

Active Damping of a DC Network with a Constant Power Load: An Adaptive Observer-based Design

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Abstract—This paper proposes a nonlinear, output feedback, adaptive controller to increase the stability margin of a direct-current (DC), small-scale, electrical network containing an *unknown* constant power load. Due to their negative incremental impedance, constant power loads are known to reduce the effective damping of a network, leading to voltage oscillations and even to network collapse. To overcome this drawback, we consider the incorporation of a controlled DC-DC power converter in parallel with the constant power load. The design of the control law for the converter is particularly challenging due to the existence of unmeasured states and unknown parameters. We propose a standard input-output linearization stage, to which a suitably tailored adaptive observer is added. The good performance of the controller is evaluated with numerical simulations.

I. INTRODUCTION

This note is concerned with the stability analysis of electrical networks with Constant Power Loads (CPLs). It is well-known that, due to their negative incremental impedance characteristic, CPLs induce voltage oscillations or even network collapse [1]. The analysis of networks with this type of loads started with [2], and has been an active research problem since then, see *e.g.*, [3], [4] and [1]; we refer the reader to [5], and the references therein, for a recent review on this topic.

Various techniques have been explored for the stabilization of DC networks with CPLs—a survey may be found in [5]. These techniques are categorized into passive and active damping methods: the former are based on open-loop hardware alterations, whereas the latter imply the modification of existing—or added—control loops. In an active damping strategy, the control loops can be modified at three different network’s positions [5]: at the source’s side, at the load’s side, and at a midpoint between them. In the present paper we are interested in the latter approach, which was firstly explored

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in [6], [7], and [8], for the stabilization of a small-scale network with a single CPL. In these references, the network’s stabilization is achieved by adding a *controlled* power converter in parallel with the load and then, designing a suitable feedback control law for it: in [6], the converter is modeled as a simple controlled current source, and a linear control law is designed to stabilize the overall network; a similar approach, but using a full model for the power converter, is used in [7], however, their stabilization result is based on linearization of the network’s dynamics; lastly, in [8] a large signal stability analysis, but using approximate techniques, such as the Takagi-Sugeno fuzzy model, is carried out to evaluate the performance of a linear controller.

The main contribution of this paper is described next. Following [6] and [7], we study the stabilization of a small-scale DC network supplying electrical energy to a CPL. First, we augment the network by placing a controlled power converter between the load and the source. Then, for the converter’s control-loop design, instead of relying on linear-feedback techniques, we propose an adaptive observer-based nonlinear control law that provably achieves the overall network’s stabilization. The control design is particularly challenging due to the existence of unmeasured states—the current of the DC network—and the unknown power of the CPL. The construction of the proposed controller is based on the use of standard *input-output linearization*, to which a suitably tailored *adaptive observer* is added; its good performance is evaluated *via* numerical experiments. We point out that the stabilization problem addressed in this manuscript was previously studied in [9], where a *state-feedback* adaptive passivity-based control was proposed; the key differences between both works are specified in Remarks 2 and 3 of this note.

The rest of the paper is structured as follows. In Section II we present the model of the system under study and summarize its stability properties. The proposed controller configuration, adopted from [6] and [7], is presented in Section III. Our main stabilization results are included in Section IV. In Section V we present the numerical implementation of our theoretical developments. The note is finalized in Section VI with a number of concluding remarks and open problems that can be studied to extend our results.

II. PROBLEM FORMULATION

A. Description of the open-loop system

The electrical network under study is shown in Fig. 1. It represents a simplified model of a DC power system and has

been used in the literature, *e.g.*, in [7], [10] and [11], to study the stability problems associated with CPLs. It is composed of a DC voltage source supplying energy to an instantaneous CPL. The transmission line is simply represented by the lossy inductor $L_1 > 0$ and the CPL is connected through the bus capacitor $C_1 > 0$. The network's dynamics are described by

$$\begin{aligned} L_1 \dot{x}_1 &= -r_1 x_1 - x_2 + E, \\ C_1 \dot{x}_2 &= x_1 - \frac{P}{x_2}, \end{aligned} \quad (1)$$

