

h as the composition of two mappings, $h = f \circ s$, such that $s : \hat{\mathcal{H}}^n \to \mathcal{R}' \subseteq \mathbb{R}$ and $f : \mathcal{R}' \to \mathcal{R}_{out} \subseteq \mathbb{R}$. The mapping h can be considered as a n-ary function parameterized by a tuple $\hat{\vec{w}} \ \epsilon \ \hat{\mathcal{H}}^n$ representing neuron's weights, i.e. $h(\hat{\vec{x}}, \hat{\vec{w}}) = f(s(\hat{\vec{x}}, \hat{\vec{w}}))$. In particular, the function s represents a *similarity* and f a squashing non-linear function with its image in $[0,1]$. Accordingly, the neuron is sensitive to the degree of similarity between its input, composed in general by a mixture of continuous and discrete quantities possibly with missing data. More precisely, s is understood as a *similarity index*, or proximity relation (transitivity considerations are put aside). That is, a binary, reflexive and symmetric function $s(x, y)$ with image on $[0,1]$ such that $s(x,x) = 1$ (strong reflexivity). The semantics of $s(x,y) > s(x,z)$ is that object y is more similar to object x than z. An instance of this model uses as s function Gower's similarity index [6]. This coefficient has its values in the real interval $[0,1]$ and for any two objects i, j given by tuples of cardinality n, is given by the expression

$$s_{ij} = \frac{\sum_{k=1}^{n} g_{ijk} \, \delta_{ijk}}{\sum_{k=1}^{n} \delta_{ijk}}$$

where:

- g_{ijk} is a similarity *score* for objects i, j according to their value for variable k. These scores are in the interval $[0,1]$ and are computed according to different schemes for numeric and qualitative variables. In particular, for a continuous variable k and any two objects i, j the following similarity score is used:

  $$g_{ijk} = 1 - \frac{|v_{ik} - v_{jk}|}{\text{range } (v_{\cdot k})}$$

  Here, v_{ik} denotes the value of object i for variable k and

  $$\text{range } (v_{\cdot k}) = \max_{i,j} (|v_{ik} - v_{jk}|)$$

  (see [6] for details on other kinds of variables).

- δ_{ijk} is a binary function expressing whether both objects are comparable or not according to their values w.r.t. variable k. It is 1 if and only if both objects have values different from $\mathcal{X}$ for variable k, and 0 otherwise.

As for the activation function, a modified version of the classical sigmoid is used, such that it maps the real interval $[0,1]$ on $(0,1)$.

$$f(x,p) = \begin{cases} \frac{-p}{(x-0.5)-a(p)} - a(p) & \text{if } x \leq 0.5 \\ \frac{-p}{(x-0.5)+a(p)} + a(p) + 1 & \text{otherwise} \end{cases}$$

$$a(p) = \frac{-0.5 + \sqrt{0.5^2 + 4 * p}}{2}$$

where $a(p)$ is an auxiliary function and $p > 0$ is a real-valued parameter controlling the curvature.

2.1 A Fuzzy Extension

A step forward in generalizing the previous specific model is a relaxation of real valued inputs, by considering more flexible situations, now tolerating imprecision. According to the conceptual setting of the family of neuron models studied based on similarity, it is natural to state a fuzzy extension following the same approach. Similarity relations from the point of view of fuzzy theory have been defined elsewhere [8], [16]. In the present case, the situation is not that of a fuzzy similarity or proximity relation defined on real values, but a relation between fuzzy entities. Let $\mathcal{F}_i$ be a family of normalized fuzzy sets from the source set and $\tilde{A}, \tilde{B} \in \mathcal{F}_i$ two fuzzy sets. The following similarity relation is used:

$$g(\tilde{A},\, \tilde{B}) = \max_x \, (\mu_{\tilde{A} \cap \tilde{B}}(x))$$

where

$$\mu_{\tilde{A} \cap \tilde{B}}(x) = \min \, (\mu_{\tilde{A}}(x),\, \mu_{\tilde{B}}(x))$$

