International Journal of Control
Vol. 00, No. 00, Month 200x, 1–24

RESEARCH ARTICLE

High-gain Interval Observer for Partially Linear Systems with Bounded Disturbances

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(*Received 00 Month 200x; final version received 00 Month 200x*)

In this paper, a high-gain interval observer is proposed for a class of partially linear systems affected by unknown but bounded additive disturbances term and measurements noise. The proposed observer is based upon a classical high-gain structure from which an interval observer for the system is designed. The proposed interval observer is designed based on suitable change of coordinates which ensure the cooperativity of the system. To prove the effectiveness of the proposed approach, two numerical examples are provided and the corresponding simulation results are presented.

Keywords: Interval observer; Uncertainty; High-gain observer; Partially linear systems

1 Introduction

High-gain observer (HGO) is an important technique in non-linear control Khalil [2017]. In Khalil and Praly [2014], a brief history of this technique is presented summarizing the main ideas and results as well as some of their applications in control. HGOs were introduced in the context of linear feedback as a tool for robust observer design. The use of HGOs in nonlinear feedback control started to appear in the late 1980s Khalil and Praly [2014]. Since then, an intense research in this topic has been developed with the main results summarized in Khalil and Praly [2014].

ISSN: 0020-7179 print/ISSN 1366-5820 online

DOI: 10.1080/0020717YYxxxxxxxx
http://www.informaworld.com

As discussed in Khalil [2017], one of the most serious challenge in implementing HGOs is the effect of measurement noise and disturbances. One way to deal with this issue, is to estimate these disturbances as proposed in Yao et al. [2018], where a disturbance observer is designed for singular Markovian jump systems. Another way is to work in a bounded-error context that assumes that the disturbances is unknown but bounded, as it is considered in our paper. Therefore, such noted challenge will be face in the context of an unknown-but-bounded description of noise and disturbances, such that instead of generating a nominal estimation an interval that bounds the set of estimated states is proposed using a interval observer approach. On the other hand, interval observers were introduced by Gouze et al. [2000] as an robust estimation approach. Since, then it has been active area of research. As, e.g. in Mazenc and Bernard [2011], time-varying exponentially stable interval observers can be constructed using the Jordan canonical form for a stable linear system with additive disturbances. In Mazenc and Dinh [2014], a new technique of construction of continuous–discrete interval observers for continuous-time systems with discrete measurements and disturbances in the measurements and the dynamics is introduced.

The principle of interval observers is to provide an interval for the state estimation that bounds the effect of noise and disturbances that are assumed to be unknown but bounded, with known bounds. There are two families of approaches for generating such interval. The first family it is based on approximating the set of states by means of some simple set (polytope, ellipsoid or zonotope) that it is computed iteratively from the the previous iteration (see, as e.g., Alamo et al. [2005], Combastel [2015]). On the other, the other approach is to design two observers one that provide respectively the upper and lower bounds of the interval that bounds the set of intervals estimated taking into account disturbance and measurement bounds. These observers are designed to satisfy the cooperativity condition that guarantee that just propagating the extreme values of uncertainty intervals is enough to produce the interval that bounds the set of estimated states Efimov and Raïssi [2016].

The design of interval observers using techniques from non-linear systems is more recent. In Raissi et al. [2010], a procedure based on interval analysis is proposed to build a guaranteed qLPV (quasi-Linear Parameter-Varying) approximation of the nonlinear model. The interval qLPV approximation makes it possible to derive two point observers which estimate respectively the lower and the upper bound of the state vector using cooperativity theory. In Efimov et al. [2013], the design of interval observers for Linear Time Varying (LTV) systems and a class of nonlinear time-varying systems in the output canonical form. In Oubabas et al. [2018], a interval sliding-mode observer is proposed. In Mazenc and Bernard [2012], input to state stability (ISS) theory is applied to the design on interval observers. In Ethabet et al. [2018], the design of interval observers for switched linear systems (SLS), a class of hybrid systems, is addressed. However, the combination of interval and high-gain observers has not been yet explored being this the main innovation of the paper.

In this paper, a high-gain interval observer (HGIO) scheme is proposed for a class of non-linear systems known as partially linear systems. The proposed approach is based on centered forms and complex intervals. Using this framework, an approach based on the design two observers, providing the upper and lower bounds of the state estimation. The proposed HGIO is designed based on suitable change of coordinates which ensure the cooperativity of the system. To prove the effectiveness of the proposed approach, two numerical examples are provided and the corresponding simulation results are presented.

The structure of the paper is a follows: In Section 2, the problem statement is formulated. In Section 3 some preliminaries regarding centered forms and complex intervals are summarized. Section 4 introduces the proposed HGIO scheme. An application example is used to illustrate the proposed approach in Section 5. Finally, Section 6 draws the conclusions and presents future research paths.