where x_1 and x_2 denote the current through L_1 and the voltage across C_1 , respectively. The *constant* parameter P corresponds to the power extracted from, or injected to, the network by the CPL, being positive in the former case and negative in the latter—in the sequel, we focus our attention on the case $P \geq 0$. The state space for this system is defined as follows

$$\mathcal{X}_1 := \{(x_1, x_2) \in \mathbb{R}^2 : x_2 > 0\}.$$

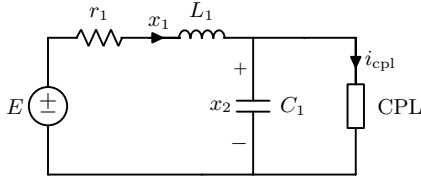


Fig. 1. A DC source supplying power to an instantaneous CPL.

B. Equilibrium analysis of the open-loop system

From [9], we enumerate the qualitative properties (QPs) of this network.

QP1 The system (1) has two real equilibria if and only if

$$E^2 - 4Pr_1 \geq 0 \Leftrightarrow P \leq \frac{E^2}{4r_1}. \quad (2)$$

QP2 One equilibrium corresponds to a high voltage/low current characteristic, which is stable only if

$$P \leq \frac{E^2 C_1 L_1 r_1}{(L_1 + C_1 r_1^2)^2}, \quad (3)$$

whenever $C_1 < \frac{L_1}{r_1^2}$.

QP3 If $C_1 > \frac{4L_1}{r_1^2}$, the strict satisfaction of (2) is sufficient for asymptotic stability of the equilibrium mentioned in QP2.

Notice that if P is negative, *i.e.*, if the load behaves as a constant power *source*, then the expressions (2) and (3) are simultaneously satisfied; consequently, this scenario poses no treat regarding voltage collapse nor network's instability, hence our focus on the case $P \geq 0$.

C. Objectives and methodology

To introduce and support our methodology, we enlist the following remarks [9].

1) Observe from QP2 that if the capacitance C_1 is not big enough then, in order to maintain the system's stability,

the power extraction from the CPL must be strictly smaller than the upper bound for existence of equilibria given in (2).

2) QP3 suggests a passive method to enlarge the domain of values of P for which stability is ensured. It consists in increasing the effective capacitance C_1 , which can be achieved with the *open-loop* parallel interconnection of a suitable capacitor and the CPL. Nonetheless, some disadvantages of this approach are reviewed in [6, Section III.A].

In view of these remarks, we specify our *control objectives* (COs) as follows.

CO1 Regulate the voltage x_2 around a constant value.

CO2 Relax the upper bound for P established in (3).

CO3 Achieve these objectives without the knowledge of P .

As originally proposed in [6] and [7], to achieve these objectives we first add a power converter in parallel with the CPL, then we design a control strategy that stabilizes the overall network to a desired equilibrium point; the detailed description of the augmented circuit is carried out in the next section, and the presentation of the control law—which is the main contribution of the paper—is done in Section IV.

III. AUGMENTED CIRCUIT MODEL

We augment the network of Fig. 1 by adding a controlled DC-DC power converter in parallel with the load [6], [7], which results in the circuit shown in Fig. 2. The converter, which in the sequel is referred to as *shunt damper*, is composed of two complementary switches u and $(1 - u)$, a lossy inductor $L_2 > 0$, a capacitor $C_2 > 0$, and a resistor $r_3 > 0$; the latter models the losses associated with the switching devices.

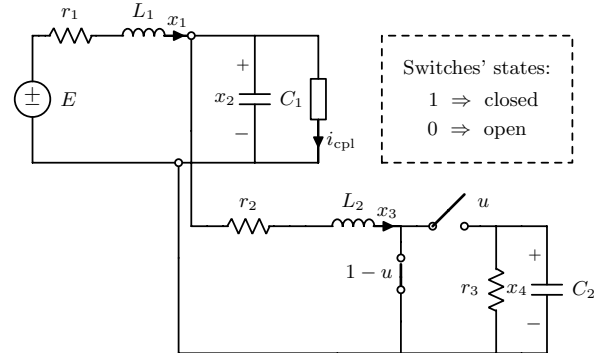


Fig. 2. The network of Fig. 1 is augmented by adding a power converter in parallel with the load.

