A Filtered Kinetic Energy Preserving Finite Volumes Scheme for Compressible Flows

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ABSTRACT

The Kinetic Energy Preserving Scheme (KEP) for compressible flows has been shown to solve 1-D shock waves without smearing them. The method does not add numerical diffusion to solutions but, as a counterpart, it needs very dense meshes in order to be Local Variations Diminishing (LVD) and stable. For 2-D and 3-D geometries, the method needs unaffordable meshes. A Filtered Kinetic Energy Preserving Method (FKEP) that partially solves this issue is presented in this document. The method filters the solution obtained by a KEP at each time step. It is shown that the use of filters does not significantly change the low frequencies of the motion while it enables the use of much coarser meshes. FKEP is tested on the 1-D shock tube.

Compressible Flow, Finite Volumes, Kinetic Energy Preserving, CFD

NOMENCLATURE

\[ \begin{align*}
E_t & \quad \text{Total Energy} \\
K & \quad \text{Kinetic energy of the whole domain} \\
L & \quad \text{Characteristic Length of the whole domain} \\
M & \quad \text{Mach number} \\
Pr & \quad \text{Prandtl Number} \\
R & \quad \text{Numerical Residue of an explicit time scheme} \\
Re & \quad \text{Reynolds Number} \\
S & \quad \text{Surface} \\
T & \quad \text{Temperature} \\
a & \quad \text{Speed of sound} \\
b & \quad \text{Dependent fluid variables} \\
c_p & \quad \text{Specific heat coefficient at constant pressure} \\
f & \quad \text{Numerical flux} \\
g & \quad \text{Convective part of numerical flux} \\
h & \quad \text{Diffusive and pressure part of numerical flux} \\
h_t & \quad \text{Total enthalpy} \\
k & \quad \text{Kinetic energy of a control volume} \\
p & \quad \text{Pressure} \\
qu & \quad \text{Heat conduction flux} \\
x & \quad \text{Residue of system of equations} \\
t & \quad \text{Time} \\
u & \quad \text{Conservative fluid variables} \\
v & \quad \text{x component of velocity} \\
w & \quad \text{y component of velocity} \\
x & \quad \text{z component of velocity} \\
x & \quad \text{Space coordinate} \\
\Omega & \quad \text{Volume of a cell} \\
\alpha & \quad \text{Time scheme main variables coefficients} \\
\beta & \quad \text{Time scheme residues coefficients} \\
\eta & \quad \text{Characteristic length of a cell} \\
\delta & \quad \text{Kronecker Delta} \\
\gamma & \quad \text{Specific heat ratio} \\
\lambda & \quad \text{Volumetric viscosity coefficient} \\
\mu & \quad \text{Viscosity coefficient} \\
\xi & \quad \text{Filter Ratio of a filter} \\
\phi & \quad \text{Scalar fluid magnitude} \\
\rho & \quad \text{Density} \\
\sigma & \quad \text{Viscous Stresses} \\
\omega & \quad \text{Averaging weight} \\
\phi & \quad \text{Filtered } \phi \text{ field} \\
\phi_{op} & \quad \text{Result of a weighted sum} \\
\end{align*} \]

Subscripts:

\[ \begin{align*}
\alpha & \quad \text{Relative to the speed of sound} \\
o, p & \quad \text{Volumes identities} \\
op & \quad \text{Interface between volumes } o \text{ and } p \\
glob & \quad \text{Relative to the whole domain} \\
Kol & \quad \text{According to Kolmogorov scale cascade theory} \\
LC & \quad \text{According to cell-length and characteristic waves based Reynolds} \\
i, j, k & \quad \text{Space coordinate} \\
q & \quad \text{Time scheme dummy index} \\
n & \quad \text{Time step} \\
\end{align*} \]

Superscripts:

\[ \begin{align*}
\hat{\phi} & \quad \text{Filtered } \phi \text{ field} \\
\tilde{\phi} & \quad \text{Result of a weighted sum} \\
\end{align*} \]
1 INTRODUCTION

Wind Turbine blades usually operate in stall or near-stall angles of attack. When modeling the flow around them, numerical methods performing well on turbulence are needed. For finite volumes and incompressible flows it has been recently shown by Lehmkuhl et al. [1] that a good choice are the kinetic energy preserving methods [2] because they do not add artificial viscosity to the flow. This allows, as it has been shown by Rodriguez et al. [3], the numerical development of flow patterns that are usually damped by artificial viscosity when simulated.

