

Measuring unidimensional inequality: a practical framework for the choice of an appropriate measure

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1. Introduction

The last decades have been marked by a growing concern over inequality and its effects, much of which is due to the huge economic, societal and environmental consequences of inequality becoming more and more apparent over time.

As with many social issues, the first challenge often lies in developing appropriate measurement tools to understand the problem before it can be addressed adequately. Over the past century, academic research has given rise to a large amount of literature dealing with the evaluation of inequality and with the properties that should characterise an inequality measure in order to obtain a measurement that accurately reflects the real situation. Inequality is sometimes examined by performing dynamic comparisons. In these cases, conclusions are drawn by comparing the distributions for different populations of the attribute under study and evaluating which one is more unequal. In the literature, this has been done on the basis of majorisation and dominance criteria. However, most of the time, dominance relationships offer only partial orderings of attributes. A way to obtain complete rankings of distributions is through the quantitative evaluation of inequality based on indexes. These measures are empirical tools that are applied to data sets of the attribute under study. This is identified as the cardinal approach and is the main focus of the present paper.

In the analysis of inequality, results rely heavily on the choice of a measurement methodology. There exists a wide variety of methods and approaches to measuring inequality and, very often, different methodologies do not lead to the same rankings. This indicates that one of the crucial steps in the analysis of inequality is the choice of a measure that is appropriate for the context in which inequality is being evaluated. To the best of the authors' knowledge, there exists no research on the development of a systematic framework for the selection of an adequate inequality measure depending on the context. It is clear that such a framework needs to be preceded by a comprehensive review of the different methods developed so far, focusing specifically on the advantages and drawbacks of each. For this purpose, the review can build upon existing seminal reviews on inequality measurement that have contributed to a better knowledge of the characteristics and conceptual implications of inequality indexes (Silber 1999, Cowell 2009 and Hao and Naiman 2010). However, in order to be able to build a framework, the review preceding it must also be carried out systematically focusing on key characteristics of each measure that will define its interrelationships in the framework.

Having said this, the main objective of this paper is to provide a framework with practical guidelines for researchers and practitioners to facilitate the task of choosing the most appropriate inequality measure for their specific needs. The framework considers seven main characteristics of the measures: their properties derived from axioms and the value judgements attached to them, the existence of an index's upper bound, its graphical interpretability, the comparative standard used, if it considers socioeconomic groups, whether it is absolute or relative and the possible sensitivity of the measure to certain parameters. Additionally, the framework accounts also for factors that are external to the index itself, such as the quantity and quality of available data. Even though a realistic evaluation of inequality would often require that different dimensions of inequality such as health and education also be taken into consideration, this paper focuses on unidimensional inequality as a first step towards a unified decision framework.

With this in mind, 204 scientific publications dealing with inequality measurement issues were selected. The results of the review were used to derive several conclusions concerning the current state of indexes of inequality and an empirical application was performed to provide a better understanding of the strengths and weaknesses of all the inequality measures. In the end, these outcomes were used to develop a framework to support the choice of an appropriate inequality measure.

The paper is organised as follows. Section 2 describes the research method. Since the outcomes of the review are essential for a proper understanding of the developed framework, these are presented in Section 3. In section 4 a numerical application of the reviewed methods using world-level data is presented. Section 5 describes the strengths and weaknesses of all the methods and proposes a decision framework for the choice of an appropriate index. Section 6 concludes.

2. Research method

2.1. Review method and development of the framework

For the development of the unified framework to support the choice of an adequate inequality index, a review of publications was first performed. The review followed a systematic process conducted through databases searches on inequality measurement. The studies of interest were published works focusing on inequality measurement, either completely or partially. The keywords used for the search were *inequality*, *unidimensional inequality* and related. The relevant publications identified at the third step were chosen according to the following criteria: (i) published before 2010 and with at least 10 citations; (ii) published between 2010 and 2012 and with at least 5 citations; (iii) published after 2012. Then, the titles and abstracts of the returned publications were examined in order to do a first judgement of the interest of the publication in question and to exclude duplicated publications. From these, the content was checked and those that were of interest for the review were included. Besides, the reference lists of the papers were screened in order to detect possible includable publications.

In total, in the end, 204 publications were reviewed, of which 70% were published before 2010, 9% between 2010 and 2012 and the remaining 21% after 2012 (see Fig. 1, where the publications have been plotted against their year of publication).

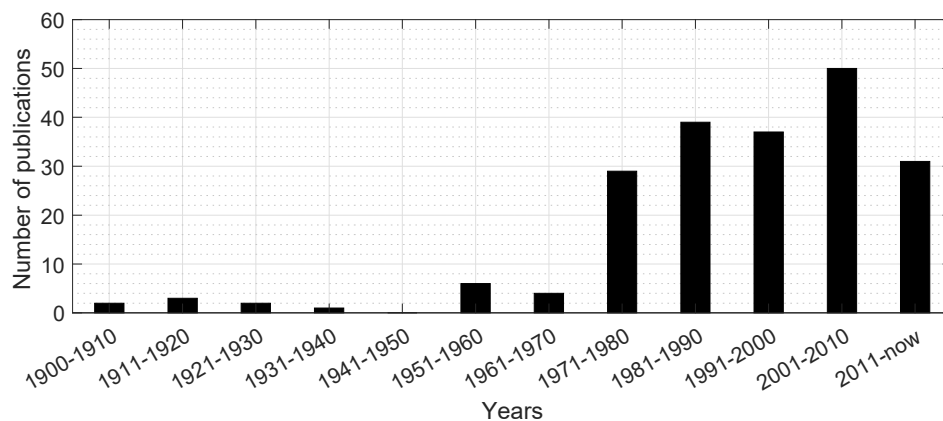


Fig. 1 Number of selected publications per year of publication

Based on a further in-depth review, the publications were grouped based on the characteristics of the approach that they took to the measurement of inequality. In the end, they were classified depending on the nature of the data (qualitative or quantitative) and on whether they measured or not inequality between socioeconomic groups. The classification made of the publications and the results from the literature review allowed, in the end, to build the unified framework.

2.2. Empirical application

Gross National Income (GNI) per capita has been used as an indicator of material wellbeing (Herrero et al. 2008) in the empirical application. The development of a new conceptual inequality framework is not the subject of this paper's discussion; therefore, from the start, it has been assumed that the economic dimension concerning inequality is the one included in the Human Development Index (HDI). For the comparison of inequality between groups, UN's country classification by geographic regions has been used (UN 1999). For the analysis of inequality between groups with a natural ordering, the indicator "mean years of schooling" has been used. For all these indicators, the core data source is the data from the United Nations Development Programme's 2016 Human Development Report (HDR).

3. Narrative literature review

Traditionally, the analysis of inequality has been classified into dominance and cardinal approaches. In the former, the distributions of the variable studied are compared in order to be able to rank them. Many efforts have been devoted to proposing conditions under which two distributions can be ranked. However, these techniques are very rarely put into practice. This is mainly because when distributions intersect, comparisons are seldom conclusive and therefore further restrictions need to be set. Additionally, even when distributions can be ordered, it is not possible to quantify by how much a distribution is more unequal than another one. This makes it difficult to monitor the progress in terms of inequality and yields less interpretable results.

In the cardinal approach, inequality is measured numerically by using indexes obtained from specific functions. Consequently, the cardinal approach has generally been preferred even though it involves the assumption of more restrictive conditions on the functional form of the index.

The present paper focuses on cardinal approaches to the measurement of inequality. There is a vast amount of literature on methodologies that take this approach. All the measures carry with them certain value judgements, but these judgments are not always explicitly attached to the measure (Decancq et al. 2009). Three main procedures have been followed for the development of this kind of measures.

First of all, some authors have proposed a collection of basic requirements that should characterise inequality measures and have then developed an inequality index that satisfied these properties. These are continuity, anonymity, symmetry axiom for population, scale invariance, principle of population, Pigou-Dalton principle and decomposability (Dalton 1920, Bourguignon 1979, Cowell 1980, Shorrocks 1980, 1982, 1984, Cowell and Kuga 1981, Toyoda 1980, Foster and Shneyerov 1999, 2000, Paul 2004, Ebert 2010, Hao and Naiman 2010, and Erreygers 2017).

Secondly, some authors have given a normative basis to inequality indexes by making explicit the value judgements attached to the index through functions. These functions are commonly referred to as social welfare functions (Sen 1997, Francois and Rojas-Romagosa 2011).

Thirdly, there are authors that have adopted measures that were developed in other fields, such as statistics, to carry out quantitative analyses of inequality.

Because in some cases the indicators used to measure inequality are qualitative instead of quantitative, there are measures specifically designed for this type of data. Consequently, knowing whether the data analysed is quantitative or qualitative is fundamental for the choice of an index. In this paper, the measures are reviewed separately for each type of data.

3.1. Quantitative data

In the literature, it is possible to distinguish measures developed for the analysis of inequality between individuals or between socioeconomic groups. Different types of socioeconomic groupings are possible, such as gender, race, income or education. Some of these groups, such as income and education levels, have an inherent ordering. This allows ranking individuals by their

socioeconomic position. Certain indexes of inequality allow considering these implicit rankings and they can't be used if there is no natural ordering of the socioeconomic groups.

In this section the measures are described by classifying them according to whether they allow reflecting or not socioeconomic groups in society and, in the second case, whether these groups have natural orderings or not.

A summary of the properties of each of the measures presented in this section and their respective mathematical expressions can be found in Table 1. The proofs of the properties can be found in the references.

a) Measures not reflecting socioeconomic groups

A first group of measures that fall into this category are mainly measures of dispersion, among which the range is the simplest one. A more commonly used index of variability is the variance, which measures the dispersion of a distribution around the mean. Derived from this measure is the coefficient of variance, which has been frequently used in the field of epidemiology (Wing et al. 1990, Gächter and Theurl 2011). Other authors have preferred the use of mean deviations instead of standard deviations. This includes measures such as the relative mean deviation and the mean log deviation (Mehran 1976, Cowell 1988, Harper and Lynch 2005, Démurger et al. 2006). Alternative dispersion statistics that have less frequently been used are the mean absolute difference, the median absolute difference, the mean pair difference and the interquartile range (Kalmijn and Veenhoven 2005, Tavits and Potter 2014, Weiss et al. 2018).

Apart from measures of dispersion, some authors have used measures of central tendency. For example, Boyce et al. 2016 used the median of different distributions and then calculated an environmental inequality index by obtaining their ratio.

All these measures lack robustness and are easily affected by outliers. Exceptions to this are the interquartile range and the median absolute difference, which are unaffected by a small number of outliers. Another drawback of some of the above-mentioned measures is their dependence on the attribute's magnitude. This is why some authors have preferred using methods yielding dimensionless indexes. This can be done, for instance, by dividing the measures by their mean. Additionally, in cases in which the distribution is heavily right-skewed, the above-mentioned measures might give more weight to the bottom part of the distribution. In order to tackle this, it can be useful to measure inequality using either the logarithmic variance or the variance of logarithms (Creedy 1977, Foster and Ok 1999, Hao and Naiman 2010). According to Hao and Naiman 2010, statistically talking it is more natural to use the latter, which is obtained as the logarithmic variance but using the geometric mean instead of the arithmetic one.

The two main merits of this type of measures are their simplicity, both of calculation and of interpretation. Nevertheless, they have traditionally lacked as much popularity as alternative indexes such as the ones described in the following sections. This lack of popularity has been more obvious among economists, who have frequently used more complex measures like the Gini index. In other fields, however, statistical measures of dispersion have been applied more often; examples of this can be found in Mehran 1976, Wing et al. 1990, Lahelma and Valkonen 1990, Gächter and Theurl 2011.

After statistical measures, quantile functions constitute the most basic inequality measures. These functions are based on the measurement of contrasts between quantiles. They are often used because of their simplicity, directness and easiness to be understood and interpreted; moreover, the data amount and computation effort needed are low. However, the information they provide is scarce in comparison with other more complex inequality measurement techniques. Besides, they neglect information about the part of the distribution that is in between the considered percentiles and about the distribution of the measures within the top and bottom percentiles. They

are also highly sensitive to extreme values and outliers. Some commonly used quantile functions are the decile ratio and the Palma ratio. The former is the ratio between the top 10% and the bottom 10%, whereas the latter is the ratio of the top 10% to the bottom 40% (Palma 2006, 2014, Cobham and Sumner 2013, Cobham et al. 2015).

A desirable characteristic of inequality measures is the existence of a graphical analogy with the index. This can enhance interpretability and help communicate results to non-experts. However, none of the groups of measures described above can be easily described graphically. This is one of the reasons why the Lorenz curve has become popular in analyses of inequality (Lorenz 1905). It is a graphical representation of a quantile function. In this representation, the cumulated percents of the population from poorest to richest are plotted along one axis while the total wealth held by these percents of the population is plotted along the other one.