Clearly it is reflexive in the strong sense and also symmetric. This is a proximity relation and can be used to include extra fuzzy components in Gower's similarity. Consider a collection of n_f extended fuzzy sets of the form $\hat{\mathcal{F}}_i = \mathcal{F}_i \cup \{\mathcal{X}\}$ and their cartesian product $\hat{\mathcal{F}}^{n_f} = \hat{\mathcal{F}}_1 \times \hat{\mathcal{F}}_2 \times \ldots \times \hat{\mathcal{F}}_{n_f}$. The resulting input set will then be $\hat{\mathcal{H}}^n =< \hat{\mathcal{R}}^{n_r}, \hat{\mathcal{F}}^{n_f}, \hat{\mathcal{O}}^{n_o}, \hat{\mathcal{M}}^{n_m} >$, where the cartesian products for the other kinds of source sets ($\hat{\mathcal{R}}_i, \hat{\mathcal{O}}_i, \hat{\mathcal{M}}_i$) are constructed in a similar straightforward way from their respective cardinalities n_r, n_o, n_m, with $\hat{\mathcal{R}}^0 = \hat{\mathcal{F}}^0 = \hat{\mathcal{O}}^0 = \hat{\mathcal{M}}^o = \hat{\mathcal{H}}^o = \phi$, $n = n_r + n_f + n_o + n_m$ and $n > 0$.

The training procedure for the resulting heterogeneous neuron –shown in Fig. 1– is based on genetic algorithms ([7], [2]) and can be devised in a natural way by extending that used for heterogeneous neurons without fuzzy inputs or weights [11]. In this extension, each fuzzy weight is characterized as a tuple of reals (instead of a single one) and this only needs a chromosome enlargement, depending on the chosen functional representation for fuzzy sets (trapezoidal, Gaussian or LR).

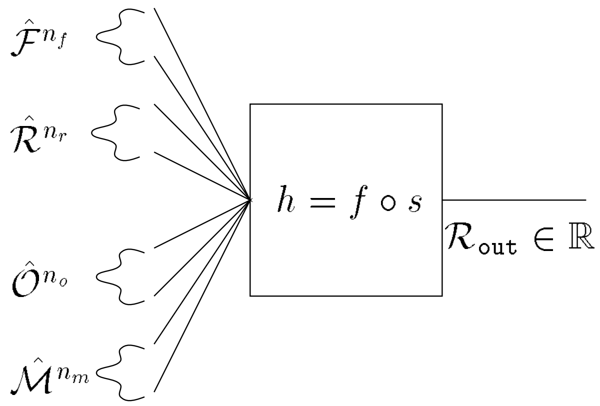

Figure 1: A fuzzy heterogeneous neuron.

3 An Example of Application in an Imprecise Domain

An environmental investigation in the tropics dealing with the detection of underground caves using geophysical measurements made at the surface of the earth was used to experiment with the extended approach described in the previous section. First, some words describing the problem are necessary.

Karstification is a peculiar geomorphological and hydrogeological phenomenon produced mostly by rock solution as the dominant process. As a consequence, earth's surface is covered by exotic irregular morphologies, like lapiaz, closed depressions (*dolinas*), sinks, potholes and the like, with the development of underground caves. This implies that the surface drainage network is usually poorly developed or simply does not exist at all, while vertical infiltration of rain waters forms an underground drainage system where water flows through fissures, galleries and caves. The studied area is located 30 km to the south of Havana City (Cuba) in the so called Havana-Matanzas Karstic Plain composed of porous, fractured and heavily karstified limestones of Middle Miocen age with abundance of a variety of clay minerals. Under the high temperatures and humidity typical of tropical conditions, weathering processes develop an overburden composed by reddish insoluble materials (*tera rossa*) coming from solution processes on the limestones.

Negative karst forms on the surface (the lapiaz, sinks, dolinas, etc.) are partially or totally covered by an overburden of variable depth. These forms often connect with caves in the underground, some of them big. Direct detection is very difficult or impossible and geophysical methods are necessary, as they are for tasks like geological mapping and construction of cross sections.

This is a very important problem from the point of view of civil engineering, geological engineering and environmental studies in general in this kind of regions.

In a selected square area (340 m side), geophysical methods complemented with a detailed topographic survey [10] were used with the purpose of characterizing the shallower horizons of the geological section and their relation with underlying karstic phenomena. Targets were zones of intense fracture and karstification, filled depressions, overburden pockets and the presence of underground caves. The set of geophysical methods included the spontaneous electric potential of earth's surface, the gamma radioactive intensity and the electromagnetic field in the VLF region of the spectrum [10]. In particular, two different surveys of spontaneous electric potential were performed, in the dry and rainy season respectively, since strong negative anomalies are due to infiltration potentials associated with electrochemical processes taking place as water infiltrates into the underground via fissures and joints. These four measurements, along with the surface topography, constitute the five variables to be used by the neural models. The complexity of these measured geophysical fields in the area is illustrated, as an example, by the distribution of gamma ray intensity and the surface topography. While radioactivity is highly noisy, topography shows few features. Both are shown in Figs. 2, 3.