On Wind Turbine offshore applications, as the blade length increases, so does the blade tip speed and the incompressibility hypothesis does not hold. On on-shore applications, one of the limits to the Wind Turbines operation is aerodynamic noise. It is necessary to improve compressible flow methods on turbulence and transition situations.

In compressible flows modeling with finite volumes, the most of the methods have been deduced from the 1-D hyperbolic equation [4]. Hence, most of them interpolate variables at interfaces giving more importance to the values on one side of it than the values on the other, adding artificial viscosity [5] to the simulated flow. Jameson [6] deduced a Kinetic Energy Preserving Scheme (KEP) for compressible flows which was not of the upwind kind and which did not take into consideration the characteristic lines when interpolating flow variables. This method gave good results on the Sod shock tube and has also been successfully tested on other flow situations at low Reynolds (∼ 100) numbers by Allaneau [7]. The drawback of the KEP is that it requires unaffordable meshes.

In this work the Jameson KEP flux scheme is used and combined with a filter on the variables after each time step. This has allowed to keep the KEP good properties on extremely coarser meshes than those demanded by the local Reynolds number stability condition. The formulation used is developed and detailed in section 2, numerical results on 1D cases are shown in section 3. Finally conclusions and future efforts directions are given in section 4.

\[ \rho \frac{\partial u}{\partial t} + \rho \frac{\partial v}{\partial x} = 0 \]

2 FORMULATION

2.1 Jameson’s Kinetic Energy Preserving scheme

Equations (1) and (2) are the basic conservative formulation of a finite volumes discretization of the Navier-Stokes is:

\[ \Omega_o \frac{\partial u_o}{\partial t} + \zeta_o = 0 \]

\[ \zeta_o = \left\{ \begin{array}{c} \rho \frac{\partial u}{\partial x} \\ \rho u \frac{\partial u}{\partial x} \\ \rho w \frac{\partial w}{\partial x} \\ \rho E_t \end{array} \right\}_o \]

\[ \Omega_o \frac{\partial v}{\partial x} = \left\{ \begin{array}{c} \rho \frac{\partial v}{\partial x} \\ \rho v \frac{\partial v}{\partial x} \\ \rho w \frac{\partial w}{\partial x} \\ \rho E_t \end{array} \right\}_o \]

\[ \frac{\partial}{\partial t} \left( \frac{\rho v^j}{2} \right) + \frac{\partial}{\partial x} \left( \mu \frac{\partial v^j}{\partial x} \right) + \frac{\partial p}{\partial x} = 0 \]

\[ \mu = \frac{\mu_0 T^3}{T + 110.3} \]

For ideal air, Equations (3) apply. For dry air, \( \mu_0 = 1.461 \cdot 10^{-6} Pa \cdot s \) but, as will be later commented, the value used in the present work is greater.

\[ p = (\gamma - 1) \rho \left( \frac{v^1 v^1}{2} \right) \]

\[ h_t = E_t + \frac{p}{\rho} \]

\[ \sigma_{ij} = \mu \left( \frac{\partial v^i}{\partial x^j} + \frac{\partial v^j}{\partial x^i} \right) + \lambda \delta_{ij} \frac{\partial v^k}{\partial x^k} \]

\[ \lambda = \frac{-2}{3} \mu \]

\[ T = \frac{h_t - \frac{1}{2} \rho v^k v^k}{c_p} \]

\[ q^j = -\frac{\mu \cdot c_p}{Pr} \frac{\partial T}{\partial x^j} \]

\[ \mu = \mu_0 \frac{T^3}{T + 110.3} \]

The main unknowns are the conservative variables of compressible fluids \( u_o \).