Many indices have been proposed based on the Lorenz curve: the Gini coefficient, the Pietra index, the Amato index, Eliazar's sociogeometric approaches and the Kolkata index. Among these, the Gini coefficient (1912) is the one that has gained more popularity. It analyses the size distribution of income and wealth by measuring the area between the Lorenz curve and the line of perfect equality. The index lies between 0 (total equality) and 1 (total inequality).

Despite the fact that the Lorenz curve and the Gini coefficient were derived as methodologies within economics, their use has extended to other fields such as epidemiology (Wagstaff et al. 1991, Le Grand and Rabin 1986, Preston et al. 1981, Le Grand 1989, Leclerc et al. 1990, Erreygers 2009a, Kobayashi et al. 2015) or environment and ecology (Ruitenbeek 1996, Fernández-Tschieder and Binkley 2018, Seidl et al. 2018, Druckman and Jackson 2008).

Generalisations of the coefficient which allow for differing aversions to inequality have also been developed. This has been done the application of weight functions to the area under the Lorenz curve (Kakwani 1980, Donaldson and Weymark 1980, Weymark 1981, Yitzhaki 1983). Also, to consider sensitivities to different transfers, Chakravarty 1988 proposed the E-Gini, an extension of the original coefficient obtained by scaling up the Lorenz curve through the distribution's mean. Other extensions to the Gini index can be found in Subramanian 2015 and Majumdar 2015.

In spite of its popularity, the Gini index has four main limitations. First of all, its computation is not straightforward. In fact, several authors have proposed simplified methods for calculating it, such as absolute differences between two random variables (Pyatt 1976), integration of cumulative distributions (Dorfman 1979), covariance formulas (Lerman and Yitzhaki 1984, Shalit 1985), absolute Lorenz curves (Hart 1975, Lambert and Aronson 1993) or matrix algebra (Silber 1989). A review on this topic can be found in Xu 2003 and Yitzhaki and Schechtman 2013.

Secondly, it does not reflect demographic changes or social and economic characteristics of the population. To solve this, Paglin 1975 and Formby and Seaks 1980 proposed a new Gini coefficient obtained by changing the line of total equality of the Lorenz curve. They proposed a new line (the P curve) that reflects lifetime incomes adjusted for each stage of the life cycle inherent to the age-income profile. This way, the inequality existing between ages is excluded from the Gini coefficient. The fourth drawback found is that the coefficient is less sensitive to stratified differences than to individual ones (Futing Liao 2006).

Thirdly, it is not additively decomposable (Shorrocks 1984). Notwithstanding this, many authors have studied its decomposition into factor components. For instance, through graphical methods (Lambert and Aronson 1993), using the covariance (Lerman and Yitzhaki 1984, Lerman and Yitzhaki 1985, Sastry and Kelkar 1994), using individual pairwise income comparisons (Dagum 1997, Mussard et al. 2003) or using the shares of total income and their relation to overall inequality (Fei et al. 1978, Silber 1989).

Finally, the most severe drawback of the Gini coefficient is that it is possible to have two totally different distributions having the same Gini coefficient. In such cases, it is possible to reach different conclusions concerning their levels of inequality depending on the relative weight that is allocated to low-income groups compared with higher income groups. This ambiguity occurs when the Lorenz curves intersect. Subramanian 2010, 2015 and Osberg 2017 empirically show this limitation of the Gini index and describe how it can be manipulated so that the coefficient yields the same value for different distributions.

The Pietra index (1915), also named Robin Hood index or Hoover index (De Maio 2007) is a geometric measure which corresponds to the maximum vertical distance between the line of perfect equality and the Lorenz curve. It shows the proportional income that should be redistributed to achieve perfect equality. Therefore, higher values of the index indicate more inequality since there is more redistribution needed in order to achieve perfect equality.

A less known index is the Amato inequality index (Amato 1968, independently rediscovered by Kakwani 1980, Eliazar and Sokolov 2012). It is also based on the Lorenz curve and is calculated through its length. Its use has been low due, in part, to the difficulty inherent to the analytic evaluation of the index (Arnold 2012). This index lies between $\sqrt{2}$ and 2, since it is a geometrical measure based on the sum of the length of the segments of the curve (see Fig. 2).

Although all these measures relate geometrically to the Lorenz curve, each indicator provides different information about the distribution analysed. Fig. 2 shows the graphic relationship between the Lorenz curve and the Gini, Pietra and Amato indexes.

Finally, apart from these three indexes, more recent indexes based on Lorenz curves have been developed. See, for instance, sociogeometric approaches to the measurement of inequality (Eliazar 2015a, 2015b) or the Kolkata index (Ghosh et al. 2014). Even though these measures have the advantage of being easily interpreted thanks to their relation with the Lorenz curves, they are less frequently used probably because they have only been recently proposed.

In addition to the Lorenz curve, there are other less known graphic representations of income distributions: the Bonferroni curve and the Zenga curve. The Bonferroni curve (Bonferroni 1930) is a minor modification of the Lorenz curve based on the comparison between partial means and the general mean of a certain distribution. It is constructed by considering as abscissa the cumulative proportion of individuals arranged in increasing size of their incomes and as ordinate the corresponding mean density of their income (see Giorgi and Crescenzi 2001 and Imedio-Olmedo et al. 2011 for more information).

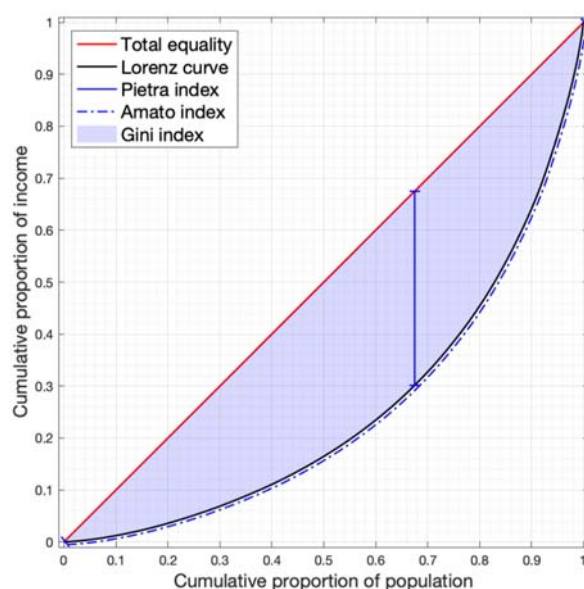


Fig. 2 Lorenz curve (black), total equality line (red) and graphic meaning of the Gini, Pietra and Amato indexes

Alternatively, the Zenga curve is as well a graphic representation of inequality. It was introduced in 1984 and improved in 2007 (Zenga 1987, 2007). This curve involves the ratio of the mean income of the $x\%$ poorest to that of the $(100-x)\%$ richest. In Poliscichio and Porro 2009, a comparison between the Lorenz and Zenga curves can be found. Other literature concerning the development and characteristics of the Zenga curve can be found in Zenga et al. 2011, Maffenini and Poliscichio 2014, Nair and Sreelakshmi 2016.

In order to show how the Lorenz, Zenga and Bonferroni curves behave for different distributions, Fig. 3 show these curves for three lognormal distributions with the same mean (mean=1.0) but different standard deviations ($\sigma = 0.5, 1, 2$). As it can be observed, the three methodologies rank the curves in the same order. Apart from this, it can be seen how the conceptual difference between these curves leads to different locations of the line of total equality and inequality.

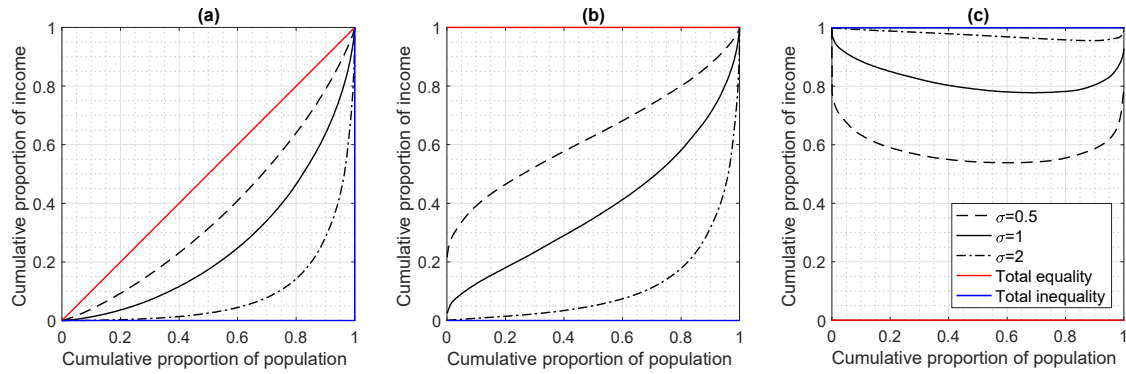


Fig. 3 Lorenz curve (a), Bonferroni curve (b) and Zenga curve (c) for lognormal distributions of population's income with standard deviations= 0.5, 1.0, 2.0 and for distributions with perfect equality (red) and total inequality (blue)

The Bonferroni index attaches more weight to the right-part of the distribution and is, therefore, more sensitive to lower values of the studied attributed. This differing sensitivity is inherent to the numerical expression of the index and consequently, it can't be modified to satisfy the needs of the analyst. Giving more weight to certain parts of the distribution might be useful to explicitly attach specific value judgements to the index. There are two families of indexes that capture the changes of the measure in particular parts of the distribution by making use of sensitivity parameters: the generalised entropy (GE) inequality indexes and Atkinson indexes (AI).

The first one was proposed in the fields of statistical physics and information theory. In this field, the statistical variability of a given dataset can be measured through the Boltzmann-Gibbs-Shannon entropy, which quantifies the divergence of the dataset's size (Eliazar 2016). Theil 1967 pioneered this approach to analysing inequality by measuring the extent to which a society deviates from a state in which every individual's share of total income equals the population's share based on a ratio of these shares. Drawing on Theil's approach, the GE inequality indexes were proposed. They used a more general definition of entropy (Toyoda 1975, Cowell and Kuga 1981, Foster and Shneyerov 1999). In this family of indexes, the sensitivity parameter α can take any real value and it represents the weight that is given to distances between incomes at different parts of the distribution: for large and positive values of alpha, the index is more sensitive to changes affecting the upper tail of the distribution, whereas for lower values the GE is more sensitive to changes in the lower tail.

The second family of indexes that allows for different sensitivities at different parts of the was initially described in Atkinson 1970 and is based on Sen 1974's definition of social welfare. It uses the concept of the equally distributed equivalent level of income, which is the level of income

per person that, if equally distributed, would enable society to reach the same level of welfare as the actual distribution.

The sensitivity parameter, ε , represents the inequality aversion level and it has been given alternative definitions in the literature. For instance, Stymne and Jackson 2000 argue that it stands for the importance that society attaches to inequality in the distribution of income, whereas Howarth and Kennedy 2016 describe it as the elasticity of the marginal utility of income under the assumption that a single elasticity applies to all individuals. Accordingly, higher values of this factor entail greater social utility or willingness by individuals to accept lower incomes in exchange for a more equal distribution.

AI are considered to be very intuitively interpreted due to their derivation from a normative approach and the use of the equally distributed equivalent income (Lasso de la Vega and Urrutia 2008, Cowell and Fiorio 2011, Gayant and Le Pape 2017). However, they don't allow decomposability and therefore it is not possible to estimate and analyse to what extent the between- and within-group components contribute to inequality (Lasso de la Vega and Urrutia 2008, Cowell and Fiorio 2011). Consequently, in spite of the fact that the GE indexes do not have a clear and intuitive significance, one of their main appeals over AI is decomposability (Bourguignon 1979, Shorrocks 1980, Cowell 1980, Lasso de la Vega 2008, Gayant 2017).

In fact, using a measure with sensitivity parameters can carry certain difficulties. First of all, because of the complexity of determining the operational value of the parameters; in the existing literature, the methods that have been established for determining it are limited to empirical assessments (Atkinson 1983, Ogaki et al. 1996, Stymne and Jackson 2000). Secondly, because the usage of these families of indexes for policy assessment might be totally conditional on the methodology used for estimating them.