2 Problem statement

In this paper, we consider the following class of partially linear systems, with a non-linear term ϕ reflecting the injection of the states, described as follows:

$$\begin{cases} \dot{x}(t) = Ax(t) + \phi(t, u(t), x(t)) + d(t) \\ y(t) = Cx(t) + V(t) \end{cases} \tag{1}$$

where $x(t) \in \mathrm{R}^n$, $u(t) \in \mathrm{R}^m$, $d(t) \in \mathrm{R}^n$, $y(t) \in \mathrm{R}^s$, and $V(t) \in \mathrm{R}^s$ represent, respectively, the state vector, the set of admissible bounded and measurable inputs, the additive disturbances, the measurements and the output disturbances. ϕ is Lipschitz w.r.t the state $x(t)$ and continuous w.r.t to the time. The disturbances are assumed to be unknown but bounded with known bounds.

For the system (1), the following HGO structure is considered Freidovich and Khalil [2008]

$$\dot{\hat{x}}(t) = A\hat{x}(t) + \phi(t, u(t), \hat{x}(t)) + L(y(t) - C\hat{x}(t)) + \hat{d}(t) \tag{2}$$

where $\hat{d}(t)$ represents the estimation of the disturbance $d(t)$ considering that, as discussed in the introduction, the only knowledge we have is about the bounds.

Following Ali et al. [2014], the observer gain is parameterized as follows $L = \theta\Delta_\theta^{-1}K$ where $\theta > 1$ and the matrix Δ_θ is a diagonal matrix n x n defined by:

$$\Delta_\theta = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & \frac{1}{\theta} & \cdots & 0 \\ 0 & 0 & \ddots & 0 \\ 0 \cdots & & 0 & \frac{1}{\theta^n} \end{bmatrix}.$$

Then, (2) can reformulated as

$$\dot{\hat{x}}(t) = (A - \theta\Delta_\theta^{-1}KC)\hat{x}(t) + \phi(t, u(t), \hat{x}(t)) + \theta\Delta_\theta^{-1}Ky(t) + \hat{d}(t) \tag{3}$$

Then, the design of the HGIO (2) involves finding two observers that provides the lower and upper interval bounds of the state estimation guaranteeing that the state of the system (1) satisfies

$$x(t) \in [\underline{x}(t), \overline{x}(t)]$$

assuming that the initial conditions are unknown but bounded inside a compact with known bounds: $x(0) \in X_0$.

As discussed in the introduction, each observer can be then constructed for system (1) under the restrictive assumptions of the cooperativity of the interval estimation error dynamics Gouze et al. [2000]. Cooperativity requires the Metzler character of the matrix $(A - \theta\Delta_\theta^{-1}KC)$. To overcome this limitation, the designed observer is based upon a time-varying change of coordinates, depending on the parameter θ, which renders the matrix $(A - \theta\Delta_\theta^{-1}KC)$ Metzler in the new base Metzler and Hurwitz by the appropriate selection of K.

3 Preliminaries

Before introducing the proposed HGIO for partially linear systems, some preliminaries about complex intervals are given. Complex intervals are usually defined by rectangles or disks in the complex plane Petkovic and Petkovic [1998], Boche [1966]. In the following, centered forms concepts are introduced as well as the main notations and the technical propositions that will further be used to derive the main results of our work. The interested reader can refer to Combastel and Raka [2011] to obtain more details about complex intervals.

The complex intervals used in this work rely on a partial order defined over C (the field of complex numbers) with three statements: $\forall(L,U) \in \mathrm{C}\times\mathrm{C}, L\star U \Leftrightarrow (L^R\star U^R)\wedge(L^I\star U^I)$ where $\star\in\{=,<,>\}$. Similar statements also hold with $\star\in\{\leq,\geq\}$. $L^R\in\mathrm{R}$ and $L^I\in\mathrm{R}$ denotes respectively the real and the imaginary part of $L\in\mathrm{C}$ (idem for U). In the following, this notation will be used to refer to the real and imaginary parts of scalar, vector or matrix complex arguments. A complex interval $[L,U]$ is defined as $[L,U]=[L^R,U^R]+i[L^I,U^I]$ if $L\leq U$, where $[L^R,U^R]=[L,U]^R$ and $[L^I,UI]=[L,U]^I$ are usual real intervals. A centered form can also be introduced using the operator $\pm$ which is defined as:

$$\begin{aligned} \pm : \mathrm{C}\times\mathrm{C}^+ &\rightarrow \mathrm{IC} \\ (c,r) \quad &\mapsto c\pm r=[c-r,c+r] \end{aligned} \tag{4}$$