Taking this in mind and defining the total numerical kinetic energy as is shown in Eq.(4),

\[ K = \sum_o \Omega_o \kappa_o = \sum_o \Omega_o \left( \frac{\rho v^j (\rho v^j)}{2 \rho} \right)_o \]

Jameson[6] deduced the KEP proceeding as in Eq.(5) and avoiding any contribution of the convective part of fluxes to change the kinetic energy Eq.(6).

\[ \frac{dK}{dt} = \sum_o \Omega_o \frac{dK}{d\kappa_o} \frac{d\kappa_o}{dt} = \sum_o \Omega_o \frac{dK}{d\kappa_o} \frac{d\kappa_o}{du_o} \frac{du_o}{dt} \frac{\partial \kappa_o}{\partial u_o} \Omega_o \]

\[ \frac{dK}{d\kappa_o} = \Omega_o \left( \begin{array}{c} \frac{\rho v^1 v^1}{2} \\ \frac{\rho v^1 v^1}{2} \\ \frac{\rho v^1 v^1}{2} \\ \frac{\rho v^1 v^1}{2} \\ \rho h_t \end{array} \right)_o \]

\[ \frac{\partial}{\partial x} \left( \frac{\rho v^j}{2} \right) + \frac{\partial}{\partial x} \left( \mu \frac{\partial v^j}{\partial x} \right) + \frac{\partial p}{\partial x} = 0 \]

\[ f^j_{op} = \left\{ \begin{array}{c} 0 \\ \frac{\rho v^j}{2} \\ \rho v^j v^j \\ \rho v^j v^j \\ \rho v^j v^j \end{array} \right\}_{op} \]
\[
\sum_o \frac{dK}{d\omega_o} \sigma_{op}^j S_{op}^j = 0 \tag{6}
\]

Jameson also forced the contribution of \( h_{op}^j \) to \( K \) to be according to the Kinetic Energy Equation applied to the whole volume. Then, resulting interface fluxes are restricted to accomplish Equation (7).

\[
(\rho u^i v^j)_{op} = (\rho u^i)_{op} \hat{v}^j_{op} \tag{7}
\]

The Eq.(7) can be got with the equalities in Eq.(8).

\[
\begin{align*}
(\rho u^i)_{op} &= \rho_{op} \hat{v}^i_{op} \\
\sigma_{op}^i &= \hat{\sigma}_{op}^i \\
h_{t_{op}} &= \hat{h}_{t_{op}} \\
q_{r_{op}} &= \hat{q}_{r_{op}}
\end{align*} \tag{8}
\]

It is not clear how to correctly compute \( \sigma \) and in practice the viscous stresses are computed by means of the usual centered on interfaces derivatives of velocity. The \( h_{t_{op}} \) and \( q_{r_{op}} \) interface values are suggested to be computed as shown only for consistency with the other quantities reasons.

The main process for computing is, supposed known \( u_o(t) \forall o \):

1. Compute \( (h_{op}^j)^n \forall op, \forall j \) according to Eqs.(8) and (2).
2. Compute \( (\varepsilon_o)^n \forall o \), with (1).
3. Apply an explicit time integration scheme:

\[
\begin{align*}
\varepsilon_{o}^{n+1} &= \Psi(u_{o}^{n}, u_{o}^{n-1}, ..., u_{o}^{n+1-N}, \\
&u_{o}^{n+1}, ..., u_{o}^{n+1-M}; \Delta t) = 0 
\end{align*} \tag{9}
\]

4. Compute \( (h_{o})^{n+1} \) from \( (u_{o})^{n+1} \)

Some tests with this scheme can be seen on Jameson [6]. In few words, tests on the Sod case show that the KEP scheme is the best on following a shock wave without dissipation.

The main problem of KEP is the need of very dense meshes that maintain the finite volume and maximum eigenvalue based Reynolds number under 2. This is the Local Cell-Based Reynolds Number Condition (LCBRC). Else, the method is not Local Variations Diminishing (LVD) and it diverges.

2.2 The Filtered Kinetic Energy Preserving scheme

This section contains the main discussion that lead to the proposal of the FKEP and the corresponding formulation.

2.2.1 KEP limits

After a power spectrum analysis on the 1-D shocktube resolved by a sub-LCBRC mesh, it was noticed that the cause of the KEP instability is the amplification of small wavelength scales of the flow, this producing instable oscillations of the fluid variables. Once small wavelengths amplified, the oscillation propagates to larger wavelengths as it grows. Finally, these oscillations instabilizes computations, and can lead to divergence. In Figures 1 and 2 it can be seen that with the Total Variations Diminishing (TVD) 3 stages Runge-Kutta time scheme, the oscillations maximum amplitude is limited enough to permit an analysis of the divergence nature. Notice that the oscillated solution is just the sum of the correct solution of the case and oscillations. This behavior suggests that if the amplitude growth of the small wavelengths was eliminated without affecting on the large wavelengths, the KEP method could be used on much coarser meshes.