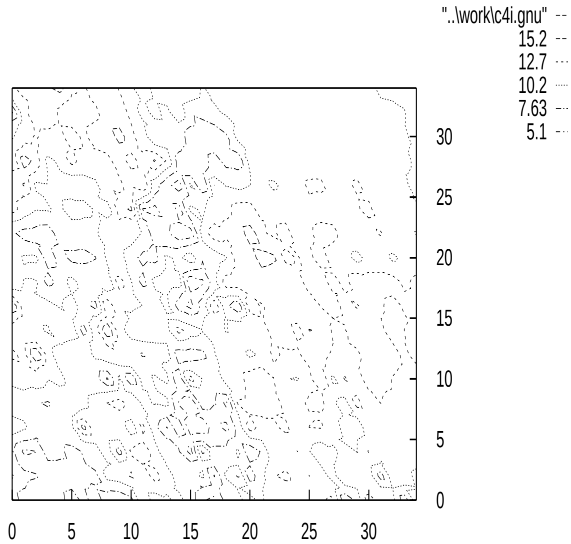

Figure 2: Distribution of gamma ray intensity in the studied area.

Geophysical survey methodologies consider independent sets of measurements in order to account for different kind of errors and the natural variability of such kind of information. In order to be considered acceptable, each survey must have an error no greater than 5% when comparing the original and the independent measurements. This means that the reported values of all geophysical fields (i.e, the available data), have an inherent uncertainty which must be considered. In the area, a gentle variation in geological conditions for both the bedrock and the overburden was suspected by geologists and also a large underground cave with a single gallery was known to exist in the central part of the area. The cave has about 300 meters long with cross sections ranging from less than one square meter in the narrowest part, to chambers having 40 meters wide and 30 meters high, reaching the surface in the form of a gorge in the bottom of a depression.

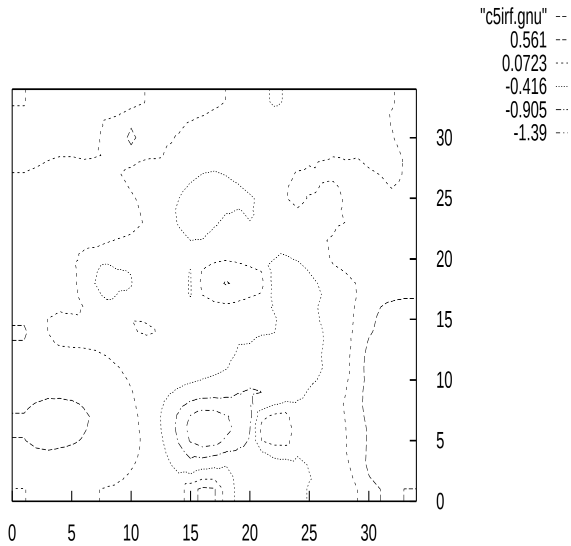

Figure 3: Surface topography of the studied area.

An isolation of the different geophysical field sources was necessary in order to focus the study on the contribution coming from underground targets, trying to minimize the influence of both the larger geological structures, and the local heterogeneities. According to the *a priori* geological ideas, each geophysical field was assumed to be described by the following additive two-dimensional model composed by trend, signal and random noise:

$$f(x,y) = t(x,y) + s(x,y) + n(x,y)$$

where f is the physical field, t is the trend, s the signal, and n the random noise component, respectively. In order to isolate an approximation of the signals produced by the underground target bodies, a linear trend term $t'(x,y) = c_0 + c_1\,x + c_2\,y$ was computed (by least squares) and subtracted from the original field. The residuals $r(x,y) = f(x,y) - t'(x,y)$ were then filtered by direct convolution with a low pass finite-extent impulse response two-dimensional filter in order to attenuate the random noise component [5]. Such convolution is given by:

$$s'(x,y) \;=\; \sum_{k_1=-N}^{N} \; \sum_{k_2=-N}^{N} \; h(k_1,k_2)\; r(x-k_1, y-k_2)$$

where $r(x,y)$ is the residual, $s'(x,y)$ is the signal approximation and $h(k_1,k_2)$ is the low-pass zero-phase shift digital filter.