where $\mathrm{C}^+=\{r\in\mathrm{C}, r\geq 0\}$ is the set of *positive* complex numbers and IC is the set of scalar complex intervals. c (resp. r) denotes the center (resp. the radius) of the complex interval $c\pm r$. Based on the partial order, which is previously introduced, it is clear that $r\geq 0 \Leftrightarrow c-r\leq c+r$. The restriction of $\pm$ to real numbers is defined by analogy to (4) with $\pm:\mathrm{R}\times\mathrm{R}^+ \rightarrow \mathrm{IR}$. Hence, $(c\pm r)^R=c^R\pm r^R$ and $(c\pm r)^I=c^I\pm r^I$.
To compute $a(c\pm r)$, two operators namely *cabs* and *ctimes*, defined in Combastel and Raka [2011], are used in this paper . Their definitions for scalar values are as follows: *cabs*:$|a|=|a^R|+i|a^I|$ and *ctimes*: $a\diamond b=|a||b|+2|a^I||b^I|$ where i and $|.|$ respectively referring to $\sqrt{-1}$ and the absolute value operator. In Combastel and Raka [2011], the element-by-element extension of these operators to vectors and matrices can be found. Then, the linear image of a complex interval matrix can be obtained by applying the following theorem that has been proved in Combastel and Raka [2011].

Theorem 3.1 : $\forall(M,C,R)\in\mathrm{C}^{n\times p}\times\mathrm{C}^{p\times q}\times\mathrm{C}^{p\times q}$,
$M(C\pm R)=(MC)\pm(M\diamond R)\in\mathrm{IC}^{n\times q}$,

where $M \diamond R = |M||R| + 2|M^I||R^I| \in (\mathrm{C}^+)^{p\times q}$ □

Three technical propositions and two corollaries, which have been proven in Combastel [2012], are recalled here since they will be used later in the paper.

Proposition 3.2

$$F \in \mathrm{C}^{n\times p}, G \in \mathrm{C}^{n\times q} : |[F, G]|\mathbf{1} = |F|\mathbf{1} + |G|\mathbf{1} \tag{5}$$

$$F \in \mathrm{C}^{n\times p} : |F|\mathbf{1} \leq \|F\|\mathbf{1}(1 + i) \tag{6}$$

$$F \in \mathrm{C}^{n\times p}, \varsigma \in (\mathrm{R}^+)^n : \|F diag(\varsigma)\|\mathbf{1} = \|F\|\varsigma \tag{7}$$

where $[.,.]$, $\|.\|$, $\mathbf{1}$ and $\leq$ respectively denote the usual horizontal concatenation operator, the element-by-element modulus of a complex matrix argument, a column vector with all elements equal to 1 and the inferior or equal element-by-element relation operator defined over complex vectors and matrices. □

Proposition 3.3 Let z: $\mathrm{R}^+ \to \mathrm{C}^n$, $z^c : \mathrm{R}^+ \to \mathrm{C}^n$ and $z^r : \mathrm{R}^+ \to (\mathrm{C}^+)^n$ be three continuous functions (with respect to time). If $\forall t \in \mathrm{R}^+, z(t) \in z^c(t) \pm z^r(t)$ with $z^r(t) > 0$, then a continuous function $\sigma : \mathrm{R}^+ \to [-1, +1]^{2n}$ returning bounded real vectors values exists and satisfies

$$\forall t \in \mathrm{R}^+, z(t) = z^c(t) + \triangle(z^r(t))\sigma(t) \tag{8}$$

where the operator $\triangle(.)$ is defined as

$$\forall \upsilon \in \mathrm{C}^n, \triangle(\upsilon) = [diag(\upsilon^R), i.diag(\upsilon^I)] \in \mathrm{C}^{n\times 2n} \tag{9}$$

□

Corollary 3.4 Let $z(t) \in \mathrm{C}^n$ be the state of the dynamic system

$$\dot{z}(t) = diag(\kappa)z(t) + \Phi(z(t), t) \tag{10}$$

where $\kappa \in \mathrm{R}^n$ and the initial condition satisfies $z(0) \geq 0$ and $\Phi : \mathrm{C}^n \times \mathrm{R}^+ \to (\mathrm{C}^+)^n$ is a positive function which locally Lipschitz w.r.t. its first argument and continuous w.r.t. the second one, i.e.,

$$\Phi(z(t), t) \geq 0, \forall t \in \mathrm{R}^{+} \tag{11}$$

Then, $z(t)$ satisfies

$$z(t) \geq 0, \forall t \in \mathrm{R}^{+} \tag{12}$$

□

In addition to the assumptions and the conclusion of Corollary 3.4, we have the following Proposition.