The averaged dynamic model of the augmented system is given by

$$\begin{aligned} L_1 \dot{x}_1 &= -r_1 x_1 - x_2 + E, \\ C_1 \dot{x}_2 &= x_1 - \frac{P}{x_2} - x_3, \\ L_2 \dot{x}_3 &= -r_2 x_3 - u x_4 + x_2, \\ C_2 \dot{x}_4 &= -\left(\frac{1}{r_3}\right) x_4 + u x_3, \end{aligned} \quad (4)$$

where x_3 is the current through L_2 , x_4 is the voltage across C_2 , and $u \in (0, 1)$ —which is the system’s control variable—represents the duty cycle. The state space of the system is given as

$$\mathcal{X} := \{x \in \mathbb{R}^4 : x_2 > 0, x_4 > 0\}.$$

We conclude this section by making the important observations that x_1 —being the current of a reduced model of the network—is *not measurable*, and the power P is *unknown*.

IV. MAIN RESULTS

In this section we propose a nonlinear, adaptive, output-feedback controller that makes the augmented network (see Fig. 2) comply with the control objectives described in Subsection II-C. First, we analyze the equilibria of (4) and establish constraints, on the system’s parameters, for their physical feasibility. Secondly, under the assumption that x_1 can be measured, and the constant parameter P is known exactly, we present a full-information input-output linearization controller [12] that asymptotically stabilizes a specified equilibrium state. Finally, this controller is complemented with an observer for x_1 and an on-line estimator for P —yielding an adaptive, output-feedback controller design.

A. Existence of equilibria

A pair $(\bar{x}, \bar{u}) \in \mathcal{X} \times (0, 1)$ is an equilibrium of (4) if and only if it belongs to the set¹

$$\mathcal{E} := \{(\bar{x}, \bar{u}) \in \mathcal{X} \times \mathbb{R} : f(\bar{x}) + g(\bar{x})\bar{u} = 0\}.$$

An important issue in the design is to ensure that the *power consumption* of the shunt damper is minimal. In order to carry-out this analysis, we find convenient to parametrize \bar{x} in terms of \bar{u} , as follows.

Proposition 1: Fix $0 < \bar{u} < 1$ as a desired steady state duty cycle of the shunt damper.

C1 $(\bar{x}, \bar{u}) \in \mathcal{E}$ if and only if

$$\Delta := E^2 \ell_2 - 4Pr_1 \ell_1 \geq 0, \quad (5)$$

where we have defined

$$\ell_1 := r_3 \bar{u}^2 + r_1 + r_2, \quad \ell_2 := r_3 \bar{u}^2 + r_2$$

and

$$\begin{aligned} \bar{x}_1 &= \frac{1}{2r_1 \ell_1} \left[E(\ell_2 + 2r_1) - \sqrt{\ell_2} \sqrt{\Delta} \right], \\ \bar{x}_2 &= \frac{1}{2\ell_1} \left[\sqrt{\ell_2} \sqrt{\Delta} + E\ell_2 \right], \\ \bar{x}_3 &= \frac{1}{2\ell_1} \left(\frac{\sqrt{\Delta}}{\sqrt{\ell_2}} + E \right), \\ \bar{x}_4 &= \frac{r_3 \bar{u}}{2\ell_1} \left(\frac{\sqrt{\Delta}}{\sqrt{\ell_2}} + E \right). \end{aligned} \quad (6)$$

¹For space reasons we omit the explicit presentation of the obvious choice for the vector fields f and g , that allow writing (4) as $\dot{x} = f(x) + g(x)u$.

C2 The power dissipated at the shunt damper in steady state, *i.e.*, the quantity

$$\begin{aligned} \mathcal{P}_L &:= r_2 \bar{x}_3^2 + \frac{1}{r_3} \bar{x}_4^2, \\ &= \frac{r_3 \bar{u}}{4\ell_1^2} \left(\frac{\sqrt{\Delta}}{\sqrt{\ell_2}} + E \right)^2 \end{aligned}$$

attains a maximum at $P = 0$ if and only if $\Delta > 0$.

