Multiphase PMSM and PMaSynRM Flux Map Model with Space Harmonics and Multiple Plane Cross Harmonic Saturation

Abstract— Multiphase Synchronous Machines vary in rotor construction and winding distribution leading to non-sinusoidal inductances along the rotor periphery. Moreover, saturation and cross-saturation effects make the precise modeling a complex task. This paper proposes a general model of multi-phase magnet-excited synchronous machines considering multi-dimensional space modeling and revealing cross-harmonic saturation. The models can predict multiphase motor behavior in any transient state, including startup. They are based on flux maps obtained from static 2D Finite-Element (FE) analysis. FE validations have been performed to confirm authenticity of the dynamic models of multiphase PMaSynRMs. Very close to FE precision is guaranteed while computation time is incomparably lower.

Keywords— Permanent Magnet Machines (PMSM), Synchronous Reluctance Machines (SRM), Finite Element Analysis (FEM), machine modeling, cross harmonic saturation, space harmonics, multiphase machines.

I. INTRODUCTION

Permanent magnet synchronous machines (PMSMs) have become extremely important in the industry because they offer very high efficiency and torque density [1]. Due to these features they are applied in various electromechanical conversion systems, such as industrial drives, renewable energy (RE) power generation, hybrid and electric vehicles (HEVs), household appliances and other propulsion systems [2], [3].

Furthermore, multiphase PMSMs have gained a lot of interest due to their inherent power division between multiple phases. They find applications where high power and reliability is demanded, such as in aerospace or ship propulsion [4]. Another attractive aspect of multi-phase systems is their fault tolerant operation [5].

PMSMs of three and more phases are divided into surface mounted (SPMSM) and interior (IPMSM) types. The first one exhibits electromagnetic alignment torque whereas the second attributes for the extra reluctance torque component due to magnetic asymmetry in the rotor construction. In the context of this classification, Permanent Magnet assisted Synchronous Reluctance Machines (PMaSynRM) can also be considered as a part of the PMSM family, with the main reluctance torque due to rotor anisotropy and an auxiliary torque, a result of magnets installed onto the rotor magnetic paths. Consistently, saturation phenomena exist in all of these types of machines. Moreover, there is an evident distinction between sinusoidal and concentrated winding machines. The latter ones contain more spatial harmonics, but the discrete sinusoidal distribution also has an impact on the inductance fluctuation, which for the sake of precise modeling should not be disregarded.

Analytical models of PMSM machines have been thoroughly studied by the scientific community to obtain reliable and precise behavior of the electric drives and generators for investigation, design optimization [6], diagnosis [7], control [8], [9], and sensorless observing purposes [10], [11]. Finite Element Analysis (FEA) is widely adopted for the geometrical optimization and performance prediction. While it requires a high computational burden, other rapid and accurate models are sought. To some extent Magnetic Equivalent Circuits (MECs) are interesting options [12]-[14], but mostly for simple rotor structures [15]. Variable inductance estimator was presented in [16], where parameters depend on current magnitude and phase angle, and are extracted from the stored energy using FEA model and motor steady state operation. Modified two-axis model was proposed in [17], in which self and cross-coupling inductances were introduced and experimentally evaluated in locked rotor condition. However, these methods do not take into account the significant amount of space harmonics present due to slotting and local saturations. The existence was filtered in [18], but still it led only to the fundamental inductance components with cross saturations. The model from [17] was extended in [19] to include current and position dependent characteristics of the flux linkages assuming...
permanent magnet flux as only position dependent. In spite of this simplification, the model gave very good matching with the FEA. A detailed model with iron losses included for the time stepping simulation based on the inverse solution of fluxes was proposed in [20]. It works on flux linkages and does not separate flux from permanent magnet on the grounds that the superposition principle does not hold under saturation conditions. The model contains two current maps and one torque map. Similar concept had been proposed in [21] (and [22] for natural abc system). Nonetheless the initial condition of the dq fluxes is not taken into account and is limited to three phases only (one dq plane).

This work proposes generalized detailed model of PMSM (PMa-SynRM) multiphase machines with any kind of rotor construction and unrestricted number of phases with start dynamics. The model extends standard two axis inverted flux maps approach to multiphase multiplane. Maps can be identified by 2D static FEA. Since the fluxes in multiple dq frames depend on multiple d and q currents and also rotor position, the model comprises discrete winding distribution, slotting and iron saturation. It is recognized that the permeability of iron is affected by the currents from PMa-SynRM multiphase machines with any kind of rotor only (one dq plane).