Proposition 3.5

Let $\overline{z}(t) \in \mathrm{C}^{n}$ be the state of the dynamic system

$$\dot{\overline{z}} = diag(\kappa)\overline{z}(t) + \overline{\Phi}(\overline{z}(t), t) \tag{13}$$

where $\kappa \in \mathrm{R}^{n}$ and the initial condition satisfies $\overline{z}(0) \geq z(0)$. Let $\overline{\Phi} : (\mathrm{C}^{+})^{n} \times \mathrm{R}^{+} \to (\mathrm{C}^{+})^{n}$ be a positive function which is locally Lipschitz w.r.t. its first argument and continuous w.r.t. the second one satisfying

$$\overline{\Phi}(z(t), t) \geq \Phi(z(t), t), \forall t \in \mathrm{R}^{+}. \tag{14}$$

$\overline{\Phi}$ is the upper bound for Φ that can also be expressed

$$\overline{\Phi}(z(t), t) = g(t) + H(1+i)z^{R}(t) + H(1+i)z^{I}(t) \tag{15}$$

where $g : \mathrm{R}^{+} \to (\mathrm{C}^{+})^{n}$ is a positive and continuous function and $H \in (\mathrm{R}_{+})^{n \times n}$. Then, the states of (10) and (13) satisfy the inequality (16) i.e. $\overline{z}(t)$ is an upper bound for $z(t)$.

$$\forall t \in \mathrm{R}^{+}, \overline{z}(t) \geq z(t) \tag{16}$$

□

It is worth noting that the Proposition 3.5 can be derived from the Proposition 13 presented in Combastel [2012] by considering the particular case where the matrices M and N in Combastel [2012] are replaced here by $H(1+i)$. Based on Corollary 14 in Combastel [2012] and using the last expression, which implies $M^{R} = N^{R} = M^{I} =$

$N^I = H$ (since $H \in \mathbb{R}_+^{n\times n}$), as well as the assumptions and conclusion of Proposition 3.5, the following Corollary can be deduced.

Corollary 3.6 If the Metzler matrix $\hat{A} = [diag(\kappa) + H, H; H, diag(\kappa) + H] \in \mathbb{R}^{2n\times 2n}$ is Hurwitz stable and g is bounded, then $\overline{z}(t)$ is bounded (when $t \to +\infty$). $z(t)$ is also bounded and its upper bound $\overline{z}(t)$ presents a stable dynamics.

□

4 High-gain interval observer

4.1 *High-gain interval observer structure for partially linear systems*

In this section, a procedure for designing a HGIO for the system (**??**) with unknown but bounded additive disturbances is presented. Noted that the measurements are not available at each instant time t. The proposed observer is defined by a centered and radius dynamics which allow to compute the upper and the lower state bounds of system (1). The design of such observer requires some properties such as monotony Gouze et al. [2000]. Unfortunately, this property is hard to satisfy in many cases. To overcome this difficulty, a time-varying change of coordinates which is based upon the diagonalizing of the observer state matrix $A - \theta\Delta_\theta^{-1}KC$ of system (1) will be introduced in this section.

Let us now choose the gain K such that $A-\theta\Delta_\theta^{-1}KC$ is C-diagonalizable and Hurwith stable. Then,

$$A - \theta\Delta_\theta^{-1}KC = \nu^{-1}\, diag(\rho + i\omega)\, \nu \tag{17}$$

where $\nu \in \mathbb{C}^{n\times n}$, $\rho \in \mathbb{R}^n$, $\omega \in \mathbb{R}^n$ respectively denote the vector containing the real and the imaginary parts of the eigenvalues of $A - \theta\Delta_\theta^{-1}KC$. The function *diag* returns a diagonal matrix from its input vector.

Then, a change of base that will guarantee the cooperativity of the transformed system will be applied. This is achieved by the following time-varying change of coordinates applied to (17)

$$z(t) = T(t)\hat{x}(t), \;\; \text{with } \; T(t) = diag(e^{-(\frac{1}{\Theta}+i\omega)t})\nu \tag{18}$$

where $\dfrac{1}{\Theta}$ is a column vector with all elements equal to $\dfrac{1}{\theta}$. The proposed transformation (18) will allow us to design our HGIO for the partially linear system (1) affected by bounded disturbances. The following Theorem 4.1 is formulated summarizing the main result of the paper.

