A NEW CREEP MODEL DIRECTLY USING TABULATED TEST DATA AND IMPLEMENTED IN ANSYS

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Abstract. Nowadays plastics are increasingly used in highly stressed structures in all kinds of constructions. The time dependency, the so-called viscosity, is a crucial part of the material behavior of plastics. A typical form of viscosity is creep. Creep is the increase of deformation under constant load. In the FE-simulation creep behavior is usually described by creep law functions. The commercial software provide many creep law functions depending on time, stress, strain, temperature and multiple material parameters. To run a creep simulation, the user must define all the parameters which requires a certain effort. Curve-fitting procedures might be of help, the results, however, often are not precise enough. For these reasons, we introduce our new creep model doing the similar job as the creep law functions but being able to directly use the tabulated data of the creep tests without curve-fitting procedures. In this paper, we use the model to create a 3D stress-creep strain-time surface based on the tabulated data like isochronous curves, which is represented by bicubically blended Coons patches to provide a good convergence due to their differentiability. This creep model supports strain hardening, which shows more realistic behavior when the load changes significantly during the simulated process.

1 INTRODUCTION

Numerous plastics show a combined material behavior like elastic, plastic and viscous one. The viscosity of plastics strongly depends on the loading rate, the loading duration and the temperature [1, 2]. Ignoring the viscosity of plastics like creep can lead to a severe failure of construction based on simulation [3]. Creep describes the increase of the deformation with time under a mechanical stress [4].

The simulation of creep behavior is based on the creep test data. The creep is directly measured from the creep and relaxation test. Creep curve provides a dependence of creep strain on time, relaxation curve provides a dependence of stress on time while creep strain can indirectly be determined. The isochronous curves are concluded from those two types of curve [5]. The isochronous curve gives a stress-strain relation at the identical time point.

Another well-established curve for creep is the creep modulus curve. The creep modulus is the quotient of the stress and total strain, thus showing a combination of creep and elastic behavior over time. Considering Young’s modulus as constant there is a direct relation to creep strain.
Besides of stress and time creep behavior of plastics also depends on temperature. Conventional creep models usually describe the temperature dependency using the Arrhenius equation showing exponentially growing creep rates with respect to the inverse of the temperature [6]. Modelling temperature dependency in this way shows limited accuracy. On the other hand creep rate indeed varies nonlinearly with the temperature. Thus, the alternative, linearly interpolating all creep parameters might better match the material behavior for the given temperatures but lacks in between.

2 MOTIVATION

Numerous creep models proposed creep law functions defining creep behavior. For those functions several more or less abstract, not directly measurable material parameters must be specified. The difficulty is to determine the parameters as accurate for general application. Often one creep curve for a single stress can be fitted accurately. The other dependencies, those of temperature and stress, then show larger errors if - which is often the case - the curves for different stress levels are not similar in the mathematical sense, i.e. not scalable. Furthermore, it has to be emphasized that usual creep models give functions not for creep strain but for strain rate. Curve fitting requires either the solution of the differential equation given in this way or the determination of strain rates from measured strain which can be difficult due to either a small number of sampling points or oscillations in the measured curves if the number of sampling points is larger. The study of [7] indicate that general curve fitting functions could create insufficiently accurate or even wrong parameters and mislead the simulation. As a consequence the user has a larger effort to determine creep parameters and will even though finally not be satisfied.

Furthermore, due to numeric (time integration scheme) creep rate models can be subject to larger initial errors, e.g. for the power function of time with negative exponent, even if the analytical use of the parameters show good accordance with tests. The latter is the reason why the authors avoid rate formulations but use incremental ones as shown below.

Despite the right selection of creep model and define its parameter, the material behavior of a creep model can also differ from the test data[8,9]. The reason for that is the fact that constant material parameters compromise the accuracy with multiple loads and temperatures. With the compromise the material behavior in simulation is only accurate in a limited range [10].

In this work, we focus our attention on the construction of a creep model with direct use of tabulated test data from the creep test. The tabulated data here are e.g. the creep-time curve, isochronous curve and creep modulus curve. This creep model works without parameter identification and the curve fitting function. Instead of the parameters it uses a three-dimensional surface to find the proper creep state for calculation. For a good differentiability and a decent extensionality, we decided to use bicubical Coons patches (also named bicubically blended Coons patches) for the surface construction.

