VISCOPLASTIC CONSTITUTIVE MODELS FOR ZERO-THICKNESS INTERFACE ELEMENTS, FORMULATION AND APPLICATIONS

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Key words: Viscoplasticity, Interface elements, Finite Element Method (FEM), Viscoplastic relaxation.

Abstract. An energy-based work-softening visco-plastic model for zero-thickness interface elements has been developed as an extension of an existing elastic-perfectly-viscoplastic formulation. In the inviscid limit the model also collapses into a well-established fracture mechanics-based elasto-plastic model. The new model is verified satisfactorily for common loading cases at interfaces such as pure tension (mode I) opening, and shear-compression (mixed-mode) sliding, with results that in the long term match the predictions of the fracture mechanics inviscid model.

1 INTRODUCTION

Zero–thickness interface elements, sometimes also called "cohesive elements", were introduced by Goodman [1] for geotechnical analysis using the FEM, and in more recent times are becoming popular for their many possible applications. In the simplest scenario of linear elastic behaviour, interface elements inserted in between standard continuum elements may be used to represent the presence of a thin, deformable layer without the need to use extremely fine meshes. If considered linear elastic but assuming sufficiently high stiffness values, interface elements may lead to stresses and deformations of the continuum elements practically identical to those that would be obtained in the same domain without any interface elements inserted. And that may be used as a means to obtain, besides the regular stresses and deformations of the continuum, also the normal and shear stress tractions transmitted across the planes in which interface elements have been inserted. Stress tractions may be of high interest for instance along interfaces in between different material layers or in between different materials. If equipped with non-linear constitutive laws exhibiting a maximum strength condition, interface elements may also be used to represent frictional sliding planes, cracks or fractures. For this purpose, the precise type and characteristics of the constitutive law are essential. A frictional contact surface may be represented with perfect plasticity, while a developing crack will require a constitutive model with softening which incorporates fracture energy parameters. In the late 90s, a model of this type was proposed by the research group [2],

and the same model was later developed further and improved [3].

A useful extension of the zero-thickness interface elements and constitutive laws is in the field of visco-plasticity [4]. Potential applications range from representation of physical time-dependent behavior (such as for instance failure under sustained load), to purely numerical strategies such as visco-plastic relaxation. In recent years, an elastic-perfectly visco-plastic interface model was proposed [5], together with a discussion on the schemes necessary for the various possible applications. In the inviscid limit (time tending to infinity) the model response approached an elastic-perfectly plastic version of the group's fracture model.

The rate-dependent (viscoplastic) formulation requires a time integration strategy to discretize time in increments and evaluate linearized relation between stress and strain increments for each time step. Usually, the algorithms proposed in the literature are strain-driven, i.e. they are based on the initial stress scheme used in Finite Elements, in which the strain increments are prescribed to the constitutive equations, which then return the resulting stress [6-7-8].

In contrast, stress-driven schemes are less common. After the original constant-stress implementation of Zienkiewicz and Cormeau [9] that may be considered the most elementary form of stress-driven viscoplastic schemes, to the knowledge of the authors, the only previous proposals of this type are [4]. The implementation of stress-prescribed schemes is conceptually much simpler and numerically advantageous (explicit integration of the constitutive equations and simple coding).

The main objective of this paper is to describe and demonstrate an energy-based softening visco-plastic model for zero-thickness interface elements using a stress-prescribed integration algorithm for Perzyna viscoplasticity in a FE framework. In the inviscid limit the predictions of the model approach those of the full version of the fracture-based elasto-plastic model previously proposed by the group.

2 ENERGY-BASED WORK-SOFTENING VISCO-PLASTIC MODEL FOR ZERO-THICKNESS INTERFACE ELEMENTS

As already mentioned, the new energy-based softening visco-plastic model for zero-thickness interface elements is developed as an extension of an existing elastic-perfectly-viscoplastic formulation [5], which is based on a hyperbolic cracking surface. This model was proposed originally for the behaviour of geotechnical interfaces [10], later modified for fracture energy-based opening and development of cracks in quasi-brittle materials (concrete, rock, etc) [2], and more recently extended to 3D and reformulated more efficiently [3].