At this point, it must be noted that there exist connections between some of the above-described inequality indexes. The GE family of inequality measures, for instance, yields the Theil index when $\theta = 1$ and the mean log deviation when $\theta = 0$. Besides, for $0 < \theta < 1$, the GE has an ordinal relationship with Atkinson indexes. Lasso de la Vega and Urrutia 2008 have provided an extension of the Atkinson indexes (the Extended Atkinson Family), which allows transforming each member of the family of GE indexes into one of the Atkinson one. In Fig. 4 a lognormal distribution (with mean 1 and standard deviation 2) has been used to illustrate the behaviour of the two families of indexes in relation with their respective sensitivity parameters. It can be seen how the extended family of Atkinson indexes allows obtaining a relationship of both tails of the GE indexes. In addition, it can be seen from both graphs that the larger the sensitivity parameter the higher the inequality index for the same distribution.

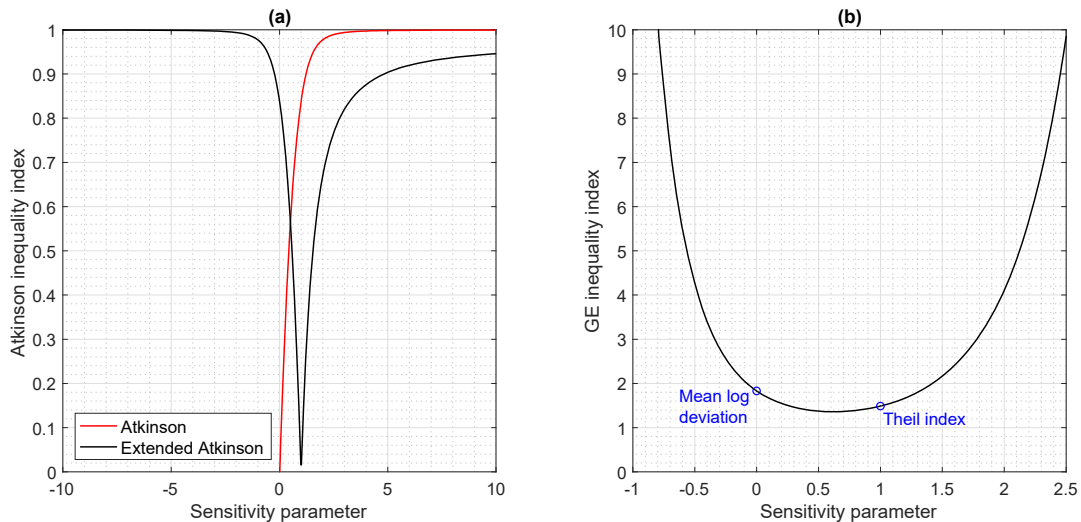


Fig. 4 Atkinson and Extended Atkinson indexes (a), and GE indexes (b) for a lognormal distribution of mean 1 and standard deviation 2

b) Measures reflecting socioeconomic groups without a natural ordering

Some measures have been developed specifically for the analysis of inequality between socioeconomic groups. The three main measures that can be included in this category are the index of dissimilarity, the index of disparity and the population attributable fraction.

First of all, the index of dissimilarity was first proposed in the analysis of the segregation of human settlements (Duncan and Duncan 1955, James and Taeuber 1985) but has since then been used in other fields, such as in epidemiology (Preston 1981, Wagstaff et al. 1991), socioeconomics (Voas and Williamson 2004, Yalonetzky 2015), demography and geography (Taeuber and Taeuber 1976), education (Hagedorn et al. 1996) and occupation (Hutchens 1991, 2015). In their publication, Duncan and Duncan 1955 also introduced the segregation curves, which are similar to Lorenz curves. These curves consider two different groups and plot the cumulative fraction of the first type against the second one, ranked from the lowest to the highest value.

Nevertheless, the index of dissimilarity was developed to compare data only between two groups and its usage is, therefore, more restrictive than some of the methods described below, which measure inequality between two or more groups. For instance, the index of disparity was first used by Percy and Keppel 2002. Its formulation allows measuring inequalities existing between different social groups relative to a reference point. It describes inequality by measuring the differences between the studied attribute's rates and a reference one. The reference measure can be chosen by the analyst, but it is common to choose the mean rate or the best group rate.

However, this index does not account for the population share of each social group and can therefore lead to inaccurate results. A measure that allows measuring inequality between more than two groups and account for the population size at the same time is the population attributable fraction. It was first used in the context of inequalities in epidemiology to calculate differences in the risk of an outcome between two different groups (normally, an exposed and an unexposed group) (Yeracaris and Kim 1978, Leon et al. 1992, Mackenbach 1992). It describes the proportional reduction in the overall proportion of the attribute analysed that would occur if everybody experienced the rates of the highest-ranked population. It is calculated as the difference between the attribute's frequency in the population and the attribute's frequency in individuals in the highest category.

Even though the above-mentioned indexes were explicitly developed to reflect inequality between socioeconomic groups, there exist other measures that can easily be adapted to do so. This can be done by using group data instead of individual data. If the formula used is not weighted with the relative population of each subgroup, then each subgroup is treated as equally sized, whereas the population sizes of each subgroup can be considered in the calculations if weighted formulas are used. Measures that can be adapted are the variance, coefficient of variation, mean log deviation, relative mean deviation and generalised entropy.

c) Measures reflecting socioeconomic groups with a natural ordering

This category of measures includes those indexes in which the indicator used needs to be grouped using categories that can be ordered. Firstly, one index that was initially developed to measure socioeconomic inequalities in health is the slope index of inequality (Pamuk 1985, Wagstaff et al. 1991). This approach involves arranging the social classes from lowest to highest ranked on a horizontal axis; for each class, the mean status of the attribute being analysed is plotted in the vertical axis as a bar whose width is proportional to the fraction of the population in the class. The slope index of inequality is obtained by measuring the slope of the regression line that models the relationship between the attribute and their relative ranks. However, this index is an absolute

measure; by dividing the data by the mean level of the attribute it is possible to obtain its relative version, also referred to as the relative index of inequality.

However, as the index is estimated as a linear regression trend, the distribution cannot have significant deviations from linearity because otherwise, the results would be biased. A measure that does not present this particular problem is the concentration index, which was developed by Wagstaff et al. 1989 drawing a parallelism with the Lorenz curve. This index is based on the concentration curves, which plot the cumulative proportion of the population ranked by some indicator of socioeconomic status (from the least to the most advantaged) against the cumulative proportion of the attribute considered (Wagstaff et al. 1991, Kakwani et al. 1997). This measure has mainly been used in epidemiology, where many researchers have analysed the relationship between health and socioeconomic status. Therefore, in the existing literature, the attribute evaluated in the curves has usually considered various dimensions of health (Van Doorslaer et al. 1997), health care utilisation (Van Doorslaer et al. 2000) or health care payments (Wagstaff et al. 1999). On the one hand, the main merits of this index are its similarity with the Gini coefficient, its easy decomposability and its visual representation through the concentration curves. On the other hand, comparisons of populations with different mean levels of the attribute studied might be problematic (Wagstaff 2005) and the obtained index may be arbitrary if the variable is qualitative rather than quantitative (Erreygers 2006). It is for these reasons that improvements and extensions of the concentration index have been proposed (Erreygers 2009b, Wagstaff 2009).

Both the concentration index and the slope index of inequality have two main advantages: they can be graphically represented and they allow to analyse relationships between socioeconomic classes and levels of inequality. The type of relationship is given by the sign of the index, which depends on the gradient between the socioeconomic level and the attribute studied. However, this may also be seen as a drawback because the application of these indexes is more restrictive than the previously described measures. This is because they can only be used with social groups that have a strict intrinsic ordering, such as health or income levels. There are many examples of socioeconomic groups that don't have a natural ordering; for instance, gender, geographical regions, races, etc. Additionally, some authors have pointed out at the fact that if the socioeconomic group in the middle of the distribution presents higher levels of the attribute studied, these inequality measures may indicate that there is no inequality (Harper and Lynch 2005).

3.2. Qualitative data

Even though there is a vast amount of inequality indices that have been developed for cardinal data, there are many variables that are categorical. This poses a limitation to the applicability of some of the previously described methodologies. As Madden 2010 states, this can be solved by three different means. First of all, one can transform the ordinal data to cardinal one and then make use of a regular inequality measure. Secondly, one can use an inequality measure expressly developed for ordinal data. Finally, it is also possible to adapt existing methods for cardinal data to ordinal variables.

The cardinalisation of ordinal data implies using a particular scale (this is, the variables are assigned a number) with which it is possible to apply a conventional index for inequality. Methodologies that have been proposed for the scaling of the variables are ordinary and weighted least squares, estimation of ordered probit regressions and interval regression approaches (Wagstaff and Van Doorslaer 1994, Van Doorslaer and Jones 2003).

However, these methodologies have an important shortcoming: inequality measures are very sensitive to how the ordinal data is scaled (Apouey 2007, Zheng 2011). Allison and Foster 2004 and Kobus and Miłoś 2012 provide some examples of how inequality measures might change depending on the scale applied. This is why some researchers have advocated using inequality

measures specifically designed for ordinal data. Kobus et al. 2018 review this set of measures, including the indexes by Abul Naga and Yalcin 2008, Kobus and Miłoś 2012, Lazar and Silber 2013, Lv et al. 2015 and Cowell and Flachaire 2017.

None of the previous indexes have a connection with the measures described in the previous section. Drawing a parallelism with the concentration index, Wagstaff 2005 proposed a method to measure inequality when the variable is binary (see also Wagstaff 2011). Besides, it allows graphically representing the results of the analysis.

To the extent of our knowledge, until the moment only Allanson 2017 has introduced an approach to rank and evaluate socioeconomic inequalities with ordinal data.

4. Empirical application

This section is devoted to present an empirical application of the methodologies described in the previous sections with the objective of highlighting their strengths and weaknesses. As it was done in section 3.2.1, this empirical application is divided into three parts based on whether socioeconomic groups are considered and whether these groups have a natural ordering.

a) Not reflecting socioeconomic classes

Firstly, the methodologies described for the measurement of inequality with quantitative data and not reflecting socioeconomic groups have been used to calculate a unidimensional index of inequality in material wellbeing (using the GNI per capita) between countries at a global level every year from 1990 to 2015. The measures used have been range (R), variance (V), coefficient of variation (CV), logarithmic variance (LV), variance of logarithms (VL), mean log deviation (MLD), relative mean deviation (RMD), decile ratio (D9D1), Palma ratio (PR), Gini index (GI), Bonferroni index (BI), Zenga index (ZI), Atkinson index (AI), Theil index (TI), Generalised entropy index (GE).

Fig. 5 shows the results of these calculations by presenting the inequality indexes calculated using each methodology and for each year between 1990 and 2015. In order to compare them, they have been standardised using the MIN/MAX formulation. The standardised results are shown in Fig. 6. Given that the resulting trends obtained by the AI and by the GE using different parameters were similar, a single value for the parameter has been chosen for the plot in Fig. 6 ($\varepsilon = 2$ for the AI and $\theta = 2$ for the GE).

First of all, the reason why the different indexes are not within the same interval is due to the fact that some of them (R, B, CV, LV, VL) do not have an upper limit. Also, while the majority of the measures are dimensionless, R and V are not. Secondly, the general trend observed in Fig. 6 is similar for all the measures: an initial middle-low inequality that reaches its maximum after 1995 and its minimum by 2015. Because of their magnitude-sensitivity, range and variance differ from this general tendency. Whereas the economic variable used increases in magnitude throughout the years (and therefore the difference between extremes is higher), the inequality measures relative to the total decreases.

These results show the heterogeneity in characteristics of the existing inequality measures. Apart from the differences in numerical formulations, also the signification that is given to the concept of inequality by each one of the methodologies differs from index to index. These two aspects were the causes of obtaining such differing results.