3 CONVENTIONAL CREEP MODEL

Numerous conventional creep models describe the creep behavior using strain rate formulation in a creep equation like:

\[
\dot{\varepsilon}^{cr} = f(\sigma, t, \varepsilon, T)
\] (1)
It shows that the creep strain rate depends on stress, time (or creep strain) and temperature. The dependencies of time and strain are normally not modelled within the same creep equation. As example, a creep equation of direct time dependency (also called time hardening) is defined as

$$\dot{\varepsilon} = C_1 \sigma^{c_2} t^{c_3} \cdot e^{-c_4/T}$$  \hspace{1cm} (2)

whereas an example for indirect time dependency (strain-hardening), which show better accuracy if the stress significantly changes (or creep process is interrupted) during the simulation, reads

$$\dot{\varepsilon} = C_1 \sigma^{c_2}(\varepsilon^{c_3})^{c_3} \cdot e^{-c_4/T}$$  \hspace{1cm} (3)

where the $C_1 \ldots C_4$ are the material parameter. These parameters are fitted to the creep test data for a number of stress, time (or creep strain) and temperature points. The creep strain rate for a single curve may be accurate. However, the parameter identification goes easily wrong considering multiple stress levels at different time point. The reason is that the stress, time and creep strain have a complex relation. Furthermore, defining the material parameter for indirect time dependency is harder than for the direct way, since the creep strain rate depends on the creep strain which makes the solution of the differential equation more difficult.

4 SURFACE CONSTRUCTION

In our new creep model, we need a three-dimensional surface to describe the creep strain depending on time and stress.\(^1\) The new creep model uses three types of creep test data for the users’ convenience. The creep test data can be the creep-time curve, isochronous curve and creep modulus curve.

The creep curve uses stress as curve parameter and describes the increase of creep strain with time. The isochronous curve connects stress-strain points at the same time level and is not a directly measured curve. These curves describe the strain-stress-time relation. Since the creep strain is our primary variable, the surface should be

$$\varepsilon^{ct} = f_{\varepsilon^{ct}}(\sigma, t)$$  \hspace{1cm} (4)

To create the surface with the given curves as tabulated data, we need to construct the curves in both $\sigma$- and $t$-directions first. The curve requires to smoothly cross all the data points in one direction due to accuracy and differentiability. Thus, the cubic spline is used to create the curve. The advantage to build the cubic spline is that the spline exactly meets selected data points, avoid an unnecessary oscillation and provide $C^2$-continuity. The spline for creep strain and time uses the logarithmic timeline due to rapid change of the creep strain at the beginning.

The surface is based on cubic splines as boundary curves. It should fit the splines and be smooth perpendicular to the edges. For this purpose, we use the bicubical Coons patches. The Coons patches use the boundary curves to generate surfaces [12, 13]. A bicubical Coons patch combines four edges from splines and the surface interpolation with the cubic Hermite Interpolation. We define the Hermite functions with cubical Bézier form as:

\(^1\)The temperature effect is discussed in section 6.
\[ B_0^3(\sigma) = (1 - \sigma)^3 \]
\[ B_1^3(\sigma) = 3\sigma(1 - \sigma)^2 \]
\[ B_2^3(\sigma) = 3\sigma^2(1 - \sigma) \]
\[ B_3^3(\sigma) = \sigma^3 \]  
\[(5)\]

So that Hermite function for \( \sigma \in [a, b] \) is:
\[ H_0^3(\sigma) = B_0^3(\sigma) + B_1^3(\sigma) \]
\[ H_1^3(\sigma) = \frac{1}{3}(b - a)B_1^3(\sigma) \]
\[ H_2^3(\sigma) = -\frac{1}{3}(b - a)B_2^3(\sigma) \]
\[ H_3^3(\sigma) = B_3^3(\sigma) + B_2^3(\sigma) \]
\[(6)\]