The idea of damping small wavelengths oscillations is sustained by the fact that these oscillations have characteristic sizes of the mesh cells, which are would be set through Eq.(10) by the LCBRC.

\[
\eta_{LC} \sim \sqrt{\frac{2}{\rho a (1 + M)}} = \frac{2L}{Re + Re_a} \forall o \forall t. \tag{10}
\]

The ratio between the requested Direct Numerical Simulation (DNS) mesh size by the Kolmogorov scale criterion based on \( \eta \) and \( \eta_{LC} \) is provided in Eq.(11). It shows that LCBRC requests a finer than DNS mesh. This gives more power to the idea of eliminating the scales of flow causing the oscillations of KEP. However, for compressible flows with shock waves and contact discontinuities the Kolmogorov criterion could not be restrictive enough because these phenomena have a characteristic thickness proportional to the viscosity coefficient. In any case, LCBRC is too restrictive for the state of the art computing technology.

\[
\frac{\eta_{Kol}}{\eta_{LC}} = \frac{L(Re)^{-3/4}}{2L \cdot (Re + Re_a)^{-1}} > Re^{1/4} \tag{11}
\]

\(^1\)A mesh too coarse to accomplish the LCBRC condition.
\(^2\)This is not totally true as fluid dynamics equations are not linear.

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2.2.2 FKEP Formulation

It is proposed to filter the discrete equation of an explicit an linear\textsuperscript{3} time scheme on a KEP spatial discretization in order to eliminate the mentioned oscillations. The problem is formulated in Eq.(12), where the parameters defined in Eq.(13) are used and $F$, which is a filter function, remains undetermined.

\[ F \left( u_o^{n+1} = \bar{u}_o^n + R_o^n \right) \]  

\[ \bar{u}_o^n = \sum_{q=1}^{q=N} \alpha_q \bar{u}_o^{n+1-q} \]

\[ R_o^n = \Delta t \Psi \left( \frac{r_o^n}{\Omega_o}, \frac{r_o^{n-1}}{\Omega_o}, \ldots, \frac{r_o^{n+1-N}}{\Omega_o} \right) \]  

(13)

The filtering function must be linear\textsuperscript{4} and can depend or not on the fluid variables. For more information on filters, consult Pope [8] or Sagaut [9]. In this work, the filtering function was controlled, for Flow Depending Filters (FDF) through the filter ratio $\xi$. This parameter remains constant for Constant Filters (CF). Filtered variables were in practice computed as in Eq.(14).

\[ F(\bar{u}_o^n) = \bar{w}_o^n = \omega_{op} \bar{u}_p \]  

(14)

After these comments, supposed known $u_o^n$ and all variables and residues of previous time steps that may be needed by the actual time scheme, the FKEP algorithm reads, for a given time integration scheme:

1. Compute $\left( \int_{t_p}^{t_j} \left( u_o^n, u_p^n, b_o^n, b_p^n \right) \right)^n \forall op, \forall j$ with Eqs.(8) and (2).
2. Compute $\bar{e}_o^n \forall o$ as in Eq.(2) and $R_o^n$ using Eq.(13).
3. Calculate $\bar{u}_o^n \forall o$ according to Eq.(13).
4. Compute $\bar{e}_o^n(\bar{u}_o^n) \forall o$ and the filter function weights $\omega_{op}(\bar{e}_o^n) \forall o, p$
5. Compute $\bar{u}_o^n = \omega_{op} \bar{u}_p^n \forall o$
6. Apply the explicit N steps time integration scheme:

\[ u_o^{n+1} = \bar{u}_o^n + R_o^n \]  

(15)

\[ 3 \text{ NUMERICAL EXPERIMENTS} \]