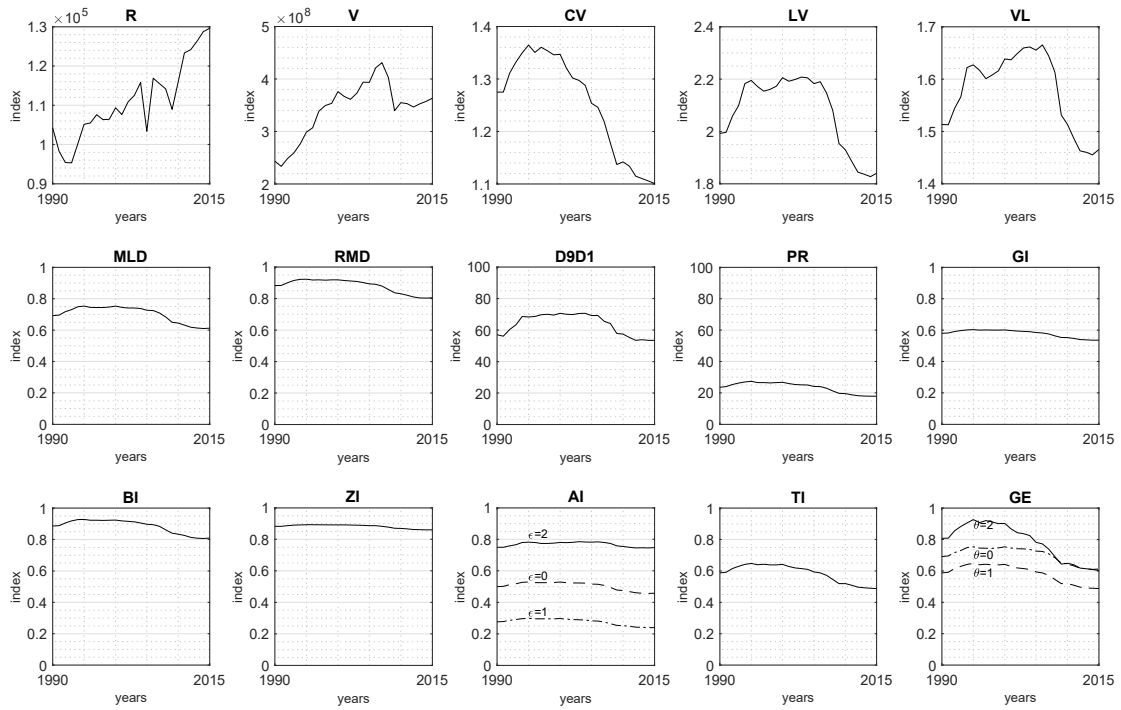


Fig. 5 Indexes of economic inequality between countries for years 1990 to 2015

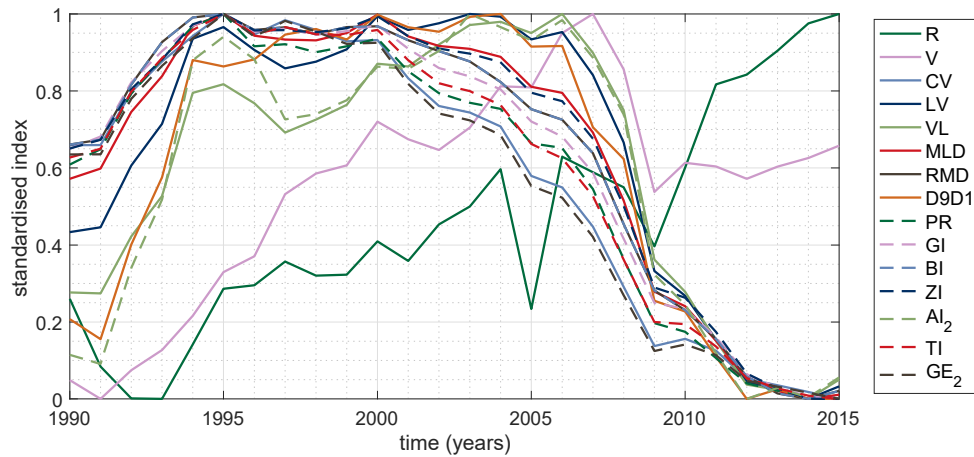


Fig. 6 Standardised indexes of economic inequality between countries for years 1990 to 2015

b) Reflecting socioeconomic groups without a natural ordering

The measures reflecting socioeconomic groups with no natural ordering have been used to calculate the same index but between groups. The grouping adopted has divided the countries into the following regional groups: Africa, Latin America and the Caribbean, Northern America, Asia, Europe and Oceania.

The measures that have been used are the index of disparity (IDsp), the population attributable fraction (PAF), the between-group variance (V), the between-group coefficient of variation (CV), the mean log deviation (MLD), the relative mean deviation (RMD) and the generalised entropy (GE). In these cases, the data is not taken individually, but grouped, albeit the formulation used is the same as in (a). The index of dissimilarity (IDsm) has not been used in this application since it addresses the measurement of inequality between two groups only.

Additionally, in those cases in which the measurement formula allows weighting or not by the population size, both the weighted and unweighted formulations have been used for better contrast between approaches. Fig. 7 shows the results of the calculations.

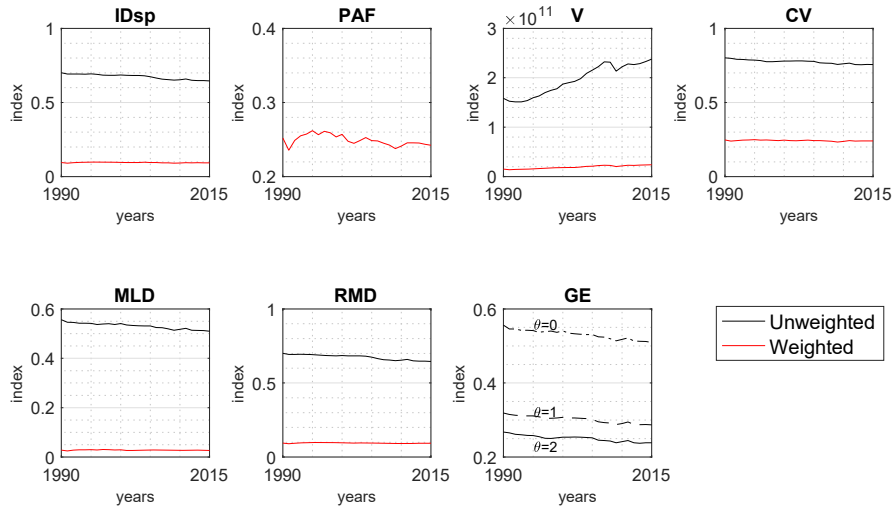


Fig. 7 Indexes of economic inequality between countries grouped by regions for years 1990 to 2015

The standardised results have been plotted in two different graphs (Fig. 8) in order to see the differences existing between results in which the index is weighted by population size and not. Due to its dependence on the indicator's units and the fact that it does not satisfy scale invariance, V follows a completely different trend in comparison to the other measures. Regarding the other measures, the trends differ depending mainly on the population weighting. Whereas the maximum levels of inequality are reached in 1990 and the minimum ones around 2015 when the results are not weighted by population (Fig. 8a), the peaks are reached in 1995 and 1997 and the lowest values in 1991 and 2009 when the results are weighted by group size (Fig. 8b).

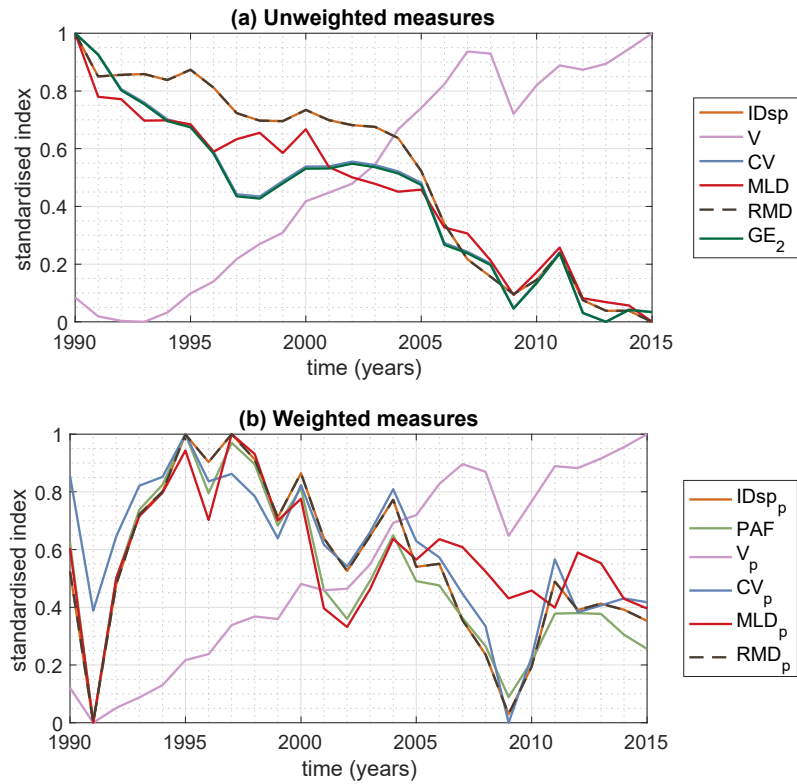


Fig. 8 Standardised indexes of economic inequality between countries grouped by regions for years 1990 to 2015

The results indicate that inequality indexes have a strong dependence on how the analysed attribute is defined and grouped (in this case, individually or grouped by regions), which might give rise to subjective evaluations and wrong interpretations of the coefficient.

c) Reflecting socioeconomic groups with a natural ordering

The use of measures that consider the ordering of the socioeconomic groups requires posing more restrictions to the kind of group defined. Consequently, the grouping used in (b) cannot be used since they don't have a natural ranking. For this reason, in the application of the slope index of inequality (SII), relative index of inequality (RII) and concentration index (CI) the countries have been grouped by education level. Countries have been arranged in five groups considering the mean number of years of schooling (see the Appendix for further information).

Fig. 9 shows the results of the calculations, which represent the inequality in material wellbeing between countries grouped by average quantity of schooling. All the indexes yield positive values, which means that there is an upward income gradient; namely, income increases with increasing education level group rank). The standardised results are shown in Fig. 10. The results reinforce the previous statement according to which inequality assessments strongly depend on how the data is grouped. The difference between SII and RII is fundamentally that the values of the former depend on the units of the attribute measured, whereas the latter is dimensionless.

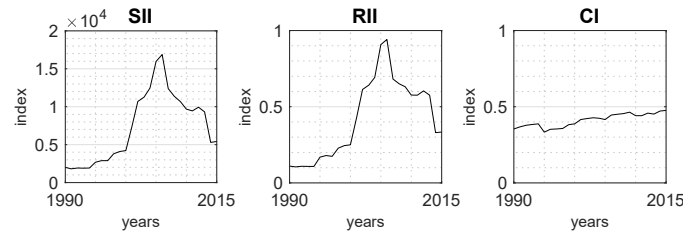


Fig. 9 Indexes of economic inequality between countries grouped by education levels for years 1990 to 2015

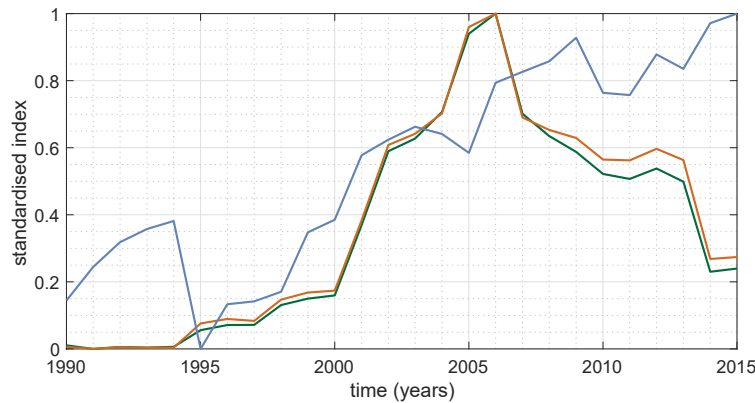


Fig. 10 Indexes of economic inequality between countries grouped by education levels for years 1990 to 2015

5. Framework

Choosing an inequality measure for the monitoring of inequality requires considering certain characteristics satisfied by the indexes, but it also involves examining the type, quality and amount of data that is available. In order to visualise these characteristics of the indexes that are important for the choice, they have been summarised in an easy and accessible way in Table 1. In this table, the different reviewed measures are presented in order of appearance in the paper. For each of them, one or two references are given; the field for which the measure was developed is also presented, along with the fields in which empirical applications of the methodology can be found. Table 1 also presents the mathematical expression of the measure, its properties and

some additional information. These properties are the elements that should be considered in the index choice and they are detailed next:

- Axiomatic properties: the collection of properties satisfied by each index. The main properties considered in the literature are scale invariance (SI), the principle of population (PP), the weak principle of transfers (WPT), the strong principle of transfers (SPT) or additive decomposability (AD).
- Upper bounds: while some measures are bounded between two points, there are some measures that are unbounded and can take any value, generally any positive number.
- Interpretability: an inequality index is considered to be more interpretable either if its formulation is easy or if it a graphic analogy.
- Comparative standard: some indexes describe how much apart the top and bottom extremes are, others measure the dispersion of the data around a reference point and others compare the current state to a reference point such as the highest-ranked level.
- Groups: some indexes quantify inequality in terms of total variation within the population, whereas others quantify it as the variation between the various groups existing in the population. In the former, the attribute studied is analysed for all members of the population. In the latter, inequality is evaluated between groups of individuals. Several issues in relation to these groups need to be born in mind. Firstly, some group categories can be naturally ordered. However, other categories do not allow for such a ranking, which might be an impediment in cases in which the index requires the groups to have an implicit ranking. Secondly, the measure of inequality can include all the existing social groups or it can just reflect inequalities between specific groups; for instance, between extreme groups.
- Relative vs absolute: relative or absolute inequality depends on whether the measure is dependent or not on the overall level of the attribute under evaluation.
- Sensitivity: some indexes have a parameter that allows researchers to pose more or less importance on certain parts of the distribution of the analysed attribute.