Remark 1: The condition $\Delta \geq 0$ in (5) is equivalent to the following upper bound for P ,

$$P \leq \frac{\ell_2 E^2}{\ell_1 4r_1},$$

which can be proved to be a strictly increasing function of \bar{u} .

Remark 2: In [9], the equilibria $(\bar{x}, \bar{u}) \in \mathcal{E}$ are parametrized in terms of \bar{x}_2 , not in terms of \bar{u} . Then, the equilibrium \bar{x} , associated to $\bar{x}_2 = \frac{E}{2}$, is singled out for stabilization. This choice allows the network’s stable operation in a wide range of values of P . Unfortunately, the steady-state shunt-damper’s power dissipation is given in this case by $E^2/4r_1 - P$, which implies a very low energetic efficiency when P is small.

The parametrization that we propose in (6) implies a more involved algebraic expression for the damper’s power losses, nonetheless, with an appropriate selection of \bar{u} these losses can be made considerable inferior with respect to the latter approach.

This discussion is illustrated in Section V.

B. Design of a full information stabilizing control law

In this subsection we present a static, state-feedback, control law that renders asymptotically stable the equilibrium point (6). For its design, we assume that $P \geq 0$ is *known*, and fix $\bar{u} \in (0, 1)$ such that (5) holds.

Following the ideas presented in [13, Section IV], we introduce the new input

$$w = x_4 u,$$

which allows rewriting the system (4) in the cascade form shown in Fig 3, where

$$\Sigma^{13} : \begin{cases} L_1 \dot{x}_1 &= -r_1 x_1 - x_2 + E \\ C_1 \dot{x}_2 &= x_1 - \frac{P}{x_2} - x_3 \\ L_2 \dot{x}_3 &= -r_2 x_3 + x_2 - w, \end{cases} \quad (7)$$

and

$$\Sigma^4 : C_2 \dot{x}_4 = -\left(\frac{1}{r_3}\right)x_4 + \frac{1}{x_4} w x_3. \quad (8)$$

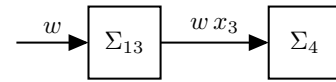


Fig. 3. Block diagram for the cascaded interconnection between the subsystems (7) and (8).

In the next proposition we present a control law that renders exponentially stable an equilibrium point of Σ^{13} .

Proposition 2: Consider the system (7) and the output signal

$$y := x_2 - \bar{x}_2. \quad (9)$$

The static-state feedback

$$w(x_1, x_2, x_3, P) = -L_2 C_1 [\beta(x_2 - \bar{x}_2) + \alpha f_2(x_1, x_2, x_3)] \\ + x_2 - r_2 x_3 - L_2 \left[f_1(x_1, x_2) + \frac{P}{x_2^2} f_2(x_1, x_2, x_3) \right], \quad (10)$$

where

$$f_1(x_1, x_2) := \frac{1}{L_1} (r_1 x_1 - E + x_2), \\ f_2(x_1, x_2, x_3) := \frac{1}{C_1} \left(-x_1 + \frac{P}{x_2} + x_3 \right),$$

is such that

$$\ddot{y} + \alpha \dot{y} + \beta y = 0. \quad (11)$$

Moreover, if $\alpha > 0$ and $\beta > 0$, the equilibrium point $(\bar{x}_1, \bar{x}_2, \bar{x}_3)$ is *locally* exponentially stable.

Next, we establish that if the subsystem Σ^{13} is stable, then, Σ^4 is also stable.

Proposition 3: Let (x_1, x_2, x_3) be any solution of the subsystem Σ^{13} in closed-loop with the state-feedback (10) and starting sufficiently close to $(\bar{x}_1, \bar{x}_2, \bar{x}_3)$. Then, x_4 , solution of Σ^4 , is positive for all time and exponentially converges to \bar{x}_4 , if it starts sufficiently close to it.

As a direct application of Propositions 2 and 3 we obtain the full-information input-output linearizing controller of the overall dynamics (4).