From the cubical spline, the positional data is available for Coons patch, the four splines are:
\[ f(a, t), f(b, t), f(\sigma, c), f(\sigma, d) \]
\[(7)\]

which \( \sigma \in [a, b], t \in [c, d] \). For purpose of continuity, first derivative information for both \( \sigma \)-
and \( t \)-direction is desired but not given. The two patches, which share the same edge in \( \sigma \)-
or \( t \)-direction, must have the same derivative in \( t \)- resp. \( \sigma \) -direction of the edge. Therefore, the
dervative is defined through a linear interpolation from two ends of the edge. Hence, there are
four derivatives for each boundary:
\[ \frac{\partial f(a, t)}{\partial \sigma}, \frac{\partial f(b, t)}{\partial \sigma}, \frac{\partial f(\sigma, c)}{\partial t}, \frac{\partial f(\sigma, d)}{\partial t} \]
\[(8)\]

Through four boundary curves and their derivatives two ruled surfaces are defined:
\[ hc(\sigma, t) = H_0^3(\sigma) \cdot f(a, t) + H_1^3(\sigma) \cdot \frac{\partial f(a, t)}{\partial \sigma} + H_2^3(\sigma) \cdot \frac{\partial f(b, t)}{\partial \sigma} + H_3^3(\sigma) \cdot f(b, t) \]
\[ hd(\sigma, t) = H_0^3(t) \cdot f(\sigma, c) + H_2^3(t) \cdot \frac{\partial f(\sigma, c)}{\partial t} + H_3^3(t) \cdot f(\sigma, d) \]
\[(9)\]

The interpolated surface \( hc(\sigma, t) \) is ruled by the splines \( f(a, t), f(b, t) \) in \( t \)-direction for \( \sigma \in [a, b] \) and \( hd(\sigma, t) \) is ruled by the splines \( f(\sigma, c), f(\sigma, d) \) in \( \sigma \)-direction for \( t \in [c, d] \). Thus, we
need \( hc(\sigma, t) \) to fix its course in \( t \)-direction and change its course in \( \sigma \)-direction like \( hd(\sigma, t) \). In this case, a new surface is used to achieve this goal. We define a surface with the corner data
for the interpolation:
\[ hcd(\sigma, t) = (H_0^3(\sigma) \ H_3^3(\sigma) H_2^3(\sigma) \ H_3^2(\sigma)) \cdot \]
\[ \begin{pmatrix}
  f(a, c) & \frac{\partial f(a, c)}{\partial t} & \frac{\partial f(a, c)}{\partial \sigma} & 0 & 0 & \frac{\partial f(a, d)}{\partial \sigma} \\
  \frac{\partial f(a, c)}{\partial \sigma} & 0 & 0 & \frac{\partial f(a, d)}{\partial \sigma} & \frac{\partial f(a, d)}{\partial t} & 0 \\
  \frac{\partial f(b, c)}{\partial \sigma} & 0 & \frac{\partial f(b, c)}{\partial t} & \frac{\partial f(b, d)}{\partial \sigma} & 0 & \frac{\partial f(b, d)}{\partial t} \\
  \frac{\partial f(b, c)}{\partial t} & \frac{\partial f(b, c)}{\partial \sigma} & \frac{\partial f(b, d)}{\partial \sigma} & 0 & 0 & \frac{\partial f(b, d)}{\partial t}
\end{pmatrix} \cdot \]
\[(10)\]

The bicubical Coons Patch is defined as:
\[ hg(\sigma, t) = hc(\sigma, t) + hd(\sigma, t) - hcd(\sigma, t) \]  
\[(11)\]
The surface we construct for isochronous curves with the Coons patches is shown in Figure 1.

![Figure 1](image)

The entire surface consists of multiple Coons patches. The cross points in the surface are the data points from the database. With this surface, the creep strain can be interpolated for one specific couple of time and stress.