A last aspect to consider is the quality and quantity of available data. The above-mentioned aspects are related to requirements posed on the index, but some restrictions might be imposed by the characteristics of the data available. If data is extremely scarce, the greater accuracy that more complex measures may decrease. Additionally, in terms of efficiency, carrying out more complicated operations will only add complexity to the problem but not additional information or precision.

All the characteristics described above contribute to the choice of a specific measure. The unified decision framework in Fig. 11 allows to visually facilitate this choice to researchers and practitioners. Note that while it can be helpful, it is still a model and therefore a simplified version which does not consider other possible factors influencing the decision.

First of all, it is necessary to identify two basic aspects: whether the analysis will evaluate one or more dimensions and whether the data available is qualitative or quantitative. As stated in the beginning, this paper has only considered a unidimensional setting; consequently, the aspects that are discussed next are restricted to studies in which only one dimension is analysed.

The first scenario is one in which the data is quantitative and socioeconomic classes need to be considered. If these groups have a natural ordering, three possible indexes can be used. The SII adequate if inequality is measured in absolute terms, and the CI and the RII if it is measured in relative terms. Both measures can be graphically interpreted, although the concentration index's graphical meaning is more straightforward.

Otherwise, if the socioeconomic groups can't be ordered, V and CV are simple measures for absolute and relative inequality, respectively. However, V does not satisfy SI; hence, when

performing analyses in which magnitudes might be subject to change, the CV is more recommendable. If the measure's main purpose is not to reach a broad public, the choice depends basically on the comparative standard needed. More complex measures of between-group analysis of inequality are the ID_{sm} , the MLD, the RMD and the GE, whose comparative standard is centred. Otherwise, the ID_{sp} and the PAF are reference-based and therefore they allow to set a specific reference level, which can be, for instance, the highest-ranked group or the ideal level of the attributed studied. Note that the ID_{sm} is only suited for analyses in which only two groups are compared. Regarding these measures, results may depend on whether the data is weighted by the groups' size. Deciding whether to use a weighted or an unweighted method will depend on the purpose and kind of analysis to be performed. Weighting the formulation by the population share of each group allows incorporating information on changes in the size of the different socioeconomic groups.

In a second scenario where it is not necessary to reflect inequality between socioeconomic groups, if the measure has a communicative purpose, we recommend the use of measures that are easily interpretable. In this sense, the GI, the R and the PR can be used. On the one hand, the GI is appealing due to its straightforward interpretation given its range from 0 to 1, which gives a quick message of whether the existing inequality is high (when values are close to 1) or low (for values that are close to 0) and to the possibility of graphically representing it through the Lorenz curve. On the other hand, we propose the R and the PR to measure variations between extremes. The R provides a measure of absolute inequality and even though it is not commonly used due to its high sensitivity to possible outliers, if the researcher's purpose is to provide an interval within which the variable fluctuates, it may be appropriate. This interval will have the order of magnitude of the original units, even though it can be made relative by dividing it by the mean. PR are more recommendable when it's necessary to compare the top and bottom proportional differences of a group of individuals. It has to be born in mind that it is insensitive to values in the middle of the distribution.

If it is not necessary for the measure to reach broad publics, the sensitivity and the graphical analogy are helpful to differentiate between indexes. If no specific sensitivities are needed, then the most suitable indexes are ZI, GI, LV, VL, MLD and RMD. Among these, ZI and GI can be graphically represented. However, if the measure requires to be sensitive to different parts of the distribution, BI is suitable if a graphical representation is necessary; if no graphical representation is needed, then AI and GE are applicable. Note that whereas AI and GE allow the researcher to adapt the sensitivity through a parameter, BI is invariably sensitive to lower parts of the distribution. With regard to GE, it is recommended when it is necessary to study the different components of inequality through the decomposition of the index. Even though it upholds all the presented axioms for inequality measures, its calculation complexity and lack of straightforward interpretation make it unattractive whenever decomposability is not necessary. As for AI, apart from the fact that it does not allow decomposition and despite its welfare assumptions, there is a difficulty implied with the determination of the sensitivity parameter, which must be done either empirically or assigned arbitrarily by the analyst.

In a third scenario, if the data is qualitative, the measure proposed by Wagstaff 2005 is suitable if the variable is binary. Otherwise, in the case in which it is necessary to consider the socioeconomic component of inequality, Allanson 2017 proposes a methodology that is appropriate for this case, whereas Abul Naga and Yalcin 2008, Kobus and Miłoś 2012, Lv et al. 2015 and Cowell and Flachaire 2017 present measures that can be used if the reflection of socioeconomic classes is not necessary.

Having said this, the justification for using one index among the existing wide variety is always contingent on the analyst and on how the study is approached. At the same time, this wide variety

of indexes is the source of potential ambiguities in the assessment of inequality because different indexes of inequality will often yield different rankings of inequality.

The choice of the index that is more adequate to the characteristics of the analysis is a key step in analysing inequality considering that the policy implications that each one has and the meaning that is given to inequality is different. The statistical choice will reflect an ethical choice as well. Once this decision is made, the measure chosen is helpful in performing a three-dimensional analysis of inequality. Not only do the indexes allow for an analysis across different dimensions by choosing multiple indicators that are relevant to the inequality framework, but they also allow for two other alternative analyses: on the one hand, carrying out the calculations at different points in time implies that an evaluation of how the index is developing can be made. This is key in assessing, for instance, the actual impact of policies aiming at reducing inequalities. On the other hand, instead of studying inequality across a given timeline, it is also possible to do it for different interest groups. Namely, given the region that is under examination, to perform an evaluation of inequality within various collectives based on factors such as gender, age, ethnic group or location. In fact, the value of the inequality index depends heavily on how the analysed attribute is defined (for example, at household or individual level, or in the case of income, excluding or not incomes from informal sectors); this might give rise to subjective evaluations and wrong interpretations of the coefficient.

6. Conclusions

A framework to support the choice of an adequate inequality index has been proposed in this paper. The framework is based on an extensive review of the different methodologies used to analyse and calculate unidimensional inequality from a cardinal approach. The methods analysed have been classified into different categories depending on the type of data for which they were developed (qualitative or quantitative) and on whether the measures consider or not socioeconomic groups. The advantages and disadvantages of each method have further been shown through empirical applications.

The seven main characteristics of inequality measures that can be used as a basis for the choice are their axiomatic properties, the existence of an upper bound, its graphical analogy, the comparative standard used, if it considers socioeconomic groups, whether it is absolute or relative and the sensitivity of the measure. As for the data, it is necessary to consider the type, amount and quality of data available.

This study can be considered as being the first step towards enhancing decision-making methods when it comes to inequality analysis. The results can be used to improve mechanisms for the choice of inequality indexes based on factors that depend on the characteristics of the index and on the data available. In addition, new applications of the indexes have been suggested through a three-dimensional analysis. This broader analysis encompasses three aspects: time, dimensions and studied groups.

Future studies could aim at addressing two main flaws that have been found in the literature concerning unidimensional measures of inequality. Firstly, further research in the measurement of inequality with ordinal data is needed; secondly, it is necessary to continue studying the evaluation of inequality between socioeconomic groups.

Finally, we believe that a quantitative analysis of inequality should always be accompanied by a qualitative one. In this way, numerical studies can be adjusted to the real conditions and qualitative research can be supported by real data-based claims. A balance of this kind is prone to lead to better conclusions on the causes and consequences of inequality while ensuring better evidence-informed policymaking to tackle it.

	Reference	Field	Application	Expression ^a	Properties	Upper bound	Graphical analogy	Comparative standard	SE groups ^b	Relative/Absolute	Sensitivity	Additional remarks
Range (R)	Hao and Naiman 2010	Statistics	Statistics, economics, health	$y_{max} - y_{min}$	PP	✖	✖	Extremes	✖	Absolute	✖	Simplicity in calculation, but excessively influenced by outliers.
Variance (V)	Hao and Naiman 2010	Statistics	Statistics, economics, health	$\frac{1}{n} \sum_{i=1}^n (y_i - \bar{y})^2$	PP, WPT, SPT, AD	✖	✖	Centre	(✖)	Absolute	✖	Influenced by the measure's magnitudes.
Coefficient of variation (CV)	Hao and Naiman 2010	Statistics	Statistics, economics, health	$\frac{\sqrt{\frac{1}{n} \sum_{i=1}^n (y_i - \bar{y})^2}}{\bar{y}}$	SI, PP, WPT, AD	✖	✖	Centre	(✖)	Relative	✖	No influence from the measure's magnitude, in comparison with variance. Half the squared CV is a special case of GE when $\theta=2$.
Logarithmic variance (LV)	Hao and Naiman 2010	Statistics	Statistics, economics, health	$\frac{1}{n} \sum_{i=1}^n (\log(y_i) - \log \bar{y})^2$	SI, PP, WPT	✖	✖	Centre	✖	Relative	✖	Useful to reduce skewness of a distribution.
Variance of logarithms (VL)	Hao and Naiman 2010	Statistics	Statistics, economics, health	$\frac{1}{n} \sum_{i=1}^n (\log(y_i) - \overline{\log(y)})^2$	SI, PP, WPT	✖	✖	Centre	✖	Relative	✖	Useful to reduce skewness of a distribution.
Mean log deviation (MLD)	Harper and Lynch 2005	Statistics	Statistics, economics, health, urbanism	$\frac{1}{n} \sum_{i=1}^n \ln \frac{\bar{y}}{y_i}$	SI, PP, WPT, SPT, AD	✖	✖	Centre	(✖)	Relative	✖	Special case of GE when $\theta=0$.
Relative mean deviation (RMD)	Kondor 1971, Mehran 1976	Statistics	Statistics, economics	$\frac{\frac{1}{n} \sum_{i=1}^n y_i - \bar{y} }{\bar{y}}$	SI, PP, WPT	✖	✖	Centre	(✖)	Relative	✖	
Percentile ratios (PR)	Hao and Naiman 2010	Statistics	Statistics, economics	$\frac{Y_{p_2}}{Y_{p_1}}$	SI, PP	✓	✖	Extremes	✖	Relative	✖	Flexibility in deciding which quantiles are the most meaningful depending on the subject studied, but insensitive to middle values.
Gini index (GI)	Lorenz 1905, Gini 1912	Economics	Economics, health, environment, ecology, opportunity, environment, sustainability	$\sum_{i=1}^n \sum_{j=1}^n \frac{ y_i - y_j }{2n \sum_{i=1}^n y_i}$	SI, PP, WPT	✓	✓	Centre	✖	Relative	✓ (in extended versions)	Intuitive interpretation thanks to its graphical analogue. However, complex computations are needed; sensitivity to transfers especially in the middle range.
Bonferroni index (BI)	Bonferroni 1930	Economics	Economics	$\frac{1}{n-1} \sum_{i=1}^n \frac{\bar{y} - y_i}{\bar{y}}$	SI, PP, WPT	✓	✓	Centre	✖	Relative	✖	Higher sensitivity to transfers in the lower end of the distribution. Complex computations are needed.
Zenga index (ZI)	Zenga 1984, 2007	Economics	Economics	$\frac{1}{n} \sum_{i=1}^n 1 - \frac{\bar{y}_{1:i}}{\bar{y}_{i:n}}$	SI, PP, WPT	✓	✓	Centre	✖	Relative	✖	Complex computations are needed.
Theil index (TI)	Theil 1967	Economics, socioeconomic s	Economics, socioeconomic s	$\frac{1}{n} \sum_{i=1}^n \frac{y_i}{\bar{y}} \left[\log \left(\frac{y_i}{\bar{y}} \right) \right]$	SI, PP, WPT, SPT, AD	✖	✖	Centre	(✖)	Relative	✖	Special case of GE when $\theta=1$.