The startup condition is included for every plane. It depends on the flux from permanent magnets which in turn is modulated by the rotor position. This means that another map is necessary for the motor startup modeling.

FEA verification of the novel model was carried out on the five phase PMA-SynRM without loss of generality. For higher number of phases more planes can be added, although with computation tradeoff. A convention of reluctance machines, with dl axis fixed to the highest permeance and q1 axis orthogonal to it, is used to denominate dq1 plane.

The proposed model allows FEA accurate, fast simulations and analysis from the startup transient of any PMSM family motor behavior with no phase number restriction for high performance drives. They can be implemented in hardware in the loop test benches and used for drive evaluation.

II. CONVENTIONAL PMASYNRM MODEL

The conventional dq model for three-phase machines is well known in the literature and it assumes infinite iron permeability, sinusoidal flux distribution, and therefore constant parameters. Variable inductances are directly projected on this model (Fig. 1) to include magnetic saturation:

\[ U_d = R_l i_d + L_d \frac{di_d}{dt} - \omega_r \left( L_q i_q - \psi_p \right) \]

\[ U_q = R_l i_q + L_q \frac{di_q}{dt} + \omega_r L_d i_d \]

\[ L_d = f_1(i_d, i_q) \]

\[ L_q = f_2(i_d, i_q) \]

\[ \psi_p = f_3(i_d) \]

\[ T_s = \frac{m}{2} \left( \psi_{pm} i_q + (L_d - L_q) i_d i_q \right) \]

where the dq inductances and permanent magnet flux linkage are functions of currents.

Since this model is simplified it cannot predict motor behavior under startup and dynamic conditions. The inductances depend sinusoidally on the rotor position (fundamental component) and become constant (“DC signals”) after Clarke-Park transform. However, with the space harmonics included these inductances become distorted. Also, time derivative of the
Fig. 3. Proposed multiphase PMSM dynamic model based on inverted flux maps (current maps)

PM flux is omitted. Eventually it is valid for three phases only as dq3 and other planes do not exist. It has been recognized that especially under saturation this model is inaccurate and particularly disadvantageous for maximum torque per ampere (MTPA) control strategy [20-22].

III. NONLINEAR MULTIPHASE PMASYNRM MODEL

To overcome the aforementioned problems with the conventional model, a reliable electromagnetic model is presented in this section. It is elaborated on a five-phase 60-slot, 6-pole PMSYNRM prototype machine with parameters specified in Table I and the cross section shown in Fig. 2. The model can be extended to any configuration of PMSM motors.

Instead of using inductances and permanent magnet flux it is possible to operate on the flux linkages only, considering dq and higher order planes (e.g. dq3 in 5-phase system) resulting in set of N equations:

\[
U_{da} = R_{da}i_{da} + \frac{d}{dt}\psi_{da} - n\omega_e\psi_{qn} \tag{4}
\]

\[
U_{qn} = R_{qn}i_{qn} + \frac{d}{dt}\psi_{qn} + n\omega_e\psi_{da} \tag{5}
\]

where the \( dn, qn \) axis flux linkages \( \psi_{da}, \psi_{qn} \) are functions of all planes \( dq \) axes currents and the rotor position \( \theta_r \) as well (5).

Therefore, the nonlinear effects of cross harmonic saturations and space harmonics are included in the flux linkage functions. These functions (5) are identified via FEM analysis performing multi-static currents and rotor position sweep covering all operating conditions. In case of the prototype machine the ranges for \( i_{d1}, i_{q1} \) are of \((-10,10)A\), for \( i_{d3}, i_{q3} \) are of \((-3,3)A\) and for \( \theta_r \) is of \((0^\circ, 60^\circ)\) (in mechanical degrees). If only one plane was considered, because of the cyclic stator teething and integer slot winding of the prototype machine, the \( \theta_r \) interval could be reduced. Due to low amplitude of \( dq3 \) magnitudes the range for this plane is significantly lower, this is advantageous in terms of reduced dataset. The resolution of the coordinate vectors depends on the step size and the number of FEM steps increases exponentially with higher resolution. Therefore, a reasonably high value of the coordinate step is preferable and the output maps can be post-processed with offline spline divisions. The resulting 5D maps can be elaborated as lookup tables (LUTs) or fitted to function of five variables.