5 NEW CREEP MODEL WITH CREEP STRAIN FORMULATION

In our new creep model, we use the incremental creep strain formulation instead of rate formulation. Since the creep strain $\varepsilon_{\text{cr}} \rightarrow 0$ at the time $t \rightarrow 0$, the creep process can start from time $t = 0$ without the numerical problem even if the strain rate tends to infinity in a rate formulation. We create a surface from the creep curve, isochronous curve as a creep strain function of time and stress. Therefore, the incremental creep strain formulation for direct time dependency is

$$\Delta \varepsilon_{\text{cr}} = f_{\text{cr}}(\sigma, t + \Delta t) - f_{\text{cr}}(\sigma, t)$$

(12)

Since the surface of creep strain is defined as a three-dimensional function $f_{\text{cr}}$, the creep strain increment is interpolated using the stress $\sigma$, the time $t$ and the time increment $\Delta t$. The term $f_{\text{cr}}(\sigma, t)$ is equal to the creep strain $\varepsilon_{\text{cr}}^{0}$ from the last converged state.

To model indirect time dependency, we must determine the point of the surface where the same creep strain $\varepsilon_{\text{cr}}^{0}$ is valid, but now for the actual stress $\sigma_{i+1}$. Since the data form triples of strain, stress and time, the time is the only differing quantity and thus is no longer the real time. Hence, the time of this point is called pseudo-time $\xi$ (at point a in Figure 2). The surface $f_{\text{cr}}$
describes the creep strain depending on time and stress. The inverse function, time as a function of stress and strain is not given. To solve for the pseudo-time $\xi$, we use a Newton-Raphson scheme for the equation
\[ h(\sigma_{i+1}, \xi) = f_{cp}(\sigma_{i+1}, \xi) - \varepsilon_0^{cr} = 0 \] (13)

Then we obtain the creep strain $\varepsilon^{cr}$ after time increment $\Delta t$ at point b in Figure 2. The difference of point a and b is the creep increment $\Delta \varepsilon^{cr}$.2
\[ \Delta \varepsilon^{cr}(\sigma_{i+1}, \xi + \Delta t) = f_{cp}(\sigma_{i+1}, \xi + \Delta t) - \varepsilon_0^{cr} \] (14)

Figure 2 creep curve with search process

Alternatively we can also create a surface from the creep modulus curve which uses the creep modulus depending on time and stress. In this case, the creep modulus is depending variable, so the surface is
\[ E^{cr} = f_{cp}(\sigma, t) \] (15)

The creep modulus is defined as the quotient of stress and total strain. The pseudo-time now is the time point where the same creep modulus applies as in the converged state but for the new stress. It is determined by a Newton-Raphson scheme for
\[ h(\sigma_{i+1}, \xi) = f_{cp}(\sigma_{i+1}, \xi) - E_0^{cr} = 0 \] (16)

where $E_0^{cr}$ is the creep modulus from last converged state. The $E_0^{cr}$ is calculated using the creep strain as:
\[ E_0^{cr} = \frac{\sigma_i}{\varepsilon^{cr}(t_i) + \varepsilon_{el}} \] (17)

As a result, the creep modulus for the time $\xi + \Delta t$ is
\[ E^{cr} = f_{cp}(\sigma_{i+1}, \xi + \Delta t) \] (18)

2 Further discussion for pseudo-time and stress out of the surface, see ([14])
To use the same local iteration of creep strain formulation (Figure 3), we convert creep modulus to creep strain

\[
\Delta \varepsilon_{cr}(\sigma_{i+1}, t + \Delta t) = \frac{\sigma_{i+1}}{E_{cr}(\sigma_{i+1}, \xi + \Delta t)} - \frac{\sigma_{i+1}}{E} - \varepsilon_{0}^{cr}
\]

where the first term is the new total strain and the second one the new elastic strain. Its partial derivative with respect to \( \sigma \) is

\[
\frac{\partial \Delta \varepsilon_{cr}(\sigma_{i+1}, t + \Delta t)}{\partial \sigma} = \frac{\partial}{\partial \sigma} \left( \frac{\sigma_{i+1}}{E_{cr}(\sigma_{i+1}, \xi + \Delta t)} \right) = \frac{1}{E_{cr}} - \frac{\sigma_{i+1}}{E_{cr}^2 \partial \sigma} - \frac{1}{E}
\]

and with respect to \( t \) :

\[
\frac{\partial \Delta \varepsilon_{cr}(\sigma_{eq,i+1}, t + \Delta t)}{\partial t} = -\frac{\sigma}{E_{cr}^2 \partial t}
\]

