The consequences of Zipf's law for syntax and symbolic reference.

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Abstract

Although many species possess rudimentary communication systems (Hauser, 1996; Újhelyi, 1996), humans seem to be unique with regard to making use of syntax (Hauser, Chomsky, & Fitch, 2002) and symbolic reference (Deacon, 1997; Donald, 1991, 1998). Recent approaches to the evolution of language formalize why syntax is selectively advantageous compared to isolated signal communication systems (Nowak & Krakauer, 1999; Nowak, Plotkin, & Jansen, 2000), but they do not explain how signals naturally combine.

Even more recent work has shown that if a communication system maximizes the communicative efficiency while minimizing the cost of communication (Ferrer i Cancho & Solé, 2003) or if a communication system constrains ambiguity in a non-trivial way while a certain entropy is maximized (Ferrer i Cancho, 2004), signal frequencies will be distributed according to Zipf's law. Here we show that such communication principles give rise not only to signals that have many traits in common with the linking words in real human languages, but also to a rudimentary sort of syntax and symbolic reference.

Keywords: Zipf's law, syntax, symbolic reference and human language.

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1 Introduction

Word frequencies in human languages tend to obey Zipf's law (Zipf, 1972), which states that, for some $\beta > 0$,

$$p_f \sim f^{-\beta},\tag{1}$$

where p_f is the proportion of words whose frequency in a given sample is f. Usually $\beta \approx 2$ is found (Ferrer i Cancho, 2004).

Zipf's law has been shown to appear when simultaneously maximizing the communicative efficiency and minimizing the cost of communication (Ferrer i Cancho & Solé, 2003). Alternatively, constraining the ambiguity of communication in a non-trivial way while a certain entropy is maximized will also lead to Zipf's law, with a wider range of exponents (Ferrer i Cancho, 2004). With Zipf's law as the basic assumption (or the communication principles leading to that law), we explore its consequences for a simple communication system.b

Our aim is to show that a basic assumption (a form of Zipf's law) naturally leads to certain consequences, in particular a certain combinatorial property of words, connectedness, that is a precondition for syntax; this is described in detail below. To this end, we shall study a highly simplified and abstracted linguistic model; both the assumption and the consequences make sense in this setting, without reference to the much more complicated details of real languages, or more realistic models for them. Our model will not, of course, be strictly realistic for any particular language, or even for the early developing human language to which we think our conclusions are most relevant, and about which very little is known. In some sense, this is the idea: we wish to show that connectedness arises naturally from Zipf's law, independently of the details of the linguistic setting. To do this, we shall consider a random model, within a class specified below. Once again, no real language is formed in this random way, but this is the point. As almost any model of the given form shows connectedness. the absence of connectedness would need further explanation; given Zipf's law, connectedness does not under a wide range of conditions.

2 The model

We assume a general communication framework, and thus define a set of signals $S = \{s_1, ..., s_i, ..., s_n\}$ and a set of objects $R = \{r_1, ..., r_j, ..., r_m\}$. A signal is a generic code that is capable of carrying meaning. The general term signal is used here to provide a high enough level of abstraction. For instance, we want to abstract from the signal medium (vocal, gestural, chemical,...) or the type of reference involved (iconic, indexical or symbolic (Deacon, 1997)). In order to exclude codes like syllables or sentences in human language from the kind of signals intended here, our signals should not be decomposable into simpler units unless such units do not have referential power. Human words can only be replaced by signals in a metaphorical sense, because human words imply symbolic reference whereas our signals are not necessarily symbols. Objects

here may be cognitive categories (Harnad, 2003; Damper & Harnad, 2000) and therefore can be modelled by a discrete set.

We define a matrix of signal-object associations $A = \{a_{ij}\}$ $(1 \leq i \leq n, 1 \leq j \leq m)$ where $a_{ij} = 1$ if the *i*-th signal and the *j*-th object are associated (the *j*-th object is a 'possible meaning' for the *i*-th signal) and $a_{ij} = 0$ otherwise. (We consider 'binary' associations only for simplicity). The matrix A defines a bipartite graph $G_{n,m}$ (Bollobás, 1998) with edges corresponding to the 1s in A. This matrix A defines signal-object associations that can be of two types: referential and non-referential. By referential we mean that the signal can refer to the object as in the link between the word 'meat' and the object 'edible organic matter' or as in the link between the verb 'eat' and the object 'the action of eating'. By non-referential we mean the remaining possible signal-object associations. For instance, the syntactic association between a verb and its argument would be realized in our model by signal-object associations between the verb and the objects representing the possible arguments. Thus, the verb 'eat' is associated not only to the object 'the action of eating' (referentially) but also to the object 'edible organic matter' (non-referentially).