Generalised entropy (GE)	Toyoda 1975, Cowell and Kuga 1981	Economics	Economics, socioeconomics	$\frac{1}{\theta^2 - \theta} \left[\frac{1}{n} \sum_{i=1}^n \left(\frac{y_i}{\bar{y}} \right)^\theta - 1 \right]$	SI, PP, WPT, SPT, AD	✖	✖	Centre	(✖)	Relative	✓ (parameter θ)	Possible decomposition into within and between group components is possible. No intuitive meaning; mathematically complex.
Atkinson index (AI)	Atkinson 1970	Economics	Economics	$1 - \left[\frac{1}{n} \sum_{i=1}^n \left(\frac{y_i}{\bar{y}} \right)^{1-\varepsilon} \right]^{\frac{1}{1-\varepsilon}}$	SI, PP, WPT	✓	✖	Centre	✖	Relative	✓ (parameter ε)	Reflection of the societal goals of equality. Difficulties involved in determining the index's parameter.
Index of dissimilarity (IDsm)	Duncan and Duncan 1955	Geography	Demography, geography, education, occupation, socio-economics, environment	$\frac{1}{2} \sum_j p_{jh} - p_{jp} $	SI, PP, WPT	✓	✖	Centre	Non-ordered	Relative	✖	Comparison between the summary measure in each group with the total population.
Index of disparity (IDsp)	Pearcy and Keppel 2002	Health	Health	$\left(\sum_{j=1}^{J-1} \frac{ r_j - r_{ref} }{J} \right) / r_{ref} \times 100$	SI, PP, WPT	✓	✖	Reference	Non-ordered	Relative	✖	Comparison between the difference between group rates and a reference rate. It yields RMD when r_{ref} is the mean value.
Population attributable fraction (PAF)	Yeracaris and Kim 1978	Health	Health	$\frac{\sum p_j (y_j - y_{ref})}{\sum p_j (y_j - y_{ref}) + y_{ref}}$	SI, PP, WPT	✖	✖	Reference	Non-ordered	Relative	✖	Comparison of the rate of an attribute in the total population with a reference group.
Concentration index (CI)	Wagstaff et al. 1991	Health	Health	$\frac{2}{\bar{y}} \sum_{i=1}^n (y_i - \bar{y}) (x_i - \frac{1}{2})$	SI, PP, WPT, AD	✓	✓	Centre	Ordered	Relative	✓ (in extended versions)	Wagstaff et al. 1991 also proposed a generalised concentration index, which gives an absolute measure of inequality. The sign indicates the gradient of the relationship between the variable and the socioeconomic rank.
Slope index of inequality (SII)	Pamuk 1985	Health	Health	$\frac{\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^n (x_i - \bar{x})^2}$	PP, WPT	✖	✓	Centre	Ordered	Absolute	✖	The sign indicates the gradient of the relationship between the variable and the socioeconomic rank.
Relative index of inequality (RII)	Pamuk 1985	Health	Health	$\frac{1}{\bar{y}} \frac{\sum_{i=1}^n (x_i - \bar{x})(y_i - \bar{y})}{\sum_{i=1}^n (x_i - \bar{x})^2}$	SI, PP, WPT	✓	✓	Centre	Ordered	Relative	✖	The sign indicates the gradient of the relationship between the variable and the socioeconomic rank.

^a y refers to the attribute studied, x is population ranked by socioeconomic status, p_j is group j 's population share, p_{jh} is group j 's population share of the population's attribute and p_{jp} is group j 's population share, r is the group's attribute rate, J is the number of groups compared, ε and θ are sensitivity parameters

^b (✖) refers to the fact that the measure can be adapted to reflect SE groups

Table 1 Characteristics of the reviewed indexes

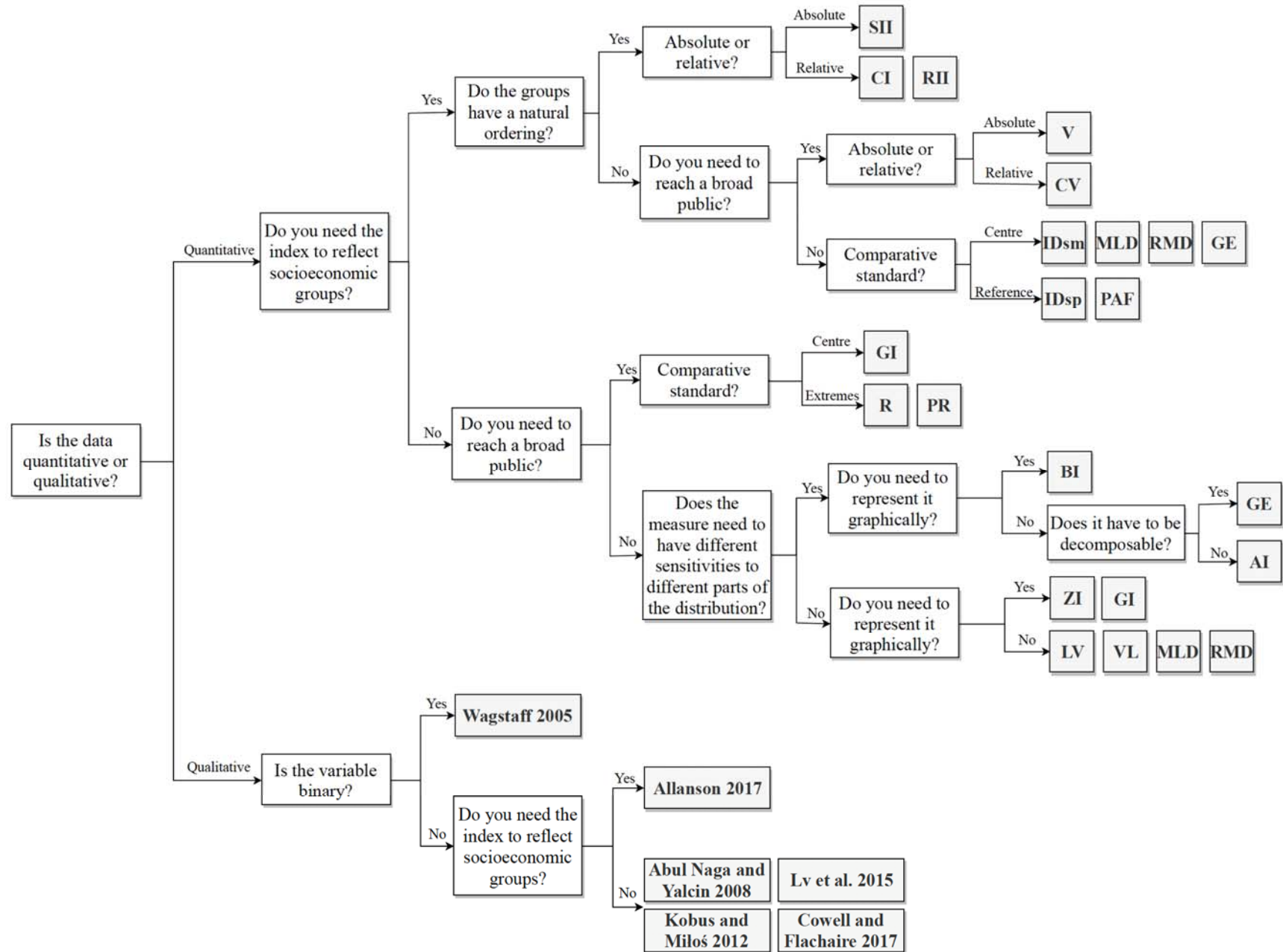


Fig. 11 Decision flowchart for the choice of an appropriate inequality index

Appendix

Table 2 shows how countries have been grouped according to the indicator “mean years of schooling”. The value of 15 has been chosen as the maximum as it is considered the projected maximum of the indicator for 2025 (United Nations 2018). The number of countries in each group is also shown for six years for illustrative purposes.

Group	Range of indicator “mean years of schooling”	Number of countries in the group					
		1990	1995	2000	2005	2010	2015
1	$0 \leq i \leq 3$	29	23	17	13	8	6
2	$3 < i \leq 6$	46	42	36	33	29	24
3	$6 < i \leq 9$	47	41	43	41	41	42
4	$9 < i \leq 12$	22	38	46	47	52	51
5	$12 < i \leq 15$	1	2	2	10	14	21

Table 2 Groups used for the empirical application of inequality between countries grouped by socioeconomic classes

References

- Abul Naga, R. H., & Yalcin, T. (2008). Inequality measurement for ordered response health data. *Journal of Health Economics*. <http://doi.org/10.1016/j.jhealeco.2008.07.015>
- Allanson, P. (2017). Monitoring income-related health differences between regions in Great Britain: A new measure for ordinal health data. *Social Science & Medicine*, 175, 72–80.
- Allison, R. A., & Foster, J. E. (2004). Measuring health inequality using qualitative data. *Journal of Health Economics*. <http://doi.org/https://doi.org/10.1016/j.jhealeco.2003.10.006>
- Amato, V. (1968). *Metodologia Statistica Strutturale*, Vol. 1. Cacucci, Bari.
- Apouey, B. (2007). Measuring health polarization with self-assessed health data. *Health Economics*. <http://doi.org/10.1002/hec.1284>
- Arnold, B. C. (2012). On the Amato inequality index. *Statistics and Probability Letters*. <http://doi.org/10.1016/j.spl.2012.04.020>
- Atkinson, A. B. (1970). On the Measurement of Inequality. *Journal of Economic Theory*, 2, 244–263.
- Atkinson, A. B. (1983). *Social Justice and Public Policy*. Wheatsheaf Books.
- Bonferroni, C. (1930). *Elementi di Statistica Generale*. Seeber, Firenze.
- Bourguignon, F. (1979). Decomposable Income Inequality Measures. *Econometrica*. <http://doi.org/10.2307/1914138>
- Chakravarty, S. R. (1988). Extended Gini Indices of Inequality. *International Economic Review*, 29(1), 147–156.
- Chiu, W. H. (2007). Intersecting Lorenz Curves, the Degree of Downside Inequality Aversion, and Tax Reforms. *Social Choice and Welfare*. <http://doi.org/10.1007/s00355-006-0170-7>
- Cobham, A., & Sumner, A. (2013). Is It All About the Tails? The Palma Measure of Income Inequality. *Center For Global Development Working Paper 343*.
- Cobham, A., Schlögl, L., & Sumner, A. (2015). Inequality and the Tails: The Palma Proposition and Ratio. *Global Policy*. <http://doi.org/10.1111/1758-5899.12320>

- Coulter, F. A. E., Cowell, F. A., & Jenkins, S. P. (1992). Differences in needs and assessment of income distributions. *Bulletin of Economic Research*. <http://doi.org/10.1111/j.1467-8586.1992.tb00538.x>
- Cowell, F. A. (1980). Generalized entropy and the measurement of distributional change. *European Economic Review*. [http://doi.org/10.1016/0014-2921\(80\)90051-3](http://doi.org/10.1016/0014-2921(80)90051-3)
- Cowell, F. A., & Kuga, K. (1981). Additivity and the entropy concept: An axiomatic approach to inequality measurement. *Journal of Economic Theory*. [http://doi.org/10.1016/0022-0531\(81\)90020-X](http://doi.org/10.1016/0022-0531(81)90020-X)
- Cowell, F. (1988). Inequality Decomposition: Three Bad Measures. *Bulletin of Economic Research*, 40(4), 309–312.
- Cowell, F. (2009). Measuring inequality. London: London School of Economics perspectives in economic analysis.
- Cowell, F. A., & Fiorio, C. V. (2011). Inequality decompositions – a reconciliation. *The Journal of Economic Inequality*. <http://doi.org/10.1007/s10888-011-9176-1>
- Cowell, F. A., & Flachaire, E. (2017). Inequality with Ordinal Data. *Economica*. <http://doi.org/10.1111/ecca.12232>
- Creedy, J. (1977). The Principle of Transfers and the Variance of Logarithms. *Oxford Bulletin of Economics and Statistics*, 39, 152-158
- Dagum, C. (1997). A New Approach to the Decomposition of the Gini Income Inequality Ratio. *Empirical Economics*, 515–531.
- Dalton, H. (1920). The Measurement of the Inequality of Incomes. *Royal Economic Society*, 30(119), 348–361.
- De Maio, F. G. (2007). Income inequality measures. *Journal of Epidemiology and Community Health*. <http://doi.org/10.1136/jech.2006.052969>
- Decancq, K., Decoster, A., & Schokkaert, E. (2009). The Evolution of World Inequality in Well-being. *World Development*. <http://doi.org/10.1016/j.worlddev.2007.12.006>
- Démurger, S., Fournier, M. & Li, S. (2006). Urban income inequality in China revisited (1988–2002). *Economics letters*. <http://doi.org/10.1016/j.econlet.2006.06.010>
- Donaldson, D., & Weymark, J. A. (1980). A single-parameter generalization of the Gini indices of inequality. *Journal of Economic Theory*. [http://doi.org/10.1016/0022-0531\(80\)90065-4](http://doi.org/10.1016/0022-0531(80)90065-4)
- Dorfman, R. (1979). A Formula for the Gini Coefficient. *The Review of Economics and Statistics*, 61(1), 146–149.
- Druckman, A., & Jackson, T. (2008). Measuring resource inequalities: The concepts and methodology for an area-based Gini coefficient. *Ecological Economics*. <http://doi.org/https://doi.org/10.1016/j.ecolecon.2007.12.013>
- Duncan, O. D., & Duncan, B. (1955). A Methodological Analysis of Segregation Indexes. *American Sociological Review*, 20(2), 210–217.
- Ebert, U. (2010). The decomposition of inequality reconsidered: Weakly decomposable measures. *Mathematical Social Sciences*. <https://doi.org/10.1016/j.mathsocsci.2010.05.001>
- Eliazar, I. (2015). The sociogeometry of inequality: Part I. *Physica A: Statistical Mechanics and its Applications*. <http://doi.org/10.1016/j.physa.2015.01.016>
- Eliazar, I. (2015). The sociogeometry of inequality: Part II. *Physica A: Statistical Mechanics and its Applications*. <http://doi.org/10.1016/j.physa.2014.12.021>
- Eliazar, I. (2016). Harnessing inequality. *Physics Reports*. <http://doi.org/10.1016/j.physrep.2016.07.005>
- Eliazar, I., & Sokolov, I. M. (2012). Measuring statistical evenness: A panoramic overview. *Physica A: Statistical Mechanics and its Applications*. <http://doi.org/10.1016/j.physa.2011.09.007>