Thus, all three types of test data curves are united by the incremental creep strain formulation. With the creep strain increment, the three-dimensional creep state is defined

\[
\Delta \varepsilon^{cr} = \Delta \varepsilon^{cr} \cdot \frac{\partial Q}{\partial \sigma}
\]

where \( Q \) is plastic potential. This equation is adopted from the flow rule of plastic material, where the plastic multiplier is replaced by the creep strain increment in this case. Like in plastics, we use the flow rule associated with the yield condition \( F \) after von Mises. There is no threshold like the yield stress, thus creep is always present if there is non-zero equivalent stress.

After the \( \Delta \varepsilon^{cr} \) is determined, the current stress state is

\[
\sigma = E \left( \varepsilon^{tot} - \varepsilon^{cr}(\sigma_{eq}, \xi + \Delta t, \sigma_{eq}(\sigma)) \right)
\]

where \( E \) is the elasticity matrix, \( \varepsilon^{tot} \) is total strain for this step. The stress state changes with the creep increment updates. The equivalent stress is defined after von Mises. In equation (23), stress tensor \( \sigma \) is defined implicitly. Therefore, a local iteration with Newton-Raphson method should take place

\[
g = \Delta \varepsilon^{cr} - \Delta \varepsilon^{cr}(\sigma_{eq}(\sigma), \xi + \Delta t) - \varepsilon^{eq}(t) = 0
\]

The derivative of \( g \) is

\[
\frac{\partial g}{\partial \Delta \varepsilon^{cr}} = 1 + \frac{\partial \Delta \varepsilon^{cr}}{\partial \sigma_{eq}} \left( \frac{\partial Q}{\partial \sigma} \right)^T E \frac{\partial Q}{\partial \sigma} - \frac{\partial \Delta \varepsilon^{cr}}{\partial \varepsilon^{cr}}
\]

For each iteration, the creep increment and its derivative is determined using the surface of Coons patches. The process for the creep increment calculation is shown in Figure 3.
Once the creep increment is determined and the local iteration is converged, the consistent tangent is requested from the global iteration. The total differential of the stress is:

$$d\sigma = E \left( de^{tot} - \frac{\partial Q}{\partial \sigma} d\Delta \varepsilon^{cr} - \Delta \varepsilon^{cr} \frac{\partial^2 Q}{\partial \sigma^2} d\sigma \right)$$  \hspace{1cm} (26)$$

The total differential of the creep increment is:

$$d\Delta \varepsilon^{cr} = \frac{\partial \Delta \varepsilon^{cr}}{\partial \sigma_{eqv}} \frac{\partial Q}{\partial \sigma} d\sigma + \frac{\partial \Delta \varepsilon^{cr}}{\partial \varepsilon^{cr}} d\Delta \varepsilon^{cr}$$  \hspace{1cm} (27)$$

Together with equation (26) and (27) a linear system of equations of order \( n+1 \) with \( n \) columns at the right hand side is obtained:

$$\begin{pmatrix} 1 + \Delta \varepsilon^{cr} E \frac{\partial^2 Q}{\partial \sigma^2} \\
\frac{\partial \Delta \varepsilon^{cr}}{\partial \sigma_{eqv}} \\
\frac{\partial \Delta \varepsilon^{cr}}{\partial \sigma} \end{pmatrix} \begin{pmatrix} \frac{\partial Q}{\partial \sigma} \\
\frac{\partial \Delta \varepsilon^{cr}}{\partial \sigma_{eqv}} \\
\frac{\partial \Delta \varepsilon^{cr}}{\partial \sigma} \end{pmatrix} \begin{pmatrix} d\sigma \\
\frac{d\sigma}{d\Delta \varepsilon^{cr}} \\
\frac{d\sigma}{d\varepsilon^{cr}} \end{pmatrix} = \begin{pmatrix} E \\
0 \end{pmatrix} \begin{pmatrix} de^{tot} \end{pmatrix}$$  \hspace{1cm} (28)$$

where \( n \) is the number of strain components. The first \( n \) rows of the solution form the consistent tangent \( \frac{d\sigma}{de^{tot}} \).