In some points, FKEP formulation is not absolutely consistent. In FKEP, it must be accepted that $u_o^{n+1}$ is properly computed as it is shown in Eq.(15). In this equation, the unfiltered value of independent variables is computed from filtered variables of previous time steps and the unfiltered time scheme residue of the actual time step. Consequently the value of $u_o^{n+1}$ is independent of filters and Eq.(15) does not exactly correspond to Eq.(12), mainly because $\bar{u}_o^n(\bar{u}_o^n) \neq \bar{u}_o^n(u_o^n)$. The necessity of computing $u_o^{n+1}$ comes from the fact that if $\bar{u}_o^n(\bar{u}_o^n)$ were used instead of $\bar{u}_o^n(u_o^n)$, Jameson’s deductions would not hold anymore and the method would lose it’s KEP property. Nevertheless Another negative point is that, as it is deduced from Eq.(16) the method is not well suited for all time schemes when FDF are used.

\[ F \text{ depends on } u_o(t) \Rightarrow F \left( \int \left( \frac{du_o(t)}{dt} + \frac{\bar{u}_o(t)}{\Omega_o} \right) dt \right) \neq \int F \left( \frac{du_o(t)}{dt} + \frac{\bar{u}_o(t)}{\Omega_o} \right) dt \]  

(16)

In this section experiences regarding the FKEP and KEP are presented. The test case is the 1D shock tube (Sod. [10]). It has been resolved with various numerical parameters. Some computations were performed before stabilizing in order to identify the divergence of KEP causes and others were performed with the aim of analyzing the FKEP dependence on filtering functions and meshes. The test case consists of a 1D $x \in [0,1]$ domain with open boundaries with a gas inside. At $t < 0$ the gas is in two different states, one to the left and the other to the right of $x = 0.5$. The left hand state is at a greater pressure and density. At $t = 0$ the imaginary membrane separating the sub-domains is removed and, consequently, a shock wave travels to the right while an expansion wave travels to the left. Initial states of the gas are in Table 1. Unless specified, the presented results correspond to the state of gas at $t = 7\epsilon - 4\sigma$. The studied cases are summarized in Table 2.

In this document, the employed gas is ideal air with Sutherland’s law for viscosity. However, the viscosity constant used in Sutherland’s equation was 100 times greater than the air value. This allowed to reduce the local $Re$ without increasing the mesh density. For filtered cases with FDF, filter ratios take values that can be equal or greater than zero, taking zero only in the case that $\phi_o$ is not a local extremum. Tests were performed computing filter ratios in two manners: using binary values for $\xi_o$ or using continuous values.

\[ \text{\footnotesize{\textsuperscript{3}}respect to the problem variables} \]
\[ \text{\footnotesize{\textsuperscript{4}}$F(\alpha_1 \phi_1 + \alpha_2 \phi_2) = \alpha_1 F(\phi_1) + \alpha_2 F(\phi_2)$}} \]
Table 1: Sod case gas at \( t = 0 \), the values for both states are gas constants.

<table>
<thead>
<tr>
<th>Magnitude</th>
<th>left</th>
<th>right</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p ) [Pa]</td>
<td>101325</td>
<td>10132.5</td>
</tr>
<tr>
<td>( \rho ) [kg/m(^3)]</td>
<td>1</td>
<td>0.125</td>
</tr>
<tr>
<td>( T ) [K]</td>
<td>344.32</td>
<td>275.46</td>
</tr>
<tr>
<td>( cp ) [J/kgK]</td>
<td>1012</td>
<td></td>
</tr>
<tr>
<td>( \gamma )</td>
<td>1.41</td>
<td></td>
</tr>
<tr>
<td>( Pr )</td>
<td>0.71</td>
<td></td>
</tr>
<tr>
<td>( \mu_0 ) [Pas]</td>
<td>1.461 ( \cdot ) 10(^{-4})</td>
<td></td>
</tr>
</tbody>
</table>

Table 2: kCV’s thousands of control volumes of the mesh .TS Time Scheme: Runge-Kutta 3 (RK3) or explicit Euler (Euler). Filters can be Constant “C”, in which case the filter ratio \( \xi \) is specified or variable “V”.