2.1 Elasto-plastic interface constitutive model

The existing fracture-based interface constitutive law, named *Normal/Shear Cracking Model*, is based on the theory of elasto-plasticity and it incorporates concepts of fracture mechanics and fracture energies. Its behavior is formulated in terms of normal and shear components on the interface plane $\boldsymbol{\sigma} = [\sigma_N, \sigma_T]^T$ and their respective relative displacements $\boldsymbol{r} = [r_N, r_T]^T$. The interface fracture model is based on a hyperbolic cracking surface [2], Fig.1a. The corresponding yield (cracking) function F is defined in terms of normal and shear stresses and three geometric parameters included in vector $\boldsymbol{p}$: the strength parameters cohesion (c), uniaxial tensile strength (χ) and internal friction angle ($tan\phi$). The algebraic expression of F

has been changed in some versions of the model, the most convenient [3] being:

$$F(\boldsymbol{\sigma}, \boldsymbol{p}) = -(c - \sigma_N tan\phi) + \sqrt{\sigma_T^2 + (c - \chi tan\phi)^2} \qquad (1)$$
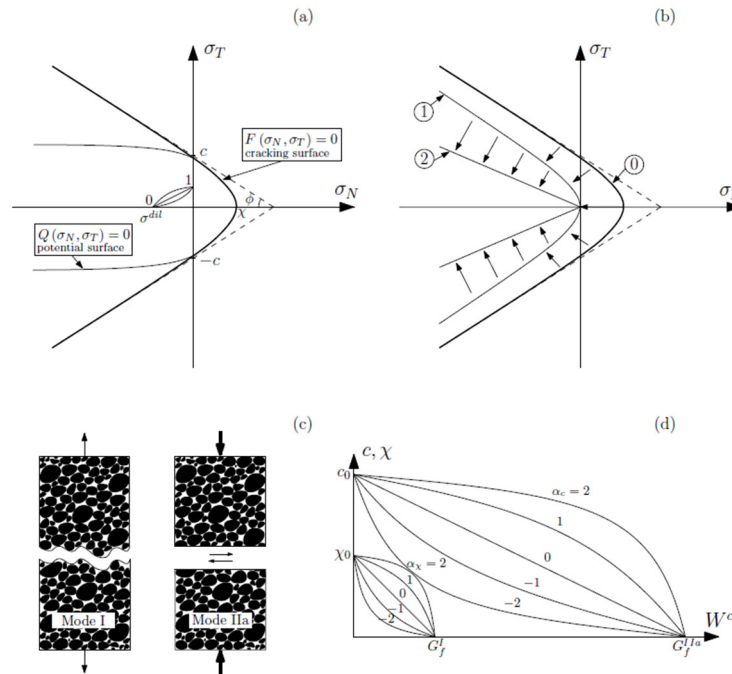

Figure 1 Interface model: (a) cracking surface and plastic potential, (b) evolution of cracking surface, (c) fundamental modes of fracture and (d) softening laws for c and χ.

The hardening/softening laws (evolution laws of the surface geometric parameters $c, \chi, tan\phi$) are formulated in terms of a single history variable, W^{cr}, defined as the energy spent in fracture processes. These laws, represented in Fig.1d, include as parameters the classical fracture energy in Mode I, G_f^I (pure tension) and a second mode named Mode IIa defined under shear and high compression without dilatancy, G_f^{IIa} (Fig.1c). The history variable work is defined incrementally as:

$$dW^{cr} = \begin{cases} \boldsymbol{\sigma} : d\boldsymbol{r}^{cr} & \sigma_N \geq 0 \; (tension) \\ (\sigma_T + \sigma_N tan\phi) dr_T^{cr} & \sigma_N < 0 \; (compression) \end{cases} \qquad (2)$$

Equations (2) show that, in the case of tensile-dominated cracking, all plastic work dissipated counts towards the history variable, while in compression-shear cracking, there is also a frictional part which is excluded from W^{cr} (Eq.2). Note that, with the definitions above, the evolution of the cracking surface is as depicted in Fig. 1b: from configuration "0" with initial tensile strength χ_0 and asymptotic cohesion c_0, as the history variable reaches $W^{cr} = G_f^I$ the surface moves to configuration "1" with zero tensile strength, and as it approaches $W^{cr} = G_f^{IIa}$ the surface moves toward configuration "2" which corresponds to a pair of straight lines of residual friction and no cohesion.