Proposition 4: Consider the system (4) in closed-loop with the static state-feedback control

$$u = \frac{1}{x_4} w(x_1, x_2, x_3, P), \quad (12)$$

where w is given in (10). Then, $\bar{x} \in \mathcal{E}$ is a locally, exponentially stable equilibrium point of the closed-loop system.

C. Stabilization assuming an unknown CPL power

We now establish an adaptive version of the previously designed controller by adding an observer for x_1 and an on-line estimator for the constant parameter P , which are now assumed to be unmeasured and unknown, respectively.

Proposition 5: Consider the system (1) and assume that x_2 is positive and belongs to the interval $[x_2^{\min}, x_2^{\max}]$. Define the adaptive observer

$$\dot{q}_1 = \frac{E}{L_1} - \frac{x_2}{L_1} - \frac{r_1}{L_1} \hat{x}_1 + k_1 \hat{P} - k_1 x_2 \hat{x}_1 + k_1 x_2 x_3, \\ \dot{q}_2 = -k_2 \hat{P} + k_2 x_2 \hat{x}_1 - k_2 x_2 x_3, \\ \hat{x}_1 = q_1 + \frac{1}{2} k_1 C_1 x_2^2, \\ \hat{P} = q_2 - \frac{1}{2} k_2 C_1 x_2^2, \quad (13)$$

where k_1 and k_2 are such that

$$-\frac{r_1}{x_2^{\max} L_1} < k_1 \leq 0, \\ T_1(x_2^{\max}) - 2T_2(x_2^{\max}) < k_2 < T_1(x_2^{\max}) + 2T_2(x_2^{\max}), \quad (14)$$

where

$$T_1(\omega) := \frac{\omega k_1 L_1 + 2r_1}{\omega^2 L_1}, \\ T_2(\omega) := \frac{1}{L_1 \omega^2} \sqrt{\omega k_1 L_1 r_1 + r_1^2}.$$

Then $|\hat{x}_1 - x_1| \rightarrow 0$ and $\hat{P} \rightarrow P$, exponentially fast.

The stability of (4) in closed-loop with an adaptive version of the control law (12) is established next.

Proposition 6: Let k_1 and k_2 be such that (14) hold. Fix \bar{u} and compute \bar{x} from (6). Define the *adaptive* control law

$$u = \frac{1}{x_4} w(\hat{x}_1, x_2, x_3, \hat{P}), \quad (15)$$

where w is given in (10), and \hat{x}_1, \hat{P} are generated by the adaptive observer (13). Then, $(x, \hat{x}_1, \hat{P}) = (\bar{x}, \bar{x}_1, P)$ is an asymptotically stable equilibrium point of the overall system.

Remark 3: A key extension with respect to the research done in [9] is the adaptive and *simultaneous* observation of the variable x_1 and the constant P . Its implementation requires the measurement of x_2 and x_3 , which is easy to obtain in a physical setup.

Remark 4: The computation of the control law (12) requires the knowledge of \bar{x}_2 , which is dependent on the unknown P ; see equation (6). For the numerical results presented in the next section, \bar{x}_2 is computed from the estimate for P , *i.e.* \hat{P} , discretely, not continuously in time. This approach is common in hierarchical and supervisory control of AC and DC microgrids [14] and prevents introducing \bar{x}_2 to the controller if an overshoot of the time-varying signal \hat{P} occurs, it also simplifies the controller design.

Remark 5: Through direct, but lengthy, computations, it can be shown that the condition (14) may be replaced by the conditions

$$0 < k_1 < \frac{8r_1(x_2^{\min} + x_2^{\max})}{L_1(x_2^{\max} - x_2^{\min})^2}, \\ T_1(x_2^{\min}) - 2T_2(x_2^{\min}) < k_2 < T_1(x_2^{\max}) + 2T_2(x_2^{\max}). \quad (16)$$

V. NUMERICAL VALIDATION

In this section we present the results of two numerical experiments aimed at validating our theoretical findings. The physical parameters of the system are taken according to Table I, and the parameters for the adaptive controller of equation (15) are taken as

$$\alpha = 3 \times 10^4, \quad \beta = \frac{\alpha^2}{4}, \quad k_1 = 10, \quad k_2 = 4 \times 10^5.$$