<table>
<thead>
<tr>
<th>Name</th>
<th>kCV’s</th>
<th>TS</th>
<th>Filter (( \xi ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>RK-40</td>
<td>40</td>
<td>RK-3</td>
<td>none</td>
</tr>
<tr>
<td>RK-15</td>
<td>15</td>
<td>RK-3</td>
<td>none</td>
</tr>
<tr>
<td>RK-15C</td>
<td>15</td>
<td>RK-3</td>
<td>C Box 1.5</td>
</tr>
<tr>
<td>RK-15V</td>
<td>15</td>
<td>RK-3</td>
<td>V Box</td>
</tr>
<tr>
<td>RK-4</td>
<td>4</td>
<td>RK-3</td>
<td>none</td>
</tr>
<tr>
<td>RK-4C</td>
<td>4</td>
<td>RK-3</td>
<td>C Box 2</td>
</tr>
<tr>
<td>RK-4V</td>
<td>4</td>
<td>RK-3</td>
<td>V Box</td>
</tr>
<tr>
<td>RK-1C1.5</td>
<td>1</td>
<td>RK-3</td>
<td>C Box 1.5</td>
</tr>
<tr>
<td>RK-1C5</td>
<td>1</td>
<td>RK-3</td>
<td>C Box 5</td>
</tr>
<tr>
<td>RK-1C</td>
<td>1</td>
<td>RK-3</td>
<td>C Box 2</td>
</tr>
<tr>
<td>RK-1V</td>
<td>1</td>
<td>RK-3</td>
<td>V Box</td>
</tr>
<tr>
<td>RK-1</td>
<td>1</td>
<td>RK-3</td>
<td>none</td>
</tr>
<tr>
<td>E-1C</td>
<td>1</td>
<td>Euler</td>
<td>C Box 2</td>
</tr>
<tr>
<td>E-1V</td>
<td>1</td>
<td>Euler</td>
<td>V Box</td>
</tr>
<tr>
<td>RK-0.3</td>
<td>0.333</td>
<td>RK-3</td>
<td>none</td>
</tr>
</tbody>
</table>

If \( pr(\phi_o) \) is the prominence\(^5\) of \( \phi_o \). On the binary values case \( \xi_o = 0 \) if \( pr(\phi_o) = 0 \) and \( \xi_o \) takes a value that makes the filter weight equally all the neighbor nodes and the studied node. On the other case, the value of \( \xi_o \) is a strictly increasing function of \( pr(\phi_o) \). In this work are presented results from binary filter ratios functions only. Concretely, if the prominence was different to zero, the filter ratio was set to be great enough to make the filter act at it’s maximum. The filtered used for all cases is the Box filter.

3.1 Spectral Analysis

With not LCBRC meshes KEP gives oscillations, which are greater for coarser meshes. This can be seen on Figures 1 and 2. Despite the oscillations, which seem to be caused by the great pressure variations within th shock wave, the basic shape of solutions is maintained although sub-LCBRC meshes are used. This was only possible with Runge Kutta TVD schemes, Euler schemes diverged. The only LCBRC case is RK-40.

![Figure 1: Results of KEP](image1)

![Figure 2: Detail of KEP results](image2)

The idea that the oscillations correspond to small wavelengths and that it’s nature is numerical is held by Figures 3 and 4. Sub-LCBRC cases are similar to RK-40 on the low-frequency-large-wavelength range, but they differ as frequency rises. Each computed mesh has a frequency range with unphysical amplitudes, the range depending on the case. Since this unstable frequencies depend on meshes, it can be set that their nature is numerical. This fact gives the idea that eliminating such unwanted phenomenon can be done by controlling the these frequencies amplitudes. Fig.4 shows that KEP can correctly resolve the relative-to-mesh low frequencies properly. Thus, once high frequencies amplitudes controlled, KEP could be stable for sub-LCBRC meshes.

3.2 Filter Influence

When putting to practice the filtering idea the most mathematically correct option would have been to compute a power spectrum of the solution after each time step, filter the stable frequency range from the

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\(^{5}\)Measure of the difference between an extremum of a set and the extremum of the previous set minus the former extremum element e. g. Let \( S = \{1, 4, 5\} \) be a set. Then \( pr(max(S)) = 1 \) and \( pr(min(S) = 3) \).
Figure 3: Power spectrum of KEP solution. RK-15, RK-4 and RK-1 are to RK-40 at low frequencies.

Figure 4: Detail of the power spectrum of P computed with KEP.

power spectrum, and go back to physical dimensions with an anti-transform operation. In LES methods this procedure is not popular as it is too computationally costly. For the 1D shock tube cases presented, although power spectrum filterings could have been performed, filters have been applied on physical space variables. The most appropriate filters are those corresponding to a low frequency-pass filter in the frequency space but only Box filters on space coordinates have been used.