Theorem 4.1 : *Given the system (1) and considering that the following assumptions hold:*

$A_1 : x(0) \in x^c(0) \pm x^r(0) \subset \mathrm{R}^n,\ z^c(0) = \nu x^c(0),\ z^r(0) = \nu \diamond x^r(0),\ x^r(0) \geq 0,\ z^r(0) \geq 0,$

$A_2 : A - \theta\Delta_\theta^{-1}KC \in \mathrm{R}^{n\times n}$ *is Hurwitz stable and* C*-diagonalizable matrix,*

$A_3 : u(t),\ V(t) \in [\underline{V}, \overline{V}]$ *and* $d(t), \hat{d}(t) \in [\underline{d}, \overline{d}]$ *are continuous w.r.t time,*

$A_4 : u(t)$ *is known and bounded,* $x^c(0)$ *and* $x^r(0)$ *are known.*

Then,

$$\begin{cases} \dot{z}^c(t) = Mz^c(t) + \Psi^c(t) \\ \dot{z}^r(t) = Mz^r(t) + \Psi^r(t) \end{cases} \tag{19}$$

is a HGIO for system the (1) where ($z^c(t)$) and the radius ($z^r(t)$) represents respectively the state dynamics of both the center and the radius in the new base ($z(t)$). The terms $\Psi^c(t)$ *and* $\Psi^r(t)$ *are defined as follows:*

$$\begin{cases} \Psi^c(t) = T(t)\phi^c(t, u(t)) + T(t)d^c(t) + T(t)\theta\Delta_\theta^{-1}KV^c(t) \\ \Psi^r(t) = |T(t)\hat{E}(t, z^r(t), z^c(t), d^r(t))|\boldsymbol{1} \\ T(t) = diag(e^{-(\frac{1}{\Theta}+i\omega)t})\nu,\ T^{-1}(t) = \nu^{-1}diag(e^{(\frac{1}{\Theta}+i\omega)t}),\ M = diag(\rho - \dfrac{1}{\Theta}) \\ \hat{E}(t, z^r(t), z^c(t), d^r(t)) = [\phi^r(t, z^c(t), \Delta(z^r(t))) \mid d^r(t) \mid \theta\Delta_\theta^{-1}KCT^{-1}(t)z^c(t) \\ \qquad \mid \theta\Delta_\theta^{-1}KCT^{-1}(t)\Delta(z^r(t)) \mid \theta\Delta_\theta^{-1}KV^r(t)] \\ d(t) = d^c(t) + d^r(t)[-1, +1]^n \\ V(t) = V^c(t) + V^r(t)[-1, +1]^s \end{cases} \tag{20}$$

In addition, the proposed HGIO (19) *satisfies the inclusion property in the new base*

$$z^r(t) \geq 0 \wedge z(t) \in z^c(t) \pm z^r(t) \subset \mathrm{C}^n, \forall t \in \mathrm{R}^+ \tag{21}$$

and also in the original base

$$\hat{x}^r(t) \geq 0 \wedge \hat{x}(t) \in \hat{x}^c(t) \pm \hat{x}^r(t) \subset \mathrm{C}^n, \forall t \in \mathrm{R}^+ \tag{22}$$

$$\hat{x}^c(t) = T^{-1}(t)z^c(t), \quad \hat{x}^r(t) = T^{-1}(t) \diamond z^r(t). \tag{23}$$

□

Remark: Note that $A3$ is satisfied because in the problem statement (Section 2), it is considered that both noise V and disturbance d are assumed unknown but with known bounds. As discussed in the introduction, another approach could be to proceed as in Ali et al. [2014] where the disturbance d is estimated by means of an augmented state system. Then, disturbance estimation error $(\hat{d}(t) - d(t))$ has been proven to be inside the ball whose radius can be made arbitrary small as the value of the parameters θ is high which is the case in our paper. In this case, it is reasonable to assume that after a certain time t_0, $\hat{d}(t) \to d(t)$.

Proof. To prove the theorem, we will follow a two steps process. In the first step, the expression of both $\Psi^c(t)$ and $\Psi^r(t)$ such that $\Psi(t) \in \Psi^c(t) \pm \Psi^r(t)$ as well as the system dynamics in the new base are detailed using the proposed time-varying change of coordinates proposed in (18). Based on the obtained expressions $(\Psi^c(t), \Psi^r(t))$ and Theorem 3 in Combastel and Raka [2011], the structure of the HGIO as well as the inclusion property are established in the last step. In what follows, we will derive the structure of the HGIO in the new base (z).

Differentiating (18) with respect to time yields to:

$$\dot{z}(t) = diag(-\frac{1}{\Theta} - iw)diag(e^{-(\frac{1}{\Theta}+iw)t})\nu\hat{x}(t) + diag(e^{-(\frac{1}{\Theta}+iw)t})\nu\dot{\hat{x}}(t) \tag{24}$$

Replacing $\dot{\hat{x}}(t)$ by its expression in (3) and using the fact that $\hat{x}(t) = T^{-1}(t)z(t)$, we derive:

$$\begin{aligned}\dot{z}(t) = {} & diag(-\frac{1}{\Theta} - iw)z(t) + T(t)(A - \theta\Delta_\theta^{-1}KC)T^{-1}(t)z(t) \\ & + T(t)\phi(t, u(t), z(t)) + T(t)\hat{d}(t) + T(t)\theta\Delta_\theta^{-1}Ky(t)\end{aligned} \tag{25}$$