- Erreygers, G. (2006). Beyond the health Concentration Index: an Atkinson alternative for the measurement of the socioeconomic inequality of health. *Paper presented at the conference Advancing Health Equity*. Helsinki: WIDER-UNU.
- Erreygers, G. (2009a). Can a single indicator measure both attainment and shortfall inequality? *Journal of Health Economics*. <http://doi.org/10.1016/j.jhealeco.2009.03.005>
- Erreygers, G. (2009b). Correcting the concentration index. *Journal of Health Economics*, 28, 504–515.
- Erreygers, G. (2009c). Correcting the concentration index: a reply to Wagstaff. *Journal of Health Economics*, 28, 521–524.
- Fei, J. C. H., Ranis, G., & Kuo, S. W. Y. (1978). Growth and the family distribution of income by factor components. *Quarterly Journal of Economics*. <http://doi.org/10.2307/1885997>
- Fernández-Tschieder, E., & Binkley, D. (2018). Linking competition with Growth Dominance and production ecology. *Forest Ecology and Management*. <http://doi.org/10.1016/j.foreco.2018.01.052>
- Formby, J. P., & Seaks, T. G. (1980). Paglin's Gini Measure of Inequality: A Modification. *American Economic Association*, 70(3), 479–482.
- Foster, J. E., & Ok, E. A. (1999). Lorenz Dominance and the Variance of Logarithms. *Econometrica*, 67(4), 901–907.
- Foster, J. E. (1985). Inequality Measurement. In H. P. Young (Ed.), *Fair Allocation*. Providence: American Mathematical Society Proceedings in Applied Mathematics.
- Foster, J. E., & Shorrocks, A. F. (1988a). Poverty Orderings. *Econometrica*, 56, 173–177.
- Foster, J. E., & Shneyerov, A. A. (1999). A general class of additively decomposable inequality measures, 111, 89–111.
- Foster, J. E., & Shneyerov, A. A. (2000). Path Independent Inequality Measures. *Journal of Economic Theory*. <http://doi.org/https://doi.org/10.1006/jeth.1999.2565>
- Foster, J.E., L. Lopez-Calva, L.F. & Székely, M. (2005). Measuring the distribution of human development: methodology and application to Mexico. *Journal of Human Development*, 6(1), 5–30.
- Francois, J., & Rojas-Romagosa, H. (2011). Household inequality, social welfare, and trade. *Journal of Development Economics*. <http://doi.org/10.1016/j.jdeveco.2010.08.006>
- Futing Liao, T. (2006). Measuring and analyzing class inequality with the Gini index informed by model-based clustering. *Sociological methodology*, 36, 201–224.
- Gächter, M., & Theurl, E. (2011). Health Status Convergence at the Local Level: Empirical Evidence from Austria. *International journal for equity in health*. <http://doi.org/10.1186/1475-9276-10-34>
- Gayant, J.-P., & Le Pape, N. (2017). Increasing Nth degree inequality. *Journal of Mathematical Economics*. <http://doi.org/10.1016/j.jmateco.2017.02.010>
- Ghosh, A., Chattopadhyay, N., & Chakrabarti, B. K. (2014). Inequality in societies, academic institutions and science journals: Gini and k-indices. *Physica A: Statistical Mechanics and Its Applications*. <http://doi.org/https://doi.org/10.1016/j.physa.2014.05.026>
- Gini, C. (1912). *Variabilità e mutabilità. Contributo allo Studio delle Distribuzioni e delle Relazioni Statistiche*. C. Cuppini, Bologna.
- Giorgi, G. M., & Crescenzi, M. (2001). A proposal of poverty measures based on the Bonferroni inequality index. *Metron*, 59(3–4), 3–16.
- Hagedorn, L., Nora, A., & Pascarella, E. (1996). Preoccupational Segregation among First-Year College Students: An Application of the Duncan Dissimilarity Index. *Journal of College Student Development*, 37, 425–437.
- Hao, L. & Naiman, D. Q. (2010). *Assessing inequality*. SAGE publications

- Harper, S. & Lynch, J. (2005). *Methods for Measuring Cancer Disparities: Using Data Relevant to Healthy People 2010 Cancer Related Objectives*. University of Michigan, Center for Social Epidemiology and Population Health.
- Hart, P. E. (1975). Moment distributions in economics: an exposition. *Journal of the Royal Statistical Society, A*, 138, Part 3, 423-434.
- Harvey, J. (2005). A note on the “natural rate of subjective inequality” hypothesis and the approximate relationship between the Gini coefficient and the Atkinson index. *Journal of Public Economics*. <http://doi.org/10.1016/j.jpubeco.2004.05.002>
- Herrero, C., Martínez, R. & Villar, A. (2018). Population structure and the Human Development Index. *Social Indicators Research*. <https://doi.org/10.1007/s11205-018-1852-0>
- Howarth, R. B., & Kennedy, K. (2016). Economic growth, inequality, and well-being. *Ecological Economics*. <http://doi.org/10.1016/j.ecolecon.2015.10.005>
- Hutchens, R. M. (1991). Segregation curves, Lorenz curves, and inequality in the distribution of people across occupations. *Mathematical Social Sciences*. [http://doi.org/https://doi.org/10.1016/0165-4896\(91\)90038-S](http://doi.org/https://doi.org/10.1016/0165-4896(91)90038-S)
- Hutchens, R. (2015). Symmetric measures of segregation, segregation curves, and Blackwell's criterion. *Mathematical Social Sciences*. <http://doi.org/10.1016/j.mathsocsci.2014.11.007>
- Imedio-Olmedo, L. J., Bárcena-Martín, E., & Parrado-Gallardo, E. M. (2011). A Class of Bonferroni Inequality Indices. *Journal of Public Economic Theory*. <http://doi.org/10.1111/j.1467-9779.2010.01494.x>
- James, D. R., & Taeuber, K. E. (1985). Measures of Segregation. *Sociological Methodology*, 15, 1–32.
- Kakwani, N., Wagstaff, A., & van Doorslaer, E. (1997). Socioeconomic inequalities in health: Measurement, computation, and statistical inference. *Journal of Econometrics*. [http://doi.org/10.1016/S0304-4076\(96\)01807-6](http://doi.org/10.1016/S0304-4076(96)01807-6)
- Kalmijn, W., & Veenhoven, R. (2005). Measuring Inequality of Happiness in Nations: In Search for Proper Statistics. *Journal of Happiness Studies*. <http://doi.org/10.1007/s10902-005-8855-7>
- Kjellsson, G., & Gerdtham, U.-G. (2013). On correcting the concentration index for binary variables. *Journal of Health Economics*. <http://doi.org/https://doi.org/10.1016/j.jhealeco.2012.10.012>
- Kobayashi, D., Otsubo, T., & Imanaka, Y. (2015). The effect of centralization of health care services on travel time and its equality. *Health Policy*. <http://doi.org/10.1016/j.healthpol.2014.11.008>
- Kobus, M., & Miłoś, P. (2012). Inequality decomposition by population subgroups for ordinal data. *Journal of Health Economics*. <http://doi.org/https://doi.org/10.1016/j.jhealeco.2011.11.005>
- Kobus, M., Półchłopek, O., & Yalonetzky, G. (2018). Inequality and Welfare in Quality of Life Among OECD Countries: Non-parametric Treatment of Ordinal Data. *Social Indicators Research*. <http://doi.org/10.1007/s11205-018-1962-8>
- Kondor, Y. (1971). An Old-New Measure of Income Inequality. *Econometrica*. <http://doi.org/10.2307/1909676>
- Lahelma, E., & Valkonen, T. (1990). Health and social inequities in Finland and elsewhere. *Social Science & Medicine*. [http://doi.org/https://doi.org/10.1016/0277-9536\(90\)90272-T](http://doi.org/https://doi.org/10.1016/0277-9536(90)90272-T)
- Lambert, P., & Aronson, J. (1993). Inequality Decomposition Analysis and the Gini Coefficient Revisited. *The Economic Journal*. <http://doi.org/10.2307/2234247>
- Lasso de la Vega, C., & Urrutia, A. (2008). The “Extended” Atkinson family: The class of multiplicatively decomposable inequality measures, and some new graphical procedures for analysts. *Journal of Economic Inequality*. <http://doi.org/10.1007/s10888-006-9053-5>
- Latty, K. (2011). Income distribution, growth and social-welfare: towards an economic solution to the growth-equality trade-off problem. *University of Sydney*. <http://hdl.handle.net/2123/8260>

- Lazar, A., & Silber, J. (2013). On the cardinal measurement of health inequality when only ordinal information is available on individual health status. *Health Economics*. <http://doi.org/10.1002/hec.1821>
- Le Grand, J., & Rabin, M. (1986). Trends in British health inequality, 1931-83. In: Culyer, A. J. and Jonsson, B., (Ed.) *Public and Private Health Services: Complementarities and Conflicts*. Basil Blackwell, Oxford, UK.
- Le Grand, J. (1989). An international comparison of distributions of ages-at-death. In J. Fox (Ed.), *Health Inequality in European Countries*. Aldershot: Gower.
- Leclerc, A., Lert, F., & Fabien, C. (1990). Differential mortality: some comparisons between England and Wales, Finland and France, based on inequality measures. *International Journal of Epidemiology*. <http://doi.org/10.1093/ije/19.4.1001>
- Leon, D., Vågerö, D. & Olausson, P. (1992). Social Class Differences in Infant Mortality in Sweden: A Comparison with England and Wales. *BMJ Clinical research*. <http://dx.doi.org/10.1136/bmj.305.6855.687>.
- Lerman, R. I., & Yitzhaki, S. (1984). A note on the calculation and interpretation of the Gini index. *Economics Letters*. [http://doi.org/10.1016/0165-1765\(84\)90126-5](http://doi.org/10.1016/0165-1765(84)90126-5)
- Lerman, R. I., & Yitzhaki, S. (1985). Income inequality effects by income source: a new approach and applications to the United States. *The Review of Economics and Statistics*, 87(1), 184–193.
- Lorenz, M. O. (1905). Methods of Measuring the Concentration of Wealth. *Publications of the American Statistical Association*, 9(70), 209–219.
- Lv, G., Wang, Y., & Xu, Y. (2015). On a new class of measures for health inequality based on ordinal data. *The Journal of Economic Inequality*. <https://doi.org/10.1007/s10888-014-9289-4>
- Mackenbach, J. P. (1992). Socio-economic health differences in the Netherlands: A review of recent empirical findings. *Social Science & Medicine*. [http://doi.org/10.1016/0277-9536\(92\)90264-Q](http://doi.org/10.1016/0277-9536(92)90264-Q)
- Madden, D. (2010). Ordinal and cardinal measures of health inequality: an empirical comparison. *Health Economics*. <http://doi.org/10.1002/hec.1472>
- Maffneni, W. & Poliscchio, M. (2014). The potential of the I(p) inequality curve in the analysis of empirical distributions. *Statistica & Applicazioni*. <https://doi.org/10.1400/229468>
- Majumdar, A. (2015). An alternative measure of economic inequality under the Lorenz curve framework in analogue to the index of refraction of geometrical optics. *Economics Bulletin*, 35, 1076–1086.
- Mehran, F. (1976). Linear measures of income inequality. *Econometrica*, 44, 805–809.
- Mussard, S., Seyte, F., & Terraza, M. (2003). Decomposition of Gini and the generalized entropy inequality measures. *Economics Bulletin*, 4(7), 1–6.
- Nair, K. R. M., & Sreelakshmi, N. (2016). The New Zenga curve in the context of reliability analysis. *Communications in Statistics – Theory and Methods*. <http://doi.org/10.1080/03610926.2013.763510>
- Ogaki, M., Ostry, J., & Reinhart, C. (1996). Saving Behavior in Low- and Middle-Income Developing Countries: A Comparison. *Staff Papers (International Monetary Fund)*. <http://doi.org/10.2307/3867352>
- Osberg, L. (2017). On the Limitations of Some Current Usages of the Gini Index. *Review of Income and Wealth*. <http://doi.org/10.1111/roiw.12256>
- Paglin, M. (1975). The Measurement and Trend of Inequality: A Basic Revision. *Economics Letters*, 65(598–609), 389–393.
- Palma, J. G. (2006). Globalizing inequality: “centrifugal” and “centripetal” forces at work. *DESA Working paper*, 35.
- Palma, J. G. (2014). Has the Income Share of the Middle and Upper-middle Been Stable around the “50/50 Rule”, or Has it Converged towards that Level? The “Palma Ratio” Revisited. *Development and Change*. <http://doi.org/10.1111/dech.12133>