4 Experiments

In order to study the behavior of these neural models, a comparison was made w.r.t. geological-geophysical accuracy of classification. This kind of knowledge, as well as results from previous non-supervised classification techniques [15] had shown the existence of two multivariate populations within the studied area: one representing more karstified zones with large interconnected underground cavities, and another in which karstification is not so intense. Since the hypothesis of two hyperspherical classes in pattern space was tenable, and the purpose of this work is to assess the relative merits of the three considered neuron models (classical, heterogeneous and fuzzy heterogeneous) in the task at hand (imprecise classification using data which are also imprecise), a network consisting of a *single* neuron was the architecture selected. Clearly, other multilayer layouts are possible and should deserve future attention, but is a good reference for initial comparisons. This, together with the small training set (relative to test), should make the problem much more difficult than it really is, so the differences should be more evident.

The experiments were conceived in two phases as follows. In phase one, a comparison is made between the classical real neuron with the heterogeneous one with real inputs and weights. In a second stage, the latter is compared to the fuzzy heterogeneous neuron. Also, the experiments were designed following geological criteria. From this point of view it is known that the number of observable caves in any karstic area is only a small fraction of the actually existing ones, making class structure itself *imprecise*, a situation usual in complex problems like those from environmental studies. Moreover, there are no sharp boundaries between rock volumes containing caves and those containing less or none. One could say that the notion of "caveness" degrades smoothly, which is another reason to use fuzzy models.

The training was supervised (in the usual mean squared error sense) by the information given by the topographic map of a large cave present in the area, so that those surface measurement points lying *exactly above* the known cave were considered as class 1 patterns and those outside as belonging to class 2 (the resulting cave is shown in figure 4). This procedure for class

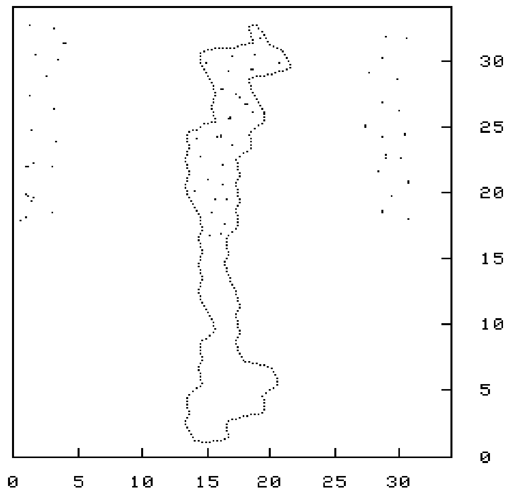

Figure 4: The known cave borders: see text for an explanation of what is considered as cave and what is not. Dots indicate the (approximate) location of the points used for training.

assignment was too conservative but, otherwise, one would have been forced to provide as output the exact caveness degree for each point. This value, besides being very difficult to estimate, would have introduced a strong subjective bias. The computation of this degree is precisely the task we want the model to perform.

Selected data from the northern half were used for training, whereas the rest was used for testing the trained network (consisting of a single neuron only). More precisely, the training set was composed by the 31 points from the northern half located exactly above the known cave (representing class 1), plus 32 others homogeneously distributed in the east-west sides. As test set we used the remaining 567 patterns from the whole area (*it est*, train = 10%, test = 90%).

4.1 Phase 1

Here we have a classical real-valued neuron (in this study, having scalar product as net input and hyperbolic tangent as a squashing activation function). The training procedure for this neuron is a combination of conjugate gradient with simulated annealing [9], whereas the heterogeneous neuron is trained using a standard genetic algorithm with the following characteristics: binary-coded values, probability of crossover: 0.6, probability of mutation: 0.01, number of individuals: 50, linear scaling with factor: 1.5, selection

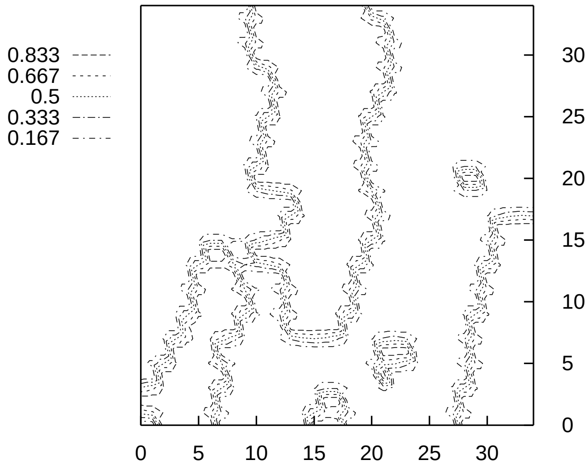

Figure 5: Results of phase 1: α-cut sets for the classical neuron.

mechanism: tournament.