Now, replacing $A - \theta\Delta_\theta^{-1}KC$ by its expression which results from its diagonalization in (17) and using the expression of the measurements y in (1), the system dynamics z can be expressed as follows:

$$\begin{aligned}\dot{z}(t) = {} & diag(-\frac{1}{\Theta})z(t) - diag(iw)z(t) + diag(\rho)z(t) + diag(iw)z(t) + T(t)\phi(t, u(t), z(t)) + T(t)\hat{d}(t) \\ & + T(t)\theta\Delta_\theta^{-1}KCT^{-1}(t)z(t) + T(t)\theta\Delta_\theta^{-1}KV(t).\end{aligned} \tag{26}$$

which can be rewritten under the following form:

$$\dot{z}(t) = Mz(t) + \Psi(t) \tag{27}$$

where $M = diag(\rho - \dfrac{1}{\Theta})$ and $\Psi(t) = T(t)\phi(t, u(t), z(t)) + T(t)\hat{d}(t) + T(t)\theta\Delta_\theta^{-1}KCT^{-1}(t)z(t) + T(t)\theta\Delta_\theta^{-1}KV(t)$.

Now, replacing in (27) the expression of $V(t) = V^c(t) + V^r(t)[-1, +1]^s$ and $\hat{d}(t) = d^c(t) + d^r(t)[-1, +1]^n$ and using the fact that the additive disturbances are bounded with known bounds then we have:

$$\begin{aligned}\Psi(t) \in\ & T(t)\phi^c(t, u(t)) + T(t)d^c(t) + T(t)\theta\Delta_\theta^{-1}KV^c(t) + +T(\phi^r(t, z^c(t), \Delta(z^r(t))) + d^r(t) \\ & + \theta\Delta_\theta^{-1}KCT^{-1}(t)z^c(t) + \theta\Delta_\theta^{-1}KCT^{-1}(t)\Delta(z^r(t)) + \theta\Delta_\theta^{-1}KV^r(t))(\mathbf{0} \pm \mathbf{1})\end{aligned} \tag{28}$$

From Theorem 3.1, $\Psi(t)$ can be expressed as: $\Psi(t) \in \Psi^c(t) \pm \Psi^r(t)$ where

$$\Psi^c(t) = T(t)\phi^c(t, u(t)) + T(t)d^c(t) + T(t)\theta\Delta_\theta^{-1}KV^c(t)$$

with

$$\Psi^r(t) = |T(t)\hat{E}(t, z^r(t), z^c(t), d^r(t))|\mathbf{1}$$

and

$$\begin{aligned}\hat{E}(t, z^r(t), z^c(t), d^r(t)) =& [\phi^r(t, z^c(t), \Delta(z^r(t))) \mid d^r(t) \mid \theta\Delta_\theta^{-1}KCT^{-1}(t)z^c(t) \\ & \mid \theta\Delta_\theta^{-1}KCT^{-1}(t)\Delta(z^r(t)) \mid \theta\Delta_\theta^{-1}KV^r(t)]\end{aligned} \tag{29}$$

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Under the assumptions considered in Theorem 4.1 are satisfied and using Theorem 3 in Combastel and Raka [2011], one can show that the proposed HGIO verify the inclusion properties (21) and (22). This ends the proof of theorem.

4.2 *Stability of the proposed observer*

In this section, we will prove the stability of the proposed HGIO which in fact derives from the stability of both the center and radius dynamics of (19).

Let first prove the stability of the center dynamics z_c. Since the matrix M is Hurwitz stable and both the input $u(t)$ and the disturbances $(d(t),\ V(t))$ are assumed to be continuous and bounded, we can easily derive that $\Psi^c(t)$ which depends only on $u(t)$, $d^c(t)$ and $V^c(t)$ that is bounded too. As a consequence, the stability of the center dynamics of $(z^c(t))$ is continuous, bounded and follows a stable dynamics.

On the other hand, the stability of the radius dynamics $(z^r(t))$ can not be directly addressed in the same way as it was addressed for the dynamic center $z^c(t)$. Indeed, the term $\Psi^r(t)$ depends on $z^r(t)$, $z^c(t)$, $d^r(t)$ and $V^r(t)$. Then, $z^c(t)$ as well as $d^r(t)$ and $V^r(t)$ can be viewed as a bounded and continuous exogenous term in the second equation of (19) since $z^r(t)$ does not affect the stability of the dynamic of $z^c(t)$. Meanwhile, $\Psi^r(t)$ still depends on the endogenous term $z^r(t)$ and thus, it is necessary to find a sufficient condition which ensures the stability of the radius dynamics $z^r(t)$. In what follows, this sufficient condition is summarized in the following Proposition.