Various theoretical approaches to syntax assume that a connection between a pair of syntactically linked words implies that the words are semantically compatible (Chomsky, 1965; Helbig, 1992). Here we assume that a connected pair of signals are connected to each other through a common object, which, acting as a rudimentary meaning, defines the semantic compatibility of the pair. Therefore, signals having a common object may or may not be synonyms (depending on whether the pair of links is referential or not). We can also model forbidden arguments. For instance, the object 'umbrella' cannot be the object of the verb 'eat', so there would be no link between this object and the verb 'eat'. In a very simplified manner, $G_{n,m}$ contains information about argument structure.

Objects are simple meanings. Words in human language have complex meanings that may involve more than one of our objects here. For instance, the word 'eat' in human language is associated to at least two objects in our view: the action of eating (referentially), and 'edible organic matter' (non-referentially). Both objects are needed to understand the meaning of the verb 'eat'.

The matrix A should be seen as a primitive association system from which different types of signal-object associations may develop. Notice that without the skeleton provided by A, complex types of associations (synonymy links, syntactic links,...) cannot emerge. A, in spite of being a simplification, may be arranged in a way that may lead to a simple form of language or in way that cannot. Here we study how and why Zipf's law leads to the former case.

Let us write p_k for the proportion of signals with k links. We make the natural simplifying assumption that the relative frequency of a signal is proportional to the number of objects it is connected to as in (Ferrer i Cancho, 2004). Under this assumption, Zipf's law, Eq. 1, is equivalent to

$$p_k \sim k^{-\beta}. (2)$$

In what follows we shall assume Eq. 2. Our model for $G_{n,m}$ will be as

follows: given the numbers n and m of signals and objects, and for each k the proportion p_k of signals connected to k objects, the graph $G_{n,m}$ is chosen uniformly at random from among all bipartite graphs with these properties. Equivalently, having decided the degree $d(s_i)$, i.e., the number of associated objects, of each signal appropriately, we join s_i to a random set of $d(s_i)$ objects, independently of the other signals. We investigate properties that the resulting graph has with high probability, noting that any such property is a very natural consequence of Zipf's law. Indeed, as noted in the introduction, the model is not complete or strictly realistic, and one cannot deduce that such a property developed in the real world only because of Zipf's law. However, Zipf's law is a sufficient explanation: given Zipf's law it would be more surprising if the property did not hold than if it did.

Note that there is a transition in the model at $\beta=2$, due to the rapid change in the number of edges as β is varied about this value. More precisely, the average degree of a signal is $\sum_{k=1}^{m} k p_k$. The infinite form of this sum converges if and only if $\beta>2$; in this range the average degree is asymptotically constant as m increases. In contrast, for $\beta=2$ the average degree grows logarithmically with m and, for $\beta<2$, as a power of m. In asymptotic analysis we shall thus consider $\beta=2+\epsilon$ for some small ϵ .

Given the signal-object graph $G_{n,m}$, we define a signal-signal graph G_n whose vertices are the signals s_i , in which two signals are joined if in $G_{n,m}$ they are joined to one or more common objects. Some links in G_n are synonymy links because they stem from two referential links in $G_{n,m}$, but the remaining links define syntactic links (e.g. verb-argument links). We define G'_n as the subgraph of G_n formed by all signals in S and all the non-synonymy links in G_n . What follows is true if the proportion of synonymy links in G_n is small.

In our simplified model, a grammatical phrase can be formed by choosing a pair of signals (u,v) in G'_n and all the signals in a path from u to v. Total freedom for forming phrases only exists when there is a path between every pair of vertices, that is, when the network is connected. Connectedness is a stronger and more realistic precondition for syntax than just combinations of a few signals as in recent formal approaches to the origins of syntax (Nowak & Krakauer, 1999; Nowak et al., 2000). For various reasons, our grammar is not a grammar in the strict sense, but rather a protogrammar, from which full human language can easily evolve. First, notice that such a grammar lacks word order (Sleator & Temperley, 1991) and link direction (Melčuk, 1989). Second, such a grammar does not imply (but allows) recursion (Hauser et al., 2002). Handling recursion implies memory resources (Lieberman, 1991) that are not necessarily available when connectedness is reached.

When $\beta=2+\epsilon$, with high probability G'_n is almost connected in the sense that almost all signals lie in a single component (the limiting proportion tends to one as $\epsilon \to 0$). See Fig. 1 A-B. Almost connectedness is easy to derive mathematically, although there is no space here for the details. We shall work in G_n rather than G'_n for simplicity. All our results carry over to G'_n if the proportion of synonymy links is small; the asymptotic results (such as that just stated) carry over for any fixed proportion of synonymy links less than one.