2.2 Perzyna viscoplastic constitutive model for the interface.

The visco-plastic version of the interface model is based on the assumption that the total visco-plastic "strain" (relative displacements) can be split into the elastic part $\boldsymbol{r}^{el}$ and the viscoplastic part $\boldsymbol{r}^{vp}$ [5]:

$$\boldsymbol{r} = \boldsymbol{r}^{el} + \boldsymbol{r}^{vp} \tag{3}$$

The elastic part is assumed to be related to stresses via isotropic linear elasticity:

$$\boldsymbol{r}^{el} = \boldsymbol{D}_0^{-1} : \boldsymbol{\sigma} \tag{4}$$

where $\boldsymbol{D}_0$ is the elastic stiffness matrix, symmetric and positive definite, and $\boldsymbol{D}_0^{-1}$ indicates its inverse, which is the elastic compliance matrix and it may be also denoted as $\boldsymbol{C}_0$.

The main change with respect to the previous perfect visco-plastic model proposed [5] is that in the current implementation the geometric parameters of the surface will evolve in a way similar to the inviscid elasto-plastic model of the previous section. A history variable W^{vp} is defined which is similar to the previous W^{cr} (except that visco-plastic "strain" replaces plastic strain), i.e.:

$$dW^{vp} = \begin{cases} \boldsymbol{\sigma} : d\boldsymbol{r}^{vp} & \sigma_N \geq 0 \ (tension) \\ (\sigma_T + \sigma_N tan\phi)dr_T^{vp} & \sigma_N < 0 \ (compression) \end{cases} \tag{5}$$

and the geometric parameters of the surface $\boldsymbol{p}$ (composed by $c, \chi, tan\phi$) are assumed to evolve with W^{vp} in a way identical as they did with W^{cr} in the inviscid elasto-plastic model (Fig. 1d).

The cracking surface $F(\boldsymbol{\sigma}, \boldsymbol{p}) = 0$ determines the limit between elastic state ($F \leq 0$) and the visco-plastic state ($F > 0$) state, in which a Perzyna visco-plastic "strain" rate is assumed:

$$\dot{\boldsymbol{r}}^{vp} = \frac{1}{\eta} \left\langle \psi \left(\frac{F(\boldsymbol{\sigma}, \boldsymbol{p})}{F_0} \right) \right\rangle \boldsymbol{m} \tag{6}$$

where η is the viscosity of the material, F_0 is a reference value of the yield surface, Q is the visco-plastic potential typical of non-associated formulations and the flow rule is $\boldsymbol{m} = \partial Q / \partial \boldsymbol{\sigma}$.

Finally, the accumulated visco-plastic strain $\Delta \boldsymbol{r}^{vp}$ (Eq.3) can be obtained by integrating in time the visco-plastic strain rate, i.e.:

$$\Delta \boldsymbol{r}^{vp} = \int_{t_0}^{t_1} \dot{\boldsymbol{r}}^{vp} \, dt = \int_{t_0}^{t_1} \frac{1}{\eta} \left\langle \left(\frac{F(\boldsymbol{\sigma}, \boldsymbol{p})}{F_0} \right) \right\rangle \boldsymbol{m} \ \ dt \tag{7}$$

For the numerical integration of the new interface visco-plastic constitutive model, the methodology is a generalization of the procedure previously proposed for perfect visco-plasticity [5]. In that integration scheme it is assumed that the stress increment $\Delta \boldsymbol{\sigma}$ (that takes place during a time increment Δt) is prescribed, and the corresponding "strain" (relative

displacement) is calculated. In this case, though, the geometric parameters of the yield function, $\boldsymbol{p}$ (composed by $c, \chi, tan\phi$) are not constant, but have to be continuously updated using the softening laws in terms of W^{vp}, which is also continuously changing. This requires a sub-stepping scheme. In the case that for a given substep $F(\boldsymbol{\sigma}_{fin}, \boldsymbol{p}_{fin}) > 0$, the integral of Eq.(7) is approximated by first order expansion as:

$$\Delta\boldsymbol{r}^{vp} = \frac{\Delta t}{\eta F_0}\left((1-\theta)F(\boldsymbol{\sigma}_{ini}, \boldsymbol{p}_{ini})\, \boldsymbol{m}(\boldsymbol{\sigma}_{ini}, \boldsymbol{p}_{ini}) + \theta F\left(\boldsymbol{\sigma}_{fin}, \boldsymbol{p}_{fin}\right)\boldsymbol{m}\left(\boldsymbol{\sigma}_{fin}, \boldsymbol{p}_{fin}\right)\right) \qquad (8)$$

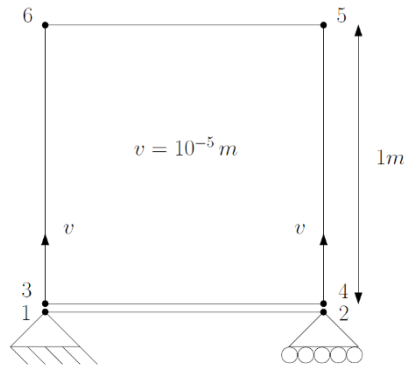where $\boldsymbol{n} = \partial F / \partial\boldsymbol{\sigma}$ and θ is a constant scalar factor that may take a fixed value between 0 and 1. This expression, properly developed, leads to the relation between $\Delta\boldsymbol{\sigma}$ and $\Delta\boldsymbol{r}$, which provides the constitutive tangential compliance and initial strain vector necessary for the iterative calculations. If $\theta = 0$, the formula above is equivalent to the forward Euler scheme. The other limit case is when $\theta = 1$ and the formula is equivalent to the traditional backward Euler scheme. Note also that, except for the case $\theta = 0$ in which all variables are known at the beginning of the increment, for any other value of $\theta > 0$ the calculation will require iterations at structural level, because the expression involves the stresses at the end of the increment, which are not known *a priori*.

3 VERIFICATION EXAMPLES

A pure tension and shear-compression simple examples have been considered in this section for verification purposes, and the results have been compared with those that would be obtained with the elastoplastic law, with which they should coincide for long visco-plastic times (inviscid limit of the visco-plastic model).

3.1 Uniaxial tension opening

The first example consists of a single interface element subject to uniaxial tension (Fig.2). A square continuum element has been also included in the discretization, although it does not play any mechanical role. Loading is composed of an alternate sequence of two kinds of steps, the first one consisting of a prescribed displacement increment (applied instantaneously), followed by the pass of a long time (which takes place at constant nodal displacements).

Interface properties

$Kn = Kt = 10^6 \, kPa/m$
$TanPhi = 0.7$
$Xi = 2.0 \, kPa$
$C = 2.0 \, kPa$
$GfI = 0.00001 \, kN/m$
$GfIIa = 0.0001 \, kN/m$
$\eta = 10^6 \, kPa \cdot s$

Figure 2. Geometry and properties of the uniaxial tension example. Prescribed displacements are imposed on nodes 3 and 4.

The results of the visco-plastic calculation are represented in Fig.3 in red color lines, together with the results obtained with the elastoplastic interface constitutive law in blue [2]. As shown in the figure, for each instantaneous load step, the visco-plastic response consists first of an elastic stress increment, which is then followed by progressive relaxation as time passes, leading in the long term to the same stress value as predicted by the fracture mechanics inviscid model for the same relative displacement. Similar behaviour is observed for each one of the prescribed displacement increments. Therefore, it can be concluded that in this case the proposed visco-plastic model in the visco-plastic limit clearly coincides with the existing elasto-plastic model.
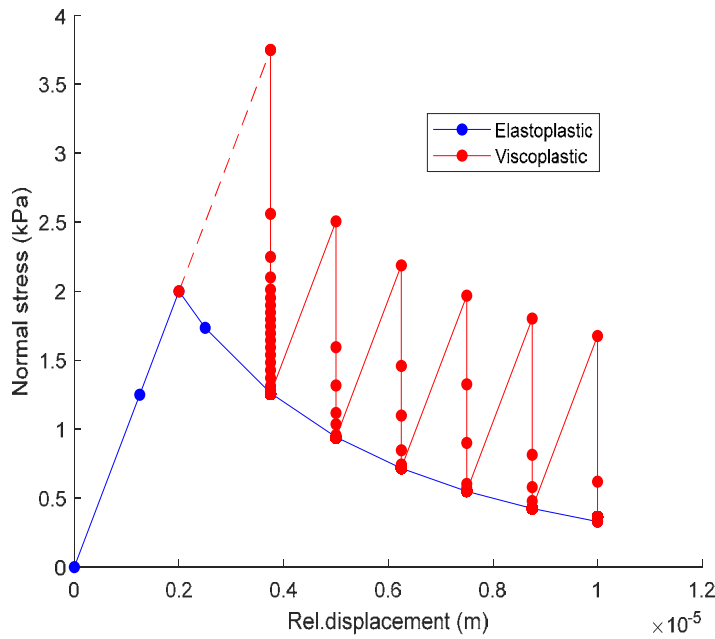

Figure 3. Normal stress-normal relative displacement evolution curve of the interface element. Viscoplastic results in the long term match the predictions of the fracture mechanics inviscid model.

The formulation proposed also provides the energy spent in the fracture process. The following table (Table 1) shows the comparison between the energies obtained with the elasto-plastic and with the visco-plastic models. As it can be seen, these values turn out very similar as expected.

Table 1: Comparison between energy values computed in both cases elastoplastic and viscoplastic. The total energy obtained in both cases are very similar as expected.

	Elasto-plastic model (kPa·m) W^{cr}	Visco-plastic model (kPa·m) W^{vp}
Energy dissipated in fracture processes	$8.35067 \cdot 10^{-6}$	$8.34314 \cdot 10^{-6}$

3.2 Shear-compression sliding

The second example consists of a shear test applied on the same geometry and with the same material parameters as in previous section. The loading (Fig.4) consists of two parts: first, applying an instantaneous vertical stress on the top face of the continuum element (Fig.4a) and, second, applying the horizontal loading. Similarly as in example one, the horizontal loading (Fig.4b) is applied as a sequence of small horizontal displacements prescribed to nodes 3 and 4 instantaneously, followed each of them by a number of time increments until stresses would be totally relaxed.
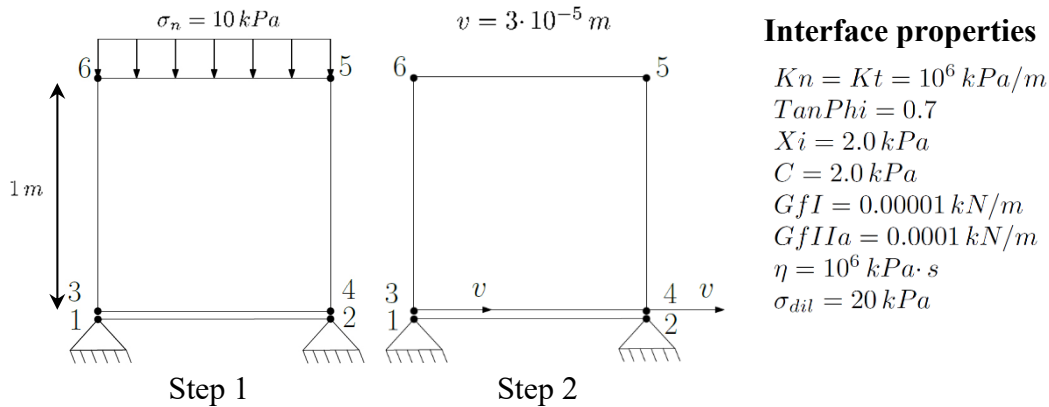

Figure 4. Geometry of the shear-compression sliding example. An instantaneous vertical stress is applied on the top face and also an instantaneous displacement is imposed on nodes 3 and 4. Also interface properties are shown.

The results of the visco-plastic calculations are represented in Fig.5a (red color lines), together with the results obtained with the existing elasto-plastic model (in blue) [2]. Fig.5b shows the evolution of the dilatancy, also computed with both elastoplastic and viscoplastic models. Similar to the previous case, the results show that in the inviscid limit the visco-plastic formulation approaches nicely the predictions of the corresponding elasto-plasticity.
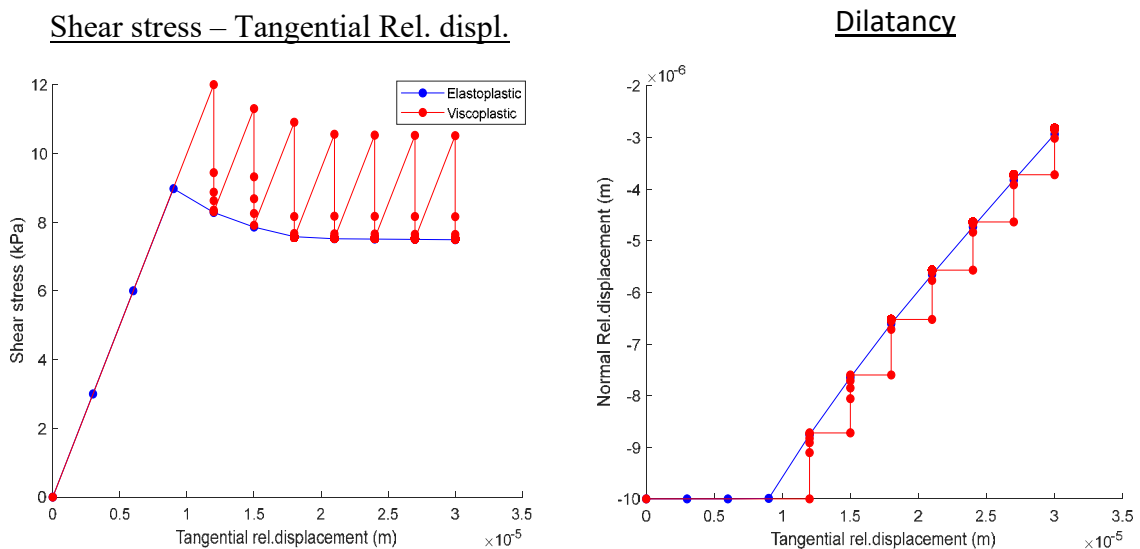

Figure 5. (a) Shear stress-tangential relative displacement evolution curve of the interface element. (b) Evolution of the dilatancy. Viscoplastic results in the long term match the predictions of the fracture mechanics inviscid model.

As in the previous example, the energy spent in the fracture process also has been calculated in this case. The following table (Table 2) shows the comparison between the energies obtained with the elastoplastic model and with the viscoplastic one, which turn out very similar as expected.

Table 2: Comparison between energy values computed in elastoplastic and viscoplastic models.

	Elasto-plastic model (kPa·m) W^{cr}	Visco-plastic model (kPa·m) W^{vp}
Energy dissipated in fracture processes	$1.56693 \cdot 10^{-5}$	$1.58374 \cdot 10^{-5}$

4 CONCLUDING REMARKS

In the present paper, an energy-based softening visco-plastic model of the Perzyna type for zero-thickness interface elements has been described, as well as its basis for numerical implementation using a stress-prescribed integration algorithm. Numerical results have been also presented for two simple examples of application, the first one consisting of a uniaxial tensile interface opening (mode I), and the second one a shear-compression (mixed-mode) failure with sliding. Both may represent academic problems of rock samples with discontinuities. The results show that in the long term the visco-plastic results match very well the predictions of the fracture mechanics inviscid model.

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