Proposition 4.2 Let $P = \|\nu\theta\Delta_\theta^{-1}KC\nu^{-1}\| + |\alpha|$, where $\alpha \in \mathrm{R}_+^*$. If $(\rho - \frac{1}{\Theta}) < 0$ (M is Hurwitz stable) and if the Metzler matrix $[M+P, P; P, M+P]$ is Hurwitz stable, then $\forall t, 0 \leq z^r(t) \leq \bar{z}^r(t) < \infty$ and $\bar{z}^r(t)$ follows a stable dynamics, so is $z^r(t)$. □

Proof. Choosing a vector gain K such that $(A - \theta\Delta_\theta^{-1}KC)$ is Hurwitz will ensure the stability of the matrix M (i.e. $(\rho - \frac{1}{\Theta}) < 0$) and consequently the stability of the center dynamics $z^c(t)$ given by the first equation in (19). Based on the fact that $\Psi^c(t)$ is continuous, bounded and does not depend on the radius dynamic $z^r(t)$, we can observe that $z^c(t)$ is also continuous and bounded with a stable dynamics. Using the fact that $z^c(t)$ has a stable dynamics, the second equation in (19) can be rewritten as in (30) since $z^c(t)$ can be considered as an exogenous input term in (31)-(32).

$$\dot{z}^r(t) = Mz^r(t) + \Psi^r(t, z^r(t)) \tag{30}$$

where

$$\Psi^r(t, z^r(t)) = |T(t)\hat{E}(t, z^r(t), z^c(t), d^r(t))|\mathbf{1} \tag{31}$$

and

$$\begin{aligned}\hat{E}(t, z^r(t), z^c(t), d^r(t)) =& [\phi^r(t, z^c(t), \Delta(z^r(t))) \mid d^r(t) \mid \theta\Delta_\theta^{-1}KCT^{-1}(t)z^c(t) \mid \\ & \theta\Delta_\theta^{-1}KCT^{-1}(t)\Delta(z^r(t)) \mid \theta\Delta_\theta^{-1}KV^r(t)].\end{aligned} \tag{32}$$

Based on both (5) in Proposition 3.2 and (9) in Proposition 3.3, the expression of $\Psi^r(t, z^r(t))$ can be rewritten as follows:

$$\Psi^r(t, z^r(t)) = g_1(t) + f_1(t) + f_2(t) + f_3(t)$$

where:

$$\begin{aligned}
g_1(t) &= |T(t)[d^r(t) \mid \theta\Delta_\theta^{-1}KCT^{-1}(t)z^c(t) \mid \theta\Delta_\theta^{-1}KV^r(t)]|\mathbf{1} \\
f_1(t) &= |T(t)\phi^r(t, z^c(t), \Delta(z^r(t))|\mathbf{1} \\
f_2(t) &= |T(t)\theta\Delta_\theta^{-1}KCT^{-1}(t)diag(z^{r,R}(t))|\mathbf{1} \\
f_3(t) &= |T(t)\theta\Delta_\theta^{-1}KCT^{-1}(t)\ i\ diag(z^{r,I}(t))|\mathbf{1}.
\end{aligned}$$

The positivity of the input term $\Psi^r(t, z^r(t))$ results from the definition of the *cabs* operator. Consequently, we have $\forall t \geq 0, \Psi(t, z^r(t)) \geq 0$ and $z^r(0) \geq 0$. Using Corollary 3.4, we can derive that $\forall t \geq 0, z^r(t) \geq 0$ (i.e. $z^{r,R}(t) \geq 0$ and $z^{r,I}(t) \geq 0$). Let us now compute the upper bounds of the terms $f_1(t), f_2(t)$ and $f_3(t)$. Consider first the term $f_2(t)$. Based on (6) and (7) in Proposition 3.2, the first inequality and equality in (33) can be deduced. The second equality in (33) can be derived from the expression of the matrix $T(t)$ (18) and using the fact that $\forall \beta \in$ R, $\|e^{i\beta}\| = 1$

$$\begin{aligned}
f_2(t) \leq \|T(t)\theta\Delta_\theta^{-1}KCT^{-1}(t)diag(z^{r,R}(t))\|\mathbf{1}(1+i) = \\
\|T(t)\theta\Delta_\theta^{-1}KCT^{-1}(t)\|z^{r,R}(t)(1+i) = \\
\|\nu\theta\Delta_\theta^{-1}KC\nu^{-1}\|z^{r,R}(t)(1+i).
\end{aligned} \tag{33}$$

Proceeding similarly, an upper bound of the term $f_3(t)$ can be obtained as follows:

$$f_3(t) \leq \|\nu\theta\Delta_\theta^{-1}KC\nu^{-1}\|z^{r,I}(t)(1+i). \tag{34}$$