A. Experiment No. 1

We carried out the simulation of the closed-loop system (4) and (15). The equilibrium point to be stabilized, \bar{x} , is computed from equation (6), fixing $\bar{u} = 0.5$. The initial condition has been taken as

$$x(0) = \bar{x}|_{P=0},$$

which corresponds to an equilibrium point in the case $P = 0$. Then, at $t = 3$ s, a step change in the CPL power, from $P = 0$ W to $P = 479$ W has been introduced, this produces

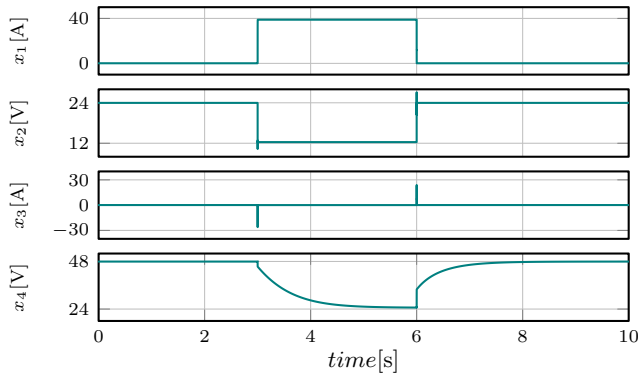


Fig. 4. Time history of x for the experiment No. 1. Convergence is achieved despite the step change in the CPL's power.

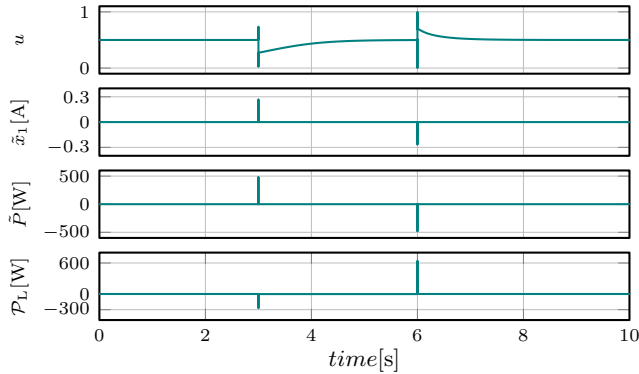


Fig. 5. Time histories of u , observation error for x_1 defined as $\tilde{x}_1 = x_1 - \hat{x}_1$, power estimation error $\tilde{P} = P - \hat{P}$ and power loss $\mathcal{P}_L = x_2x_3$ for the experiment No. 1.

a *new* equilibrium point to be stabilized: $\bar{x}|_{P=479}$. Lastly, at $t = 6$ s, the CPL is stepped down to 0 W once again.

In Fig. 4 we show the time history of each component of the state variable x . The time histories of the control variable u , the error in the estimate of x_1 , *i.e.*, $\tilde{x}_1 = x_1 - \hat{x}_1$, the error $\tilde{P} = P - \hat{P}$, and the power consumption of the shunt damper, *i.e.*, the product $\mathcal{P}_L = x_2x_3$, are plotted in Fig. 5.

B. Experiment No. 2

The setup is the same as for the previous experiment except that now we want to stabilize the equilibrium point \bar{x} with $\bar{x}_2 = \frac{E}{2}$ as done in [9]; see also Remark 2. The objective is to compare the power consumption of the power converter in both scenarios. The results are illustrated in Figures 6 and 7.

C. Experiment's analysis

Regarding the numerical experiments, the following remarks are in order.