As was expected, Figure 5 shows that using FKEP enables the control of the aforementioned unstable frequencies. The counterpart of this depends on the kind of filter used. For CF the flow is well resolved except for the addition of diffusion in all the domain, as it can be seen on Figure 6. Furthermore, the greater the filter ratio is, more diffusion is added.

The variable filter was tested in order to reduce the diffusion. It accomplished its finality but it delayed the shock wave and slightly changed the intermediate state pressure level (Figure 7). The first hypothesis about the cause of the shock wave delay are the weakness of the formulation, already pointed in Eq.(16), or some bad filter property.

Figure 5: Results of FKEP on a 1000 CV mesh.

Figure 6: Constant filters are diffusive at the expansion.

Figure 7: Variable filters do not reproduce well the shock wave.

It can be stated that FKEP eliminates the KEP instabilizes while it resolves well the low frequencies. This is shown in Figure 8.

When KEP was first envisaged by Jameson one of his main goals was to capture, without smearing, shock waves. This would be a very good property for FKEP because it would beat the Upwind-based methods. For Figure 9 a case was launched with Runge-Kutta 3
time integration, and $\xi = 2$ constant filter ratio on a 10000 CV mesh with a length of 10 m, resulting the same mesh density and filter as for RK-1C. Various instantaneous pressure fields corresponding to different times were saved. They show the evolution of the shock tube. These results were later represented joining all the shock waves in order to see the shock wave evolution in time. Figure 10 shows that no relevant smearing of the shock wave was produced (the shock waves are parallel).

Figure 9: Time evolution of a shock wave with length 10 resolved with FKEP.

Figure 11: Detail of FKEP computed with different meshes. More CV, better results at the shock wave.

Figure 12: Detail of FKEP computed with different meshes. More CV, better results at the shock wave.

3.3 Mesh dependence

For a numerical method it is a must to improve its performance when the used mesh is more dense. Figures 11 and 12 show that an increase on the number of CV leads to a better resolution in both sides of the domain.

4 CONCLUSIONS AND FUTURE WORKS

In compressible flows, the common characteristic of all of the traditionally used numerical fluxes used is that they are of the upwind kind in a more or less
dramatic manner. Then, they are always dissipative for all the motion scales of any flow. Moreover, the less dissipative fluxes are all based in Finite Differences (as the vast majority of methods for compressible fluxes) and they do not clearly admit unstructured meshes.

The Jameson’s KEP scheme is well formulated for unstructured meshes and has shown to have the best behavior when capturing shock waves. In the other hand, it requires such large meshes that nowadays it can not be pretended to use on engineering real problems. Jameson has demonstrated that his scheme, contrary to the traditional ones, does not add diffusion to the numerical fluxes.

Furthermore, the idea of conserving kinetic energy has been shown to give the best results in incompressible flows (Lehmkuhl et al.[1] and Rodriguez et al. [3]). Hence, it is expected that FKEP will work well when resolving turbulence.

The FKEP scheme has shown to hold the good features of the KEP referring to the shock wave and avoided some of its bad characteristics related with the sub-LCBRC meshes. Although a very rude filter has been used, the low frequencies of the Sod case were correctly resolved. Simulations with CF have resolved well the zone of the domain under the influence of the shock wave while those with FDF have performed better on the expansion zone.

The sub-lying idea of FKEP is that instead of eliminating the instability via upwinding, it could be left as part of the method and eliminate it every time it appears. A great variety of LVD discretization methods without the LCBRC restriction can be envisaged from this starting point.

At this moment, no mathematical proof of the consistency of FKEP has been attempted, neither conservation tests have been performed. Future work must include some proof that FKEP conserves the fluid conservative variables and the kinetic energy. Such theoretical study could impose restrictions to the suitable filters and clarify the shock wave’s filter dependence. Another issue to study is if FKEP is well adaptable to variable-sized meshes. Finally, although the generalization of FKEP to 2D and 3D is straightforward in terms of formulation (in fact all computations presented have been performed with a 3D code), it may lead to filter anisotropy problems and incompatibility with the usual boundary conditions.

In Conclusion, FKEP is an incipient promising option for the resolution of the compressible Navier-Stokes set of equations. With it, the expansion affected sub domain of the shock-tube problem has been resolved with almost any diffusion added (FDF). However, there still remain many unresolved questions about the it.

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