Finally, the term $f_1(t)$ can be bounded considering that $\exists \alpha \geq 0$ such that

$$|T(t)\phi^r(t, z^c(t), \Delta(z^r(t))|\mathbf{1} \leq |\alpha\overline{z}^c(t)| + |\alpha\Delta(z^r(t))|\mathbf{1}$$

where $\overline{z}^c(t)$ is the upper bound of the center dynamics $z^c(t)$. Note that this assumption is not restrictive since the function $\phi^r(t, z^c(t), \Delta(z^r(t))$ is Lipschitz w.r.t $z(t)$ and z^c is bounded. Thus, based on the Proposition 3.2 and the definition of the operator Δ in (9) of the Proposition 3.3, the upper bound of the term $f_1(t)$ can be expressed as follows:

$$f_1(t) \leq |\alpha \overline{z}^c(t)| + |\alpha|(1+i)(z^{r,R}(t) + z^{r,I}(t)) \tag{35}$$

Replacing now the terms $f_1(t)$, $f_2(t)$ and $f_3(t)$ by their respective upper bounds, an upper bound for the input term $\Psi^r(t, z^r(t))$ can be given as follows:

$$\Psi^r(t, z^r(t)) \leq \overline{\Psi}^r(t, z^r(t)) = g(t) + P(1+i)(z^{r,R}(t) + z^{r,I}(t)) \tag{36}$$

where $g(t) = g_1(t) + |\alpha \overline{z}^c(t)|$ and $P = \|\nu\theta\Delta_\theta^{-1}KC\nu^{-1}\| + |\alpha|$.

Proposition proof is completed by recalling that the matrix P is the same matrix defined in Proposition 4.2 and by applying the Proposition 3.5 and Corollary 3.6 where $diag(\kappa)$, z, $\overline{z}$, Φ, $\overline{\Phi}$ are replaced respectively by M, z^r, $\overline{z}^r$, Ψ^r, $\overline{\Psi}^r$ and $H = P$. $\square$

4.3 *Particular case of LTI systems*

In this section, the HGIO design procedure presented in Section 4.1 is applied to the particular case of the LTI systems given by

$$\begin{cases} \dot{x}(t) = Ax(t) + Bu(t) + F(t) + d(t) \\ y(t) = Cx(t) + V(t) \end{cases} \tag{37}$$

where $x(t) \in \mathrm{R}^n$, $u(t) \in \mathrm{R}^m$, $y(t) \in \mathrm{R}^s$ and $F(t) \in \mathrm{R}^n$ and $d(t) \in \mathrm{R}^n$ are respectively the state vector, the known input, the measurements vector, the unknown but bounded input and the additive disturbances. The disturbances as well as the initial state are assumed to be unknown but bounded with known bounds.

Theorem 4.1 can be adapted for designing a HGIO of the LTI system (37) as follows.

Theorem 4.3 : *Given a system described by (37) and assuming that the following hypotheses hold:*

$A_1 : x(0) \in x^c(0) \pm x^r(0) \subset \mathrm{R}^n,\ z^c(0) = \nu x^c(0),\ z^r(0) = \nu \diamond x^r(0),\ x^r(0) \geq 0,\ z^r(0) \geq 0,$

$A_2 : A - \theta\Delta_\theta^{-1}KC \in \mathrm{R}^{n\times n}$ *is a known* C*-diagonalizable matrix,*

$A_3 : u(t),\ F(t) \in [\underline{F}, \overline{F}],\ V(t) \in [\underline{V}, \overline{V}],\ d(t), \hat{d} \in [\underline{d}, \overline{d}]$ *are continuous w.r.t. time,*

$A_4 : u(t)$ *is known,* $F(t)$ *unknown but bounded,* $x^c(0)$ *and* $x^r(0)$ *are known. Then, the* *following system (38)*

$$\begin{cases} \dot{z}^c(t) = Mz^c(t) + \Psi^c(t) \\ \dot{z}^r(t) = Mz^r(t) + \Psi^r(t) \end{cases} \tag{38}$$

with $\Psi^c(t)$ *and* $\Psi^r(t)$ *defined as follows*

$$\begin{cases} \Psi^c(t) = T(t)Bu(t) + T(t)F^c(t) + T(t)d^c(t) + T(t)\theta\Delta_\theta^{-1}KV^c(t) \\ \Psi^r(t) = |T(t)\hat{E}(t, z^r(t), z^c(t), F^r(t), d^r(t))|\mathbf{1} \\ T(t) = diag(e^{-(\frac{1}{\Theta}+i\omega)t})\nu,\ T^{-1}(t) = \nu^{-1}diag(e^{(\frac{1}{\Theta}+i\omega)t}),\ M = diag(\rho - \dfrac{1}{\Theta}) \\ \hat{E}(t, z^r(t), z^c(t), F^r(t), d