- 1) x converges to the desired equilibrium point in both cases.
- 2) $u \in (0, 1)$ for all $t \geq 0$ in both experiments, making it physically feasible for the DC-DC converter. Nonetheless, a saturation function for u , to bound it between

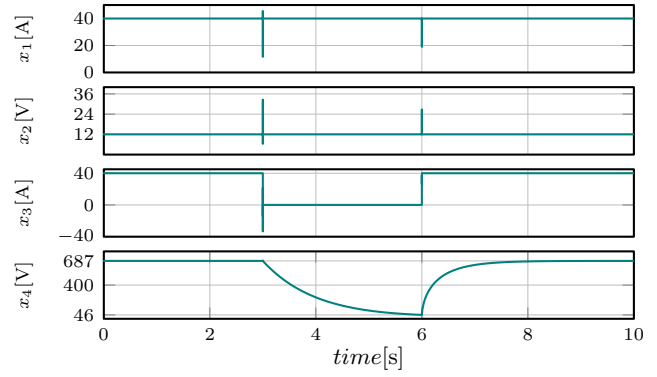


Fig. 6. Time history of x for the experiment No. 2. Convergence is achieved despite the step change in the CPL's power.

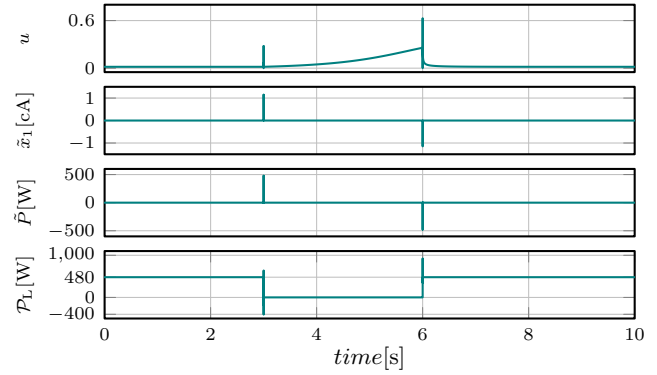


Fig. 7. Time histories of u , observation error for x_1 defined as $\tilde{x}_1 = x_1 - \hat{x}_1$, power estimation error $\tilde{P} = P - \hat{P}$ and power loss $\mathcal{P}_L = x_2x_3$ for the experiment No. 2.

zero and one, has been used; this prevents the control signal from overshooting.

- 3) The CPL changes its power discontinuously from $P = 0$ W to $P = 479$ W. This last value is well above the *necessary* bound for stability of the network *without* the shunt damper given in (3), which reads as

$$\frac{E^2 C_1 L_1 r_1}{(L_1 + C_1 r_1^2)^2} = 276.9 \text{ W.}$$

Also, it satisfies the necessary and sufficient condition for existence of equilibria established in Subsection II-B.

- 4) The observer states \hat{x}_1 and \hat{P} rapidly converge to x_1 and to P , respectively.
- 5) The power consumption of the shunt damper, \mathcal{P}_L , is drastically higher for the experiment No. 2. Furthermore, in the same experiment, we can appreciate that the shunt damper output voltage, x_4 , reaches prohibitively high values. In both senses, the controller's performance in the experiment No. 1 is superior.

VI. CONCLUSIONS AND FUTURE WORK

In this paper we have proposed a nonlinear stabilization method for a DC small-scale power system supplying electric energy to a CPL. By adding a controlled DC-DC power

TABLE I
PARAMETERS FOR THE CIRCUIT IN FIG. 2

$r_1 = 0.3 \Omega$	$L_1 = 85.0 \mu\text{H}$	$C_1 = 200 \mu\text{F}$	$E = 24.0 \text{V}$
$r_2 = 5 \text{m}\Omega$	$L_2 = 100 \mu\text{H}$	$C_2 = 1.0 \text{mF}$	$r_3 = 1 \text{k}\Omega$

converter in parallel with the load, we have been able to design, using standard input-output linearization, and a suitable tailored adaptive observer, a nonlinear adaptive control law for the stabilization of the overall network. Furthermore, our design permits the stable operation of the network for a wide range of values of the CPL and is able to relax some necessary stability bounds that are imposed if the system were to be operated in open-loop. Finally, satisfactory numerical experiments validate our theoretical results.

The results of this paper can be extended in the following directions.

- Explicitly compute estimates for the region of attraction of the equilibrium of the closed-loop system.
- Theoretically evaluate the robustness, against parameter uncertainty, of the proposed adaptive control.
- Extend our analysis to multi-port networks and with a distributed array of CPLs.

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