- Pamuk, E. R. (1985). Social Class Inequality in Mortality From 1921 to 1972 in England and Wales. *Population Studies*. <http://doi.org/10.1080/0032472031000141256>
- Paul, S. (2004). Income sources effects on inequality. *Journal of Development Economics*. <http://doi.org/10.1016/j.jdeveco.2003.02.004>
- Pearcy, J. N., & Keppel, K. G. (2002). A summary measure of health disparity. *Public health reports*, 117(3), 273-80.
- Pietra, G. (1915) Delle relazioni tra gli indici di variabilità. Nota I. *Atti Regio Istituto Veneto* 74, Parte II: 775-792.
- Polisicchio, M., & Porro, F. (2009). A comparison between Lorenz L(p) curve and Zenga I(p) curve. *Statistica Applicata – Italian Journal of Applied Statistics*, 21(3–4), 289–301.
- Preston S. H. Haines M. R. & Pamuk E. (1981). Effects of industrialization and urbanization on mortality in developed countries. In Solicited Papers Vol 2, HJSSP 19th International Population Conference, Manila. IUSSP, Liege.
- Pyatt, G. (1976). On the Interpretation and Disaggregation of Gini Coefficients. *The Economic Journal*. <http://doi.org/10.2307/2230745>
- Ruitenbeek, H. J. (1996). Distribution of ecological entitlements: Implications for economic security and population movement. *Ecological Economics*. [http://doi.org/https://doi.org/10.1016/0921-8009\(95\)00103-4](http://doi.org/https://doi.org/10.1016/0921-8009(95)00103-4).
- Sastry, D.V.S., & Kelkar, U. R. (1994). Note on the Decomposition of Gini Inequality. *The Review of Economics and Statistics*, 76(3), 584–586.
- Seidl, R., Albrich, K., Thom, D., & Rammer, W. (2018). Harnessing landscape heterogeneity for managing future disturbance risks in forest ecosystems. *Journal of Environmental Management*. <http://doi.org/10.1016/j.jenvman.2017.12.014>
- Sen, A. (1974). Informational bases of alternative welfare approaches. Aggregation and income distribution. *Journal of Public Economics*. [http://doi.org/10.1016/0047-2727\(74\)90006-1](http://doi.org/10.1016/0047-2727(74)90006-1)
- Sen, A. (1997). From Income Inequality to Economic Inequality. *Southern Economic Journal*. <http://doi.org/10.2307/1060857>
- Shalit, H. (1985). Calculating The Gini Index Of inequality For Individual Data. *Oxford Bulletin of Economics and Statistics*. <http://doi.org/10.1111/j.1468-0084.1985.mp47002006.x>
- Shorrocks, A. F. (1980). The class of additively decomposable inequality measures. *Econometrica*, 48(3), 613–625.
- Shorrocks, A. F. (1982). Inequality Decomposition by Factor Components. *Econometrica*. 50(1): 193-211.
- Shorrocks, A. F. (1984). Inequality Decomposition by Population Subgroups. *Econometrica*. <http://doi.org/10.2307/1913511>
- Silber, J. (1989). Factor Components, Population Subgroups and the Computation of the Gini Index of Inequality, 71(1), 107–115.
- Silber, J. (1999). *Handbook of Income Inequality Measurement*. New York: Springer.
- Stymne, S., & Jackson, T. (2000). Intra-generational equity and sustainable welfare: A time series analysis for the UK and Sweden. *Ecological Economics*. [http://doi.org/10.1016/S0921-8009\(99\)00144-5](http://doi.org/10.1016/S0921-8009(99)00144-5)
- Subramanian, S. (2010). Tricks with the Lorenz curve. *Economics Bulletin*, 30(2), 1594–1602.
- Subramanian, S. (2015). More tricks with the Lorenz curve. *Economics Bulletin*, 35(1), 580–589.
- Taeuber, K. E., & Taeuber, A. F. (1976). A Practitioner's Perspective on the Index of Dissimilarity. *American Sociological Review*, 41(5), 884–889.
- Tavits, M., & Potter, J. D. (2014). The Effect of Inequality and Social Identity on Party Strategies. *American Journal of Political Science*. <https://doi.org/10.1111/ajps.12144>
- Theil, H. (1967). *Economics and information theory*. North-Holland Pub. Co.

- Toyoda, T. (1975). Inequalities of income distributions: their comparisons and inequality measures. *Kokumin Keizai*, 134, 15-41. In Japanese.
- Toyoda, T. (1980). Decomposability of inequality measures. *Economic Studies Quarterly*, 31, 207–216.
- United Nations (1999). Standard country or area codes for statistical use. ST/ESA/STAT/SER.M/49/Rev.4
- United Nations Development Programme (2016). Human Development Report 2016: Human Development for Everyone. Resource document. http://hdr.undp.org/sites/default/files/2016_human_development_report.pdf. Accessed 25 July 2018.
- United Nations (2018). Human development indices and indicators: 2018 statistical update. Technical notes. [https://unstats.un.org/unsd/publication/SeriesM/Series_M49_Rev3\(1996\)_en.pdf](https://unstats.un.org/unsd/publication/SeriesM/Series_M49_Rev3(1996)_en.pdf). Accessed 10 September 2019.
- van Doorslaer, E., Wagstaff, A., Bleichrodt, H., Calonge, S., Gerdtham, U.-G., Gerfin, M., Winkelhake, O. (1997). Income-related inequalities in health: some international comparisons. *Journal of Health Economics*. [http://doi.org/10.1016/S0167-6296\(96\)00532-2](http://doi.org/10.1016/S0167-6296(96)00532-2)
- van Doorslaer, E., Wagstaff, A., van der Burg, H., Christiansen, T., Graeve, D. De, Duchesne, I., Winkelhake, O. (2000). Equity in the delivery of health care in Europe and the US. *Journal of Health Economics*. [http://doi.org/10.1016/S0167-6296\(00\)00050-3](http://doi.org/10.1016/S0167-6296(00)00050-3)
- van Doorslaer, E., & Jones, A. M. (2003). Inequalities in self-reported health: validation of a new approach to measurement. *Journal of Health Economics*. [http://doi.org/10.1016/S0167-6296\(02\)00080-2](http://doi.org/10.1016/S0167-6296(02)00080-2)
- Voas, D., & Williamson, P. (2004). The Scale of Dissimilarity: Concepts, Measurement and an Application to Socio-Economic Variation Across England and Wales. *Transactions of the Institute of British Geographers*. <http://doi.org/10.1111/j.0020-2754.2000.00465.x>
- Wagstaff, A., Van Doorslaer, E., & Paci, P. (1989). Equity in the finance and delivery of health care: some tentative cross-country comparisons. *Oxford Review of Economic Policy*. <http://doi.org/10.1093/oxrep/5.1.89>
- Wagstaff, A., Paci, P., & Van Doorslaer, E. (1991). On the measurement of inequalities in health. *Social Science & Medicine*. [http://doi.org/https://doi.org/10.1016/0277-9536\(91\)90212-U](http://doi.org/https://doi.org/10.1016/0277-9536(91)90212-U).
- Wagstaff, A. & Van Doorslaer, E. (1994), Measuring inequalities in health in the presence of multiple - category morbidity indicators. *Health Economics*. <http://doi.org/10.1002/hec.4730030409>
- Wagstaff, A., van Doorslaer, E., van der Burg, H., Calonge, S., Christiansen, T., Citoni, G., Gerdtham, U., Gerfin, M., Gross, L., Häkinnen, U., Johnson, P., John, J., Klavus, J., Lachaud, C., Lauritsen, J., Leu, R., Nolan, B., Perán, E., Pereira, J., Propper, C., Puffer, F., Rochaix, L., Rodríguez, M., Schellhorn, M., Sundberg, G., Winkelhake, O. (1999). Equity in the finance of health care: some further international comparisons *Journal of Health Economics*. [http://doi.org/10.1016/S0167-6296\(98\)00044-7](http://doi.org/10.1016/S0167-6296(98)00044-7)
- Wagstaff, A. (2005) The Bounds of the Concentration Index When the Variable of Interest Is Binary, with an Application to Immunization Inequality. *Health Economics*. <http://dx.doi.org/10.1002/hec.953>
- Wagstaff, A. (2009). Correcting the concentration index: a comment. *Journal of Health Economics*, 28, 516–520.
- Wagstaff, A. (2011). The concentration index of binary outcome revisited. *Health Economics*, 20, 1155–1160.
- Weiss, D. J., Nelson, A., Gibson, H. S., Temperley, W., Peedell, S., Lieber, A. & Gething, P. W. (2018). A global map of travel time to cities to assess inequalities in accessibility in 2015. *Nature*. <http://doi.org/10.1038/nature25181>

- Weymark, J. A. (1981). Generalized Gini inequality indices. *Mathematical Social Sciences*, [http://doi.org/10.1016/0165-4896\(81\)90018-4](http://doi.org/10.1016/0165-4896(81)90018-4)
- Wing, S., Casper, M., Davis, W., Hayes, C., Riggan, W., & Tyroler, H.A. (1990). Trends in the geographic inequality of cardiovascular disease mortality in the United States, 1962-1982. *Social science & medicine* (1982). [http://doi.org/10.1016/0277-9536\(90\)90181-Q](http://doi.org/10.1016/0277-9536(90)90181-Q).
- Xu, K. (2003). How has the Literature on Gini's Index Evolved in the Past 80 Years? *SSRN Electronic Journal*, <http://doi.org/10.2139/ssrn.423200>
- Yalonetzky, G. (2015). Index of Dissimilarity. In *The Wiley Blackwell Encyclopedia of Race, Ethnicity, and Nationalism*. American Cancer Society. <http://doi.org/10.1002/9781118663202.wberen484>
- Yeracaris, C. A., & Kim, J. H. (1978). Socioeconomic differentials in selected causes of death. *American Journal of Public Health*, 68(4), 342–351.
- Yitzhaki, S. (1983). On an Extension of the Gini Inequality Index. *International Economic Review*, <http://doi.org/10.2307/2648789>
- Yitzhaki, S., & Schechtman, E. (2013). The Gini Methodology: A Primer on a Statistical Methodology. *Springer Series in Statistics*. <http://doi.org/10.1007/978-1-4614-4720-7>
- Zenga, M. (1984). Tendenza alla massima ed alla minima concentrazione per variabili casuali continue. *Statistica*, (44): 3–27.
- Zenga, M. (2007). Inequality curve and inequality index based on the ratios between lower and upper arithmetic means. *Statistica & Applicazioni*, (5): 3–28
- Zheng, B. (2007). Unit-consistent decomposable inequality measures. *Economica*, <http://doi.org/10.1111/j.1468-0335.2006.00524.x>
- Zenga, M., Pasquazzi, L., Poliscchio, M., & Zenga, M. (2011). More on M.M. Zenga's new three-parameter distribution for nonnegative variables. *Statistica & Applicazioni*, 9(1), 5-33.
- Zheng, B. (2008). Measuring inequality with ordinal data: a note. In: Bishop, J., Zheng, B. (Eds.), *Research on Economic Inequality*, vol. 16. Emerald Group, Bingley, pp. 177–188.