Finally, torque may be calculated from the co-energy gradient (9). However, for the sake of numerical precision full torque can be directly extracted from FEM and written similarly to flux linkages as a function of \( dq1, dq3 \) (and higher) axes currents and rotor position \( \theta_r \) and it can also be stored in a LUT.

\[
T' = f_r(i_{d1}, i_{q1}, i_{d3}, i_{q3}, \ldots, i_{dn}, i_{qn}, \theta_r) \tag{10}
\]

where \( n \) subscript denotes one of the existing \( dq \) planes reference and \( n \in \{1,3,\ldots, N\} \). It should be noted with regards to the startup modeling, that it is necessary for these integrals to be supplied with the initial condition. This in turn comes from the flux crossing the airgap thrown by the permanent magnets,

Spanish Ministry of Economy and Competitiveness TRA2016-80472-R
which obviously depends on the rotor position $\theta$. It is clear that this information is already obtained by (5) such that:

$$\psi_{pmq}^p(\theta) = \psi_{pm}^p(0,0,...,0,\theta)$$  \hspace{1cm} (12)

However, to determine all $dq$ currents from all $dq$ flux linkages the inverse functions of those in (5) are needed:

$$i_{dx} = f_{dsq}^{-1}(\psi_{dx}, \psi_{dq}, \psi_{dx}, \psi_{dq}, ..., \psi_{dx}, \psi_{dq}, \theta)$$

$$i_{qs} = f_{qs}^{-1}(\psi_{dx}, \psi_{dp}, \psi_{dx}, \psi_{dq}, ..., \psi_{dx}, \psi_{dq}, \theta)$$ \hspace{1cm} (13)

In general, it is a very difficult (N-1)D (because every rotor angle can be treated separately) inverse problem to solve. It can be accomplished for the complete span of the flux linkages in all $dq$ axes for every rotor position by iterative process that seeks minimum error between a seed and the interpolated map value [20]. This operation, to establish current maps, takes several hours on a mid-class PC station (for case of five phases). The method of inverted grid intersections [22] is also applicable here.

The model schematic is shown in Fig. 3. It is based on (4) and (5). The flux linkages are calculated with (11), the currents and the torque are obtained from functions/maps of (13) and (10) respectively. The flux (5) map variations with respect to one axis current and mechanical angle are shown on Fig. 4 for $dl$ and $q1$ axes and in Fig. 6 for $dq3$ plane. Torque (10) map sub-plane is depicted in Fig. 7. The initial condition from permanent magnets with no current excitation is explicitly shown in Fig. 3 and is computed from the function/map of (12) illustrated in Fig. 5.
Altogether one 2D and five 5D maps are required to run the five phase model.

IV. MODEL VALIDATION

To corroborate proposed model, a comparison with FEM and validation was performed.

First, the models were evaluated in Matlab 2018a Simulink (block oriented) and Simscape (physical system oriented) environments in dynamic short circuit conditions. FEM validation was made with one 2D geometry in order to make it simpler and faster; furthermore, not all packages allow simulations of discrete skew machines in multi-slice mode. Complete dataset of maps was previously extracted using Altair Flux 12.3 FEA software on the professional portable personal computer based on 8 core Core i7-3740QM CPU and equipped with 32 GB of RAM memory. FEM model problem solving, result post-processing, and data export for the studied case of multi-plane maps takes about 38 hours. Then, in simulation, the motor was accelerated from standstill to 200 rpms to catch the initial condition, transient response and the steady state fluctuations of currents (Fig. 8.), fluxes in rotating (Fig. 9.) and stationary (Fig. 10.) reference frames and electromagnetic torque (Fig. 11.). The comparisons are made with 2D FEM with the same scenario. Close matching can be observed for the multi-plane fluxes with their space and slotting effects easily observable. Electromagnetic torque oscillation is very well reflected by the Simulink model and currents are virtually the same. Therefore, the proposed modeling established by the multi-plane space and saturation harmonics is validated by means of FEM simulations on 2D motor. Slight deviations are observable due to step size and interpolations. They can be reduced with higher resolutions of the map extraction.

V. CONCLUSION

The equivalent and detailed electromagnetic model of multi-phase synchronous machine was described on five-phase
Non-sinusoidal winding distribution and cogging torque is also planes together with space harmonics and initial conditions. Cross-coupling but also traverse coupling of higher harmonic PMaSynRM example. The model includes not only the classical

Acknowledgment

This work was supported in part by Spanish Ministry of Economy and Competitiveness under TRA2016-80472-R Research Project and Secretaría d'Universitats i Recerca del Departament d'Empresa i Coneixement de la Generalitat de Catalunya.

References