There are two key requirements for almost connectedness. Firstly, the 'expected neighbourhood expansion factor' f must be greater than one. Roughly speaking, f is the average number of nodes (here signals) within distance $\ell+1$ of a given node s, divided by the number within distance ℓ , for ℓ in a suitable range. If ℓ is neither too small nor too large and t is a node at distance ℓ from s, then the expected number of neighbours of t at distance $\ell+1$ from s is essentially independent of ℓ . Here, noting that one 'step' in G_n corresponds to two in $G_{n,m}$, one can check that

$$f = \frac{n}{m} \sum_{k} (k-1)kp_k.$$

For m=n this is greater than 1 for $\beta < 3.54...$, and in particular for $\beta \approx 2$. Given that f>1, standard methods show that there will be a single 'giant' component, and that all other signals are in 'small' components with only a few vertices. In fact, for $\beta=2$ this is true for $m \ll n \log n$. For $\beta=2+\epsilon$, one can easily check that asymptotically order $c(\epsilon)n$ signals are in small components, and the rest of G_n is connected. More precisely, this is true for $m \ll n/\epsilon$. Here $c(\epsilon)$ is a constant depending on ϵ and approaching zero as $\epsilon \to 0$.

Connectedness or near connectedness also implies a higher order reference where signal-signal referential associations emerge from signal-object associations. If s_i and r_j are linked and r_j and s_k are also linked ($i \neq k$), then there is a signal-signal association between s_i and s_k formed via r_j in only two steps. Signal-signal referential associations are the basis of a rudimentary form of symbolic reference. Symbolic reference is about how a word not only evokes a certain 'meaning', but also how that word evokes other words (Deacon, 1997).

We will show that the degree distribution in syntactic dependency networks (Ferrer i Cancho, Solé, & Köhler, 2003) easily follows from assuming Eq. 2. In short, these networks are formed by words as vertices, and two words are linked if they have been syntactically combined in a collection of sentences. The links in the sentence 'John eats apples' consist of two syntactic dependency links, one between 'John' and 'eats' and another between 'eats' and 'apples' (the former between the subject of the sentence and its verb and the latter between the verb and its object; see (Melčuk, 1989) for a description of the syntactic dependency formalism). These links would belong to the syntactic dependency network if the sentence were one of the sentences in the collection.

We define q_k as the proportion of signals having degree k in G_n , recalling that two signals are joined in G_n if they are associated with at least one common object in $G_{n,m}$. Let Z be the degree in G_n of a random signal s_i , so $q_k = \Pr(Z = k)$. With $\beta = 2 + \epsilon$ it is very unlikely that two given signals are joined to two or more common objects, so Z is essentially

$$\sum_{r_j \sim s_i} d(r_j) - 1,$$

where $d(r_j)$ is the degree in $G_{n,m}$ (number of associated signals) of an object r_j , and the sum is over all objects associated to s_i . Now, as whether a signal

other than s_i is associated to r_j is independent of $s_i \sim r_j$, the terms $d(r_j) - 1$ in the sum behave like essentially independent Poisson distributions, each with mean $\lambda = (n/m) \sum_k k p_k$, which tends to a constant as $n, m \to \infty$ with n/m constant. The distribution of Z does not have a very simple form, but its tail does: the sum of Poisson distributions is again Poisson, and is very unlikely to exceed its mean, here $\lambda d(s_i)$, by any given factor when the mean is large. Thus, one can check that as $k \to \infty$ (keeping n/m fixed) we have

$$q_k \sim ck^{-\beta},$$
 (3)

with c a positive constant. Thus, while the exact distribution of Z is not a power law, Z does have a power-law tail, with the same exponent β as the signal degrees and the signal frequency distribution (Eq. 1). Eq. 3 is consistent with the analysis of real syntactic dependency networks, where the proportion of words having k syntactic links with other words is $\sim k^{-\gamma}$ with $\gamma \approx 2.2$ (Ferrer i Cancho et al., 2003). Note that γ is in turn close to the typical Zipf's law exponent.

Syntactic theory regards certain function words such as prepositions and conjunctions as linkers (Melčuk, 1989), that is words serving as combining words for forming complex sentences. The most connected signals in $G_{n,m}$ share many features with real linking words (Fig. 1). Linkers in human language have (a) poor (or absent) referential power (Givón, 2002), (b) high frequency (Baayen, 2001) and (c) many connections with referentially powerful words. (a-b) are satisfied by the most connected words in $G_{n,m}$ based only on two basic axioms: (1) Zipf's law in the distribution of the number of connections per signal in $G_{n,m}$ and (2) a proportionality relationship between signal frequency of use and number of connections in $G_{n,m}$. (c) requires a further axiom: (3) two vertices in G'_n are linked if they have at least one common object in $G_{n,m}$. High degree vertices in G'_n satisfy (a) since the uncertainty associated to the interpretation of a signal grows with its number of links in $G_{n,m}$ (Ferrer i Cancho, 2004). The most connected links in G_n are also the most connected links in G'_n . Satisfying (b) follows trivially from (1-2). (c) follows from the skewed and heavy-tailed distributions for q_k , which is in turn a consequence of (1).