The results obtained by both models are shown in figures 5 and 6, respectively, where caveness prediction is plotted in five equally spaced α-cut sets. Clearly, the distribution of the two-dimensional sets for the heterogeneous neuron reflects much better the distribution of the known cave than the classical neuron, for various reasons. First, the classical neuron fails to detect the southernmost part of the known cave, whereas the heterogeneous counterpart does.

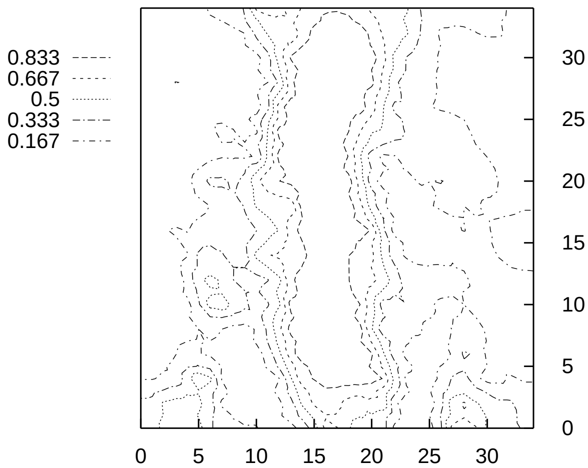

Figure 6: Results of phase 1: α-cut sets for the heterogeneous neuron.

Second, the classical neuron predicts complete cave areas in the south-east and south-west zones, which are misleading. These are also signaled by the heterogeneous neuron, but always with a degree of 0.5 or less. The only exception is a small area located in coordinates $(7-8, 12-15)$, where other geophysical methods (seismic and DC-resistivity) not used in this study had signaled cave anomalies. And third, the general layout of the actual cave (north-south main axis, slightly bended and narrower in the middle part) is better reflected by the heterogeneous neuron.

4.2 Phase 2

In a second stage, a fuzzy heterogeneous neuron was trained in the same experiment setting, but this time using fuzzy inputs. This means that all neuron weights were *fuzzy sets* (actually triangular fuzzy numbers), and both training and test vectors represented by fuzzy numbers (the mode was given by the corresponding observed value, and the spread a $\pm 5\%$ of it). This is in accordance with the upper bound of the measurement errors reported for the geophysical field surveys made. It should be noted that this criteria was conservative, since some surveys actually have had less than 5% of error.

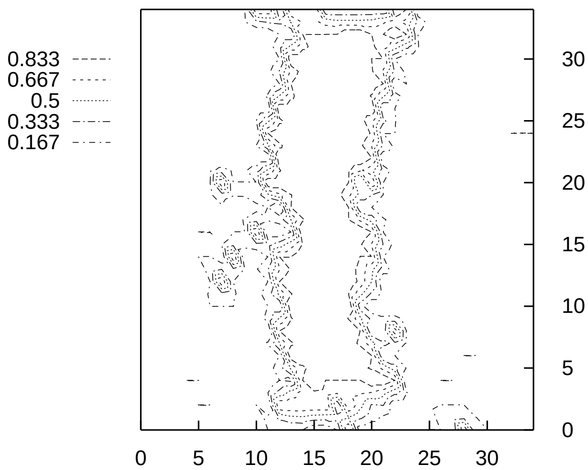

Figure 7: Results of phase 2: α-cut sets for the fuzzy heterogeneous neuron.

The results (shown in figure 7) are again qualitatively satisfactory, in what regards to the general layout of the cave. But now a quantitative factor comes into the picture: the cave is much more neatly defined, a fact that shows in two ways: first, the different α-cut sets are closer, indicating a gradual (they are clearly distinguishable) but firm transition from 0 to 1 of 2 units in the map on average, equal to about 20m in the field, a very reasonable value. That is, this narrow belt w.r.t. the trace of the known cave represents the transition zone between the rock volumes, more and lesser affected by big underground cavities. Second, the extensive anomalous zones predicted by the heterogeneous neuron in the eastern and south-western zones completely disappear, with the exception of a small region in coordinates $(25-30, 0-2)$, which should be specifically checked. What is more, the strongest region where the presence of a secondary cave is signaled by the fuzzy heterogeneous neuron is precisely the one aforementioned and confirmed to exist by other means. This a nice result, since allowing imprecise inputs and weights for all of the five variables does not degrade the overall performance. On the contrary, the results can be said to be even more accurate. Notice that all of the neurons are using the same small training set but, in practice, this situation is less favourable for the fuzzy neuron, which would need an enlarged training set to compensate for the imprecision.