6 TEMPERATURE EFFECT

In conventional creep models the Arrhenius equation is often used to characterize the temperature effect.

$$\varepsilon^{cr} = f(\sigma, t, \varepsilon) \cdot e^{-\frac{c}{T}}$$  \hspace{1cm} (29)$$

The constant \( c \) is the quotient of activation energy and universal gas constant \( R \). However, the constant \( c \) is not a measurable parameter during a creep test. The normal way to determine this parameter is using a curve fitting or solving the equation for selected data points. For the MaterialPBT-GB30 (Ultradur B 4300 K6\(^3\)), we obtain the creep data like:

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\(^3\) Creep data from [11], the isochronous curves is interpolated using Coons patches. The original data was measured form different load classes. In order to explain the temperature effect without other disruption, a constant load is assumed.
for $\sigma = 5$ MPa, $t = 10h$. Hence the parameter $c$ can be determined from two data points in the Table 1:

\[
\epsilon_i = \frac{\ln \frac{\varepsilon^{cr}_i}{\varepsilon^{cr}_{i+1}}}{\frac{1}{T_i} - \frac{1}{T_{i+1}}}
\]  

Using this parameter $c$ for the temperature outside of these two data points causes an error (Figure 4).

A linear interpolation for parameter $c$ can improve the behavior (Figure 4) but does not solve the problem that the creep curves for different temperatures can show different shape.

The new model for this creep strain-temperature relation uses the Coons patches (Figure 5). Once the stress and the time are set, we obtain the creep strain $\varepsilon^{cr}$ from the surface of Coons patch as interpolated points. A cubical spline connects interpolated points and creates a creep strain-temperature function. Thus, the creep strain could be determined from a specific temperature within the spline. This solution creates a smooth creep strain-temperature curve (Figure 4). Since temperature is a given value during the simulation no derivative with respect to it is requested. Thus, lower order piecewise interpolation is possible. It might be the more accurate the more temperature points are available from test data. For a smaller number of temperatures spline interpolation is preferable.

![Figure 4 Creep strain-temperature relation using interpolation](image-url)
If the temperature changes during the test, this interpolation takes place in every step. If the simulation takes place at a specific constant temperature but not matching a test temperature we first process the creep strain-temperature function for the corner points and create a new surface for creep strain-stress-time relation at this temperature before the simulation starts.

**Figure 5** new model illustration for interpolation of the temperature effect

5 RESULTS

Figure 6 shows the results of this new creep model. To compute these results, a solid element with 8 nodes is used. The curve “00iso” uses the isochronous curve and the curve “00cm” uses the creep modulus curve. Both curves start the creep at $t = 0 \text{ h}$. They match the data point exactly. The maximum difference between the two curves amounts to 1.12%.

**Figure 6** creep curve using the new model in compare with data points

The other two results (05iso and 05cm) start the creep calculation after 30min with indirect time dependency. Until then the creep strain keeps zero. After 30 minutes, they start with the
same curve as “00iso” and “00cm”. This result shows that the indirect time dependency work properly. For the direct time dependency the curve “05iso direct” is considering the creep process as already done for 30 minutes, so it starts with the range of the curve “00iso” beginning at the same time point. Thus, this creep model works as it should be.

We implement this creep model into the commercial software ANSYS. Figure 7 shows two examples for the use of the creep model. First model is a tensile specimen. One side of the specimen is fixed, on the other side a tensile load of 10 MPa is applied. As we expect, the creep strain concentrate on the reduced cross section. The second model is a plate with a hole in it. We apply the same boundary conditions to this model. In this model, we observe the creep concentration and the gradient of the creep strain.

These results show that the new creep model works properly with a complex geometry and multiple elements. It provides also a convergence like other creep models. The huge advantage of this new creep model is that:

- Only tabulated creep test data are used as input data
- No material parameters, i.e. no curve fitting must take place
- The surface of Coons patches meets every data point.

![Figure 7](image.png) two examples: a tensile specimen and a plate with hole

REFERENCES