3 Discussion

We have seen that, in our simplified model, Zipf's law is a sufficient condition for almost connectedness provided the number of signals and objects are similar, and that Zipf's law with almost connectedness implies the existence of linking words. Almost connectedness in signal-object associations is a necessary precondition for full syntax and going beyond mere simple signal-object associations.

The two-level organization of linguistic structure, with a limited set of words created by combinations of meaningless syllables at one level, and a limitless set of sentences created by combining words at the other, is a critical feature

of language, sometimes termed 'duality of patterning' (Hockett, 1960). Duality of patterning has not been fully considered as requisite in recent models of the origins of syntax (Nowak & Krakauer, 1999; Nowak, Krakauer, & Dress, 1999; Nowak et al., 2000; Nowak, 2000). Here, the essential requirement of connectedness is consistent with duality of patterning. We have defined phrases as paths in G'_n . Different paths in G'_n correspond to syntactically different phrases, whereas different paths in $G_{n,m}$ starting and ending at signals correspond to semantically different phrases. Note that a certain path in G'_n corresponds to at least one path in $G_{n,m}$, so, assuming the proportion of synonymy links is small, the expressivity is given by the number of signal-signal paths in $G_{n,m}$. Approximating semantically different phrases by signal-signal paths in $G_{n,m}$, the number of phrases formed by paths allowed to pass more than once through the same vertex is of course infinite; this is the usual sense in which the number of possible sentences in a language is considered to be infinite. Since paths repeating vertices are to some extent redundant, perhaps the more interesting case is that of paths passing at most once through each vertex. In this case, it is easy to show that, whenever there is a giant component, there is a constant c > 1 such that the expected number of these paths is at least c^n . To sum up, although our model is obviously much simpler than present-day languages, it provides a basis for the astronomically large number of sentences that human speakers can produce and process.

While researchers are divided when considering syntax (Hauser et al., 2002; Nowak & Krakauer, 1999; Nowak et al., 2000) or symbolic reference (Deacon, 1997; Donald, 1991, 1998) as the essence of human language, we hypothesize that syntax and forms of reference higher than mere signal-object associations are two sides of the same coin, i.e., connectedness in signal-signal associations. A communication system maximizing the information transfer (i.e. minimizing the effort for the hearer) by mapping every object to a distinctive signal (Ferrer i Cancho & Solé, 2003) implies that two signals in $G_{n,m}$ never share the same object, so G_n (and G'_n) has no links at all. Therefore, a perfect communication system cannot be connected, or even almost connected in any sense. Such a system cannot satisfy the simple precondition for syntax and complex reference that Zipf's law provides. Many non-human species seem to be close to a perfect communication system for two reasons. One is practical: those species have difficulties in dealing with signal ambiguity (Deacon, 1997). The other one is theoretical: when minimizing hearer and speaker needs simultaneously, there seem to be only two possible basic configurations: no communication and perfect communication. Zipf's law (with non-extremal exponents (Ferrer i Cancho, 2004)) and therefore human language appears in a very narrow domain between these two configurations, so non-human communication is more likely to be in the perfect communication phase than in the narrow Zipfian domain (Ferrer i Cancho & Solé, 2003). As far as we know, no non-human species arranges its meaningful signals according to Eq. 1 with $\beta = 2$.

Zipf's law provides connectedness, an essential precondition for syntax and complex reference, for free. Hence, as language developed, the transition to syntax and complex types of reference may perhaps have been as abrupt as the

transition to Zipf's law (Ferrer i Cancho & Solé, 2003). While some researchers consider Zipf's law a meaningless pattern in human language (Mandelbrot, 1953; Miller & Chomsky, 1963; Li, 1992; Nowak et al., 2000), we have shown that Zipf's law provides language with fundamental traits that do not arise in perfect communication systems.

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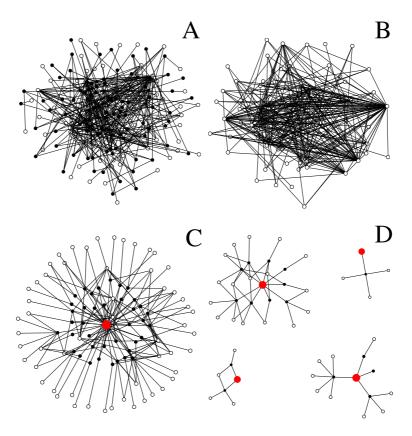


Figure 1: Examples of $G_{n,m}$ (A) and G_n (B) for $\beta=2$ and n=m=100. White and black circles are signals and objects, respectively. First and second neighbours of the most connected signal (red circle) in A (C). This and other highly connected signals are the forerunners of linking words (e.g. prepositions and conjunctions) in human language. First and second neighbours of other signals (red circles) in A (D).