5 Conclusions

A theoretical framework for a class of heterogeneous fuzzy neuron models and concrete instances and realizations of these have been set forth. These models are characterized by their built-in treatment of information coming from heterogeneous sources (perhaps missing) and make use of an explicit *similarity* measure between entities, specific for each source. Other realizations of these models have been presented elsewhere, in which their possibilities are further explored, ranging from classification benchmarking [11], time-series prediction [12], [13] and system identification [14]. In the work presented, experiments made with complex multivariate space-dependent data –coming from a real world problem in the domain of environmental studies– have shown that allowing imprecise inputs and using heterogeneous fuzzy neurons based on similarity yields models more accurate (because of their greater flexibility) than those from classical crisp real-valued models, in a problem for which one is not so much interested in crude train/test set classification errors (which could well have been presented) but in its ability to model the imprecise structure of the domain. This represents only a preliminary although promising class of models that is serving as an initial standpoint which deserves further investigation.

References

[1] Chandon, J.L, Pinson, S: Analyse Typologique. Théorie et Applications. Masson, (1981), 254.

[2] Davis, L.D.: Handbook of Genetic Algorithms. Van Nostrand Reinhold, (1991).

[3] Dubois D., Esteva F., García P., Godo L., Prade H.: A logical approach to interpolation based on similarity relations. Instituto de Investigación en Inteligencia Artificial. Consejo Superior de Investigaciones Científicas, Barcelona, España. Reporte de Investigación IIIA 96/07, (1996).

[4] Dubois D., Prade H., Esteva F., García P., Godo L., López de Mántaras R: Fuzzy set modeling in case-based reasoning. *Intl.. Journal of Intelligent Systems* (to appear) (1997).

[5] D.E. Dudgeon, R.M. Mersereau. Multidimensional Signal Processing. Prentice Hall, New Jersey, 1984.

[6] Gower, J.C. A General Coefficient of Similarity and some of its Properties. *Biometrics 27, 857-871*, 1971

[7] Goldberg, D.E.: Genetic Algorithms for Search, Optimization & Machine Learning. Addison-Wesley, (1989) .

[8] G.J. Klir, T.A. Folger. Fuzzy Sets, Uncertainty and Information. Prentice Hall Int. Editions, 1988.

[9] Ackley, D. *A connectionist machine for genetic hillclimbing* Kluwer Acad. Press, 1987.

[10] Valdés J.J, Gil J. L. Joint use of geophysical and geomathematical methods in the study of experimental karst areas. *XXVII International Geological Congress*, pp 214, Moscow, 1984.

[11] Valdés J.J, García R. A model for heterogeneous neurons and its use in configuring neural networks for classification problems. *International World Conference Artificial and Natural Neuron Networks* IWANN'97, 19-26 (1997) .

[12] Belanche, Ll., Valdés, J.J., Using Fuzzy Heterogeneous Neural Networks to Learn a Model of the Central Nervous System Control. Accepted for publication at *EUFIT'98, 6th European Congress on Intelligent Techniques and Soft Computing.* Aachen, Germany.

[13] Belanche, Ll., Valdés, J.J., Alquézar, R. Fuzzy Heterogeneous Neural Networks for Signal Forecasting. Accepted for publication at *ICANN'98, Intl. Conf. on Natural and Artificial Neural Networks.* Skövde, Sweden.

[14] Belanche, Ll., Valdés, J.J, Comas, J., Roda, I., Poch, M. Modeling the Input-Output Behaviour of Wastewater Treatment Plants using

Soft Computing Techniques. Accepted for publication at *BESAI'98, Binding Environmental Sciences and AI*, W7 held as part of *ECAI'98, European Conference on Artificial Intelligence*. Brighton, UK.

[15] Valdés J.J. Fuzzy clustering and multidimensional signal processing in environmental studies: detection of underground caves in the tropics. *European Congress on Intelligent Techniques and Soft Computing. Proceedings EUFIT'97*, Aachen, (1997) .

[16] Zimmermann H.J.: Fuzzy set theory and its applications. Kluver Academic Publishers, pp 399, 1992.