XIV International Conference on Computational Plasticity. Fundamentals and Applications
COMPLAS XIV
E. Oñate, D.R.J. Owen, D. Peric & M. Chiumenti (Eds)

LATTICE MODEL FOR FAILURE BASED ON EMBEDDED STRONG DISCONTINUITIES IN DYNAMIC FRAMEWORK

Mijo Nikolic*, Jakov Cesic†, Adnan Ibrahimbegovic†† and Zeljana Nikolic†

* University of Split
Faculty of Civil Engineering, Architecture and Geodesy
Matice hrvatske 15, 21000 Split, Croatia
e-mail: mijo.nikolic@gradst.hr

†University of Split
Faculty of Civil Engineering, Architecture and Geodesy
Matice hrvatske 15, 21000 Split, Croatia

††Chair for Computational Mechanics
UT Compiègne/Sorbonne Universités
Laboratoire Roberval de Mécanique
Centre de Recherches Royallieu, 60200 Compiègne, France

Key words: localized failure, lattice model, embedded strong discontinuities, dynamics

Abstract. Identifying the failure of materials and structures is still a challenging task and there is no unique approach or model to tackle this problem. Complexities that arise in failure modelling are numerous, starting from mesh dependency for softening to various numerical difficulties and instabilities, tracking algorithms, multiple cracking with crack interactions etc. Failure in the dynamic framework is even more challenging bringing inertial effects, crack branching etc. In this paper, lattice model for dynamic failure is presented. The final goal is to simulate crack initiation and propagation in 2D brittle and quasi-brittle structures exposed to dynamic environment. The strength of the lattice models is in their successful representation of failure mechanisms. The presented model is based on a triangular lattice of Timoshenko beams which act as cohesive links between the Voronoi cells used to compute beam cross sections. The embedded strong discontinuities in axial and transversal beam directions serve for representation of failure mechanisms in modes I and II, while mass and inertial effects are included into lattice network.

1 INTRODUCTION

Modeling of failure mechanisms and fracture are still challenging topics and are important for many applications. The main difficulties arise from non-smooth solution character and one needs to deal with discontinuities in displacement (or strain) fields [1]. Among different approaches to tackle fracture and failure mechanisms, lattice element models are a class of discrete models which have been widely used to simulate failure in terms of localized crack initiation or propagation [2]. Moreover, it is possible to account for heterogeneities in material structure, multiple crack propagation, crack coalescence with respect to heterogeneities. This is suitable for failure of heterogeneous materials, such as rocks or concrete, but the approach can also be used to simulate the failure of structures or solids. The main idea is to use spatial beam elements, which are geometrically built using Delaunay triangulation inside the domain of interest. The Delaunay edges in triangulation can be converted into lattice elements representing cohesive links between the Voronoi cells which are dual to Delaunay triangulation (Figure 1.a). Such Voronoi cells represent units of material in the domain, while lattice elements are here Timoshenko beams. Each beam in domain has its own geometrical properties; namely cross section is extracted from common area between the two neighboring Voronoi cells (Figure 1.b). In order to represent failure of single cohesive link between the Voronoi cells, we enhance the Timoshenko beams with embedded strong discontinuities, which provide jump in the displacement fields [1]. Introduction of embedded strong discontinuity into beams axial direction is related to mode I opening, while mode II is related to discontinuity in transversal direction. Such model for quasi-static crack propagation is given in [3, 4, 5]. The focus in this work is to enhance the existing formulation and to give the framework for dynamic crack propagation. Notable papers which deal with dynamic crack propagation where displacement fields are enriched to provide non-smooth discontinuous solutions with XFEM (Extended Finite Element Method) are [6, 7], or with embedded strong discontinuities in solid elements [8]. Lattice model with Reissner's beams as lattice elements for dynamic crack propagation is given in [9]. The outline is as follows. Section 2 explains the numerical model for crack propagation in dynamic environment. Section 3 provides representative numerical simulation. Conclusions are given at the end of paper.

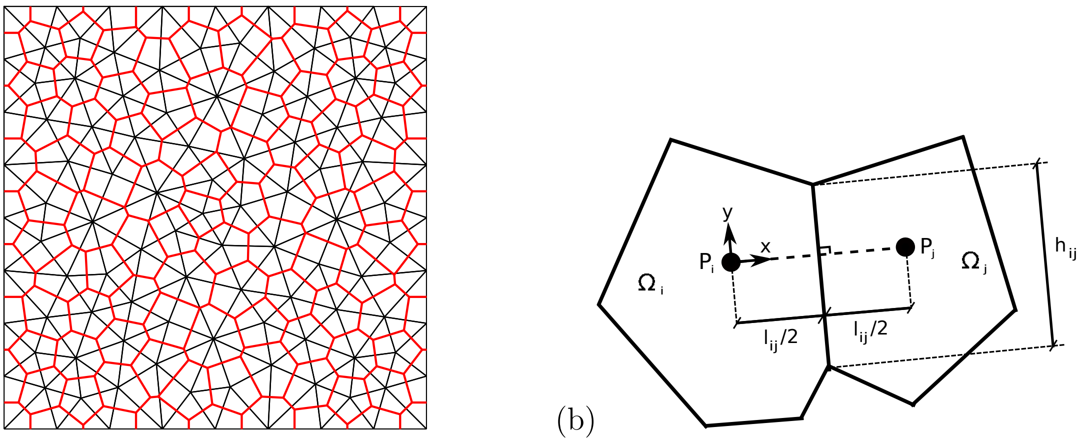
(a) 

Figure 1: (a) Structure of discrete lattice model with Voronoi cells as units of heterogeneous material and cohesive links between them (b) two neighbouring Voronoi cells

2 NUMERICAL MODEL

As already indicated above, model is assembled from Voronoi cells which are kept together by Timoshenko beams as cohesive links. Failure of Timoshenko beams occur when certain threshold is reached. Furthermore, plasticity softening regime is triggered together with activation of embedded discontinuities in axial and transversal direction of beams. The strains for standard Timoshenko beams of length l_e and cross section A are

$$\boldsymbol{\epsilon}(x) = \begin{bmatrix} \epsilon(x) = \frac{du}{dx} \\ \gamma(x) = \frac{dv}{dx} - \theta \\ \kappa(x) = \frac{d\theta}{dx} \end{bmatrix} \tag{1}$$

Standard kinematics is enhanced with additional degrees of freedom in the element interior multiplied by Dirac function. Thus, enhanced beam element with standard and additional degrees of freedom is constructed (Figure 2.) The non-regular strain field can be written

$$\boldsymbol{\epsilon}(x) = \bar{\boldsymbol{\epsilon}}(x) + \boldsymbol{\alpha}\delta_{x_c} = \begin{bmatrix} \bar{\epsilon}(x) \\ \bar{\gamma}(x) \\ \bar{\kappa}(x) \end{bmatrix} + \begin{bmatrix} \alpha_u \\ \alpha_v \\ 0 \end{bmatrix} \delta_{x_c} \tag{2}$$

Finite element interpolation for beam element with interpolation functions $\{N_1(x) = 1 - \frac{x}{l_e},\, N_2(x) = \frac{x}{l_e}\}$ and their derivatives $\{B_1^d(x) = -\frac{1}{l_e},\, B_2^d(x) = \frac{1}{l_e}\}$ produces enhanced strain field which can be written in matrix form

$$\boldsymbol{\epsilon} = \mathbf{B}\mathbf{d} + \mathbf{G}\boldsymbol{\alpha}, \tag{3}$$

where $\mathbf{B}$ is a strain displacement matrix and matrix $\mathbf{G}$ contains interpolation function G for discontinuity (Figure 2).

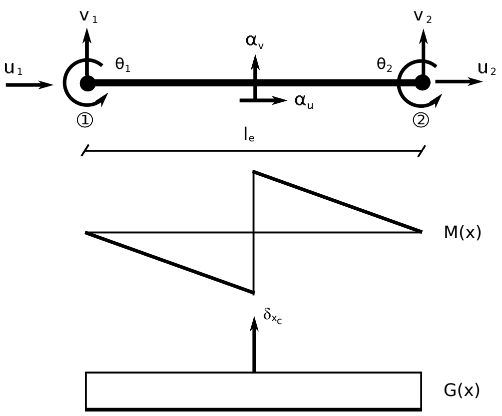


Figure 2: Enhanced Timoshenko beam element with standard and additional degrees of freedom. M and G are interpolation functions for discontinuity

Such enhancement of strain field produces the localized failure of cohesive links between Voronoi cells (Figure 3.) Virtual strain field can be written with the same interpolations

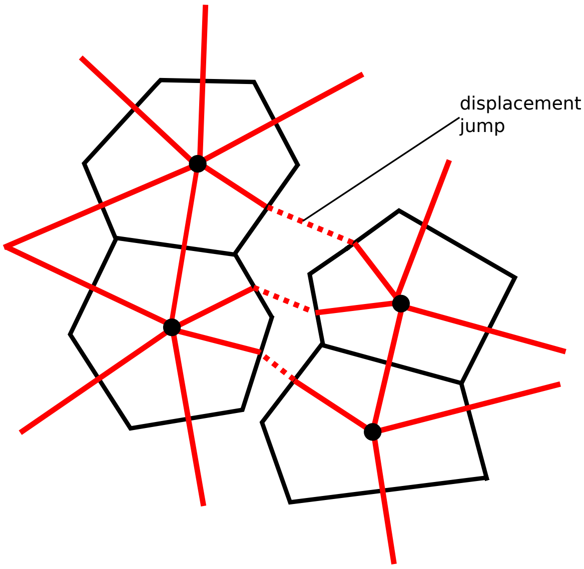


Figure 3: Localized failure of cohesive links when threshold is reached and discontinuity is activated

as the real strain field resulting with enhanced virtual work with G^{int} as internal work

$$G^{int} = \int_{l_e} (\mathbf{B}\delta\mathbf{d})^T \boldsymbol{\sigma} dx + \underbrace{\int_{l_e} \delta\boldsymbol{\alpha}^T (\overline{\mathbf{G}} + \delta_{x_c}) \boldsymbol{\sigma} dx}_{\mathrm{h(e)=0}}. \tag{4}$$

Standard internal force vector and the local residual vector due to discontinuity are obtained from enhanced virtual work

$$\begin{aligned} \mathbf{F}^{int} &= \int_0^{l_e} \mathbf{B}^{d,T} \boldsymbol{\sigma} dx \\ \mathbf{h}^{(e)} &= \int_0^{l_e} (\overline{\mathbf{G}} + \delta_{x_c}) \boldsymbol{\sigma} dx. \end{aligned} \tag{5}$$

Vector of internal forces can be obtained through the regular part of the enhanced local function

$$\mathbf{t} = -\int_0^{l_e} \overline{\mathbf{G}} \boldsymbol{\sigma} dx, \quad \mathbf{t} = (t_u, t_v, 0)^T \tag{6}$$

Softening plasticity law is implemented in computation of traction forces, while the internal variables are obtained with local return mapping algorithm for softening. Reader is referred to [1, 2, 3, 4] for more details. Linearization of enhanced virtual work and local character of embedded discontinuities allow to perform static condensation of stiffness matrix resulting with single element contribution to the FE assembly

$$A_{e=1}^{n_{el}} \left(\widehat{K}_{n+1}^{(e),(i)} \Delta \mathbf{d}_{n+1}^{(e),(i)} \right) = A_{e=1}^{n_{el}} \left(\mathbf{F}^{ext,(e)} - \mathbf{F}^{int,(e),(i-1)} \right) \tag{7}$$

where $\widehat{K}_{n+1}^{(e),(i)}$ is statically condensed stiffness matrix.

The extension of quasi-static system from above (7) towards dynamic regime can be achieved taking into consideration that local problem with embedded strong discontinuities is not directly affected by dynamic effects [8]. Thus, standard finite element procedure for dynamic case results with typical inertial terms based on a global mass matrix

$$M_n a_{n+1}^{(i)} + K_n \Delta d^{(i)} = F_{n+1}^{ext} - F_{n+1}^{(i-1)} \tag{8}$$

where M is a global mass matrix, K global stiffness matrix assembled from statically condensed local matrices due to embedded discontinuities, n and i denote time step and iteration respectively. The velocity-dependent damping is not considered presently. Direct time integration procedure is applied here for the analysis of presented nonlinear dynamic problem, with trapezoidal rule, or yet called the average acceleration method. By applying the trapezoidal rule (e.g. [10]) to the equations of motion, we obtain second order approximation to evolution equations for displacement d and velocity v, which can be written:

$$\begin{array}{lll} \dot{d}(t) = v(t) & \Longrightarrow & d_{n+1} - d_n = \frac{h}{2}(v_n + v_{n+1}) \\ \dot{v}(t) = a(t) & \Longrightarrow & v_{n+1} - v_n = \frac{h}{2}(a_n + a_{n+1}) \end{array} \tag{9}$$

Rewriting the result in $(9)_1$ we can obtain the corresponding approximation for the velocity vector in terms of displacement increment:

$$v_{n+1} = -v_n + \frac{2}{h}(d_{n+1} - d_n) \tag{10}$$

Acceleration vector at time t_{n+1} can then be obtained similarly from $(9)_2$ using the same kind of approximation

$$a_{n+1} = -a_n - \frac{4}{h}v_n + \frac{4}{h^2}(d_{n+1} - d_n) \tag{11}$$

Both of these approximations are implicit in the sense that they depend upon the displacement value at time t_{n+1}. If we implement Newton iterative algorithm, we can rewrite equations from (10) and (11) in sense of iterations

$$\begin{array}{l} v_{n+1}^{(i)} = -v_n + \frac{2}{h}(d_{n+1}^{(i-1)} + \Delta d^{(i)} - d_n) \\ a_{n+1}^{(i)} = -a_n - \frac{4}{h}v_n + \frac{4}{h^2}(d_{n+1}^{(i-1)} + \Delta d^{(i)} - d_n) \end{array} \tag{12}$$

We have to solve at each iteration the consistently linearized form of the system, which can be written as

$$[\frac{4}{h^2}M + K_n]d_{n+1}^{(i)} = F_{n+1}^{ext} - F_{n+1}^{(i-1)} - M \underbrace{[-a_n - \frac{4}{h}v_n + \frac{4}{h^2}(d_{n+1}^{(i-1)} - d_n)]}_{a_{n+1}^{(i-1)}} \tag{13}$$

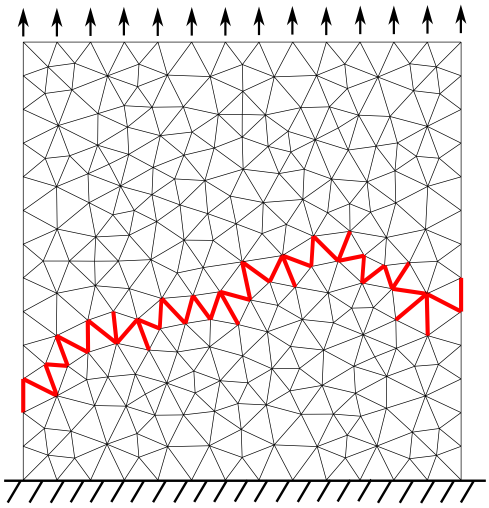


Figure 4: Uniaxial tension test and final failure pattern. Increasing softening elements at the end of simulation are red coloured

3 NUMERICAL SIMULATION

We consider here the uniaxial tension test of specimen from Figure 4. It is a heterogeneous specimen constructed from two phase elements with equal volume fraction randomly distributed throughout the domain, where one phase is strong and the other is weak [3]. The dimensions of the specimen are 10x10 cm, with moduli of elasticity being $7000kN/cm^2$ and $1000kN/cm^2$ for strong and weak phase elements and Poisson ratio 0.2. Weak phase elements are allowed to break, trigering the softening behaviour, with failure threshold for tension failure (mode I) being $0.2kN/cm^2$ and for shear failure (mode II) $0.13kN/cm^2$. Fracture energies for softening behaviour for weak elements are $0.0001kN/cm$ for tension failure case and $0.0005kN/cm$ for shear failure case. The test is conducted with imposed displacement on the upper side of the specimen, while the sum of all reactions is monitored and plotted (Figure 5) providing the macroscopic response of the specimen. Three tests are performed, including quasi-static case and two dynamic cases with differently imposed displacement rates.

It can be noted that specimen is completely broken with macro crack which propagated through the specimen (Figure 5), while in dynamic regime macroscopic curves oscillate around static response due to inertial effects. It is also observed that global softening is triggered in the time step when macro-crack found its way throughout complete specimen (Figure 4).

4 CONCLUSIONS

Lattice element model for localized failure of heterogeneous materials and structures is presented, with its extension towards dynamic regime. The main advantage of the approach is that inertial effects can be considered without changing formulation for stat-

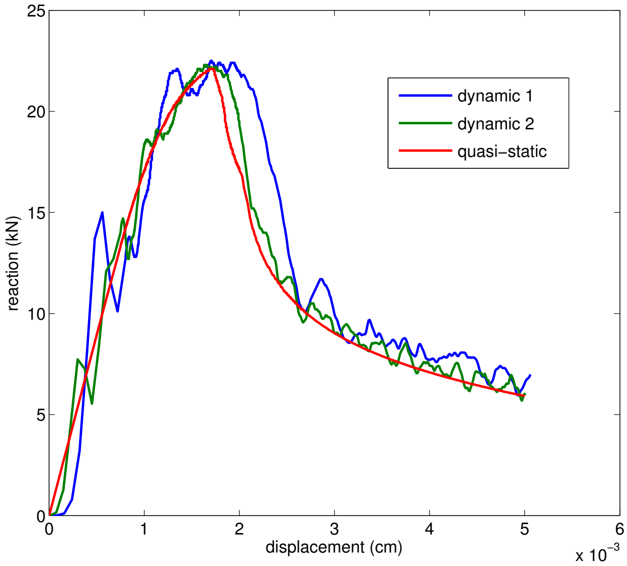


Figure 5: Macroscopic response in uniaxial tension test. Static response versus dynamic1 and dynamic2 (imposed displacement rate is two times higher for dynamic1 than for dynamic2 load)

ically condensed stiffness matrix which arises due to local discontinuous enhancements. Representative numerical simulation is performed comparing the results for static and dynamic analysis.

Acknowledgements

This work has been fully supported by Croatian Science Foundation under the project *Development of numerical models for reinforced-concrete and stone masonry structures under seismic loading based on discrete cracks* (IP-2014-09-2319).

REFERENCES

[1] Ibrahimbegovic, A. *Nonlinear Solid Mechanics: Theoretical Formulations and Finite Element Solution Methods*. Springer, London, (2009).

[2] Nikolic, M., Karavelic, E., Ibrahimbegovic, A. and Miscevic, P. Lattice Element Models and Their Peculiarities. *Arch. Comput. Meth. Eng.* (2017) doi:10.1007/s11831-017-9210-y

[3] Nikolic, M., Ibrahimbegovic, A. and Miscevic, P. Brittle and ductile failure of rocks: Embedded discontinuity approach for representing mode I and mode II failure mechanisms. *Int. J. Num. Meth. Engng.* (2015) **102**:1507–1526.

[4] Nikolic, M. and Ibrahimbegovic, A. Rock mechanics model capable of representing initial heterogeneities and full set of 3D failure mechanisms. *Comput. Methods Appl. Mech. Eng.* (2015) **290**:209-227.

[5] Nikolic, M., Ibrahimbegovic, A. and Miscevic, P. Discrete element model for the analysis of fluid-saturated fractured poro-plastic medium based on sharp crack representation with embedded strong discontinuities. *Comput. Methods Appl. Mech. Eng.* (2016) **298**:407-427.

[6] Rethore, J. Gravouil, A. and Combescure A. An energy-conserving scheme for dynamic crack growth using the eXtended finite element method. *Int. J. Num. Meth. Engng.* (2005) **63**:631-659.

[7] Menouillard, T., Rethore, J., Moes, N., Combescure, A. and Bung, H. Mass lumping strategies for X-FEM explicit dynamics: Application to crack propagation. *Int. J. Num. Meth. Engng.* (2008) **74**:447-474.

[8] Armero, F. and Linder, C. Numerical simulation of dynamic fracture using finite elements with embedded discontinuities. *Int. J. Fract.* (2009) **160**:119-141

[9] Ibrahimbegovic, A. and Delaplace, A. Microscale and mesoscale discrete models for dynamic fracture of structures built of brittle. *Comput. Struct.* (2003) **81**:1255-1265

[10] Bathe, K.J. *Finite Element Procedures.* Klaus-Jurgen Bathe, 2nd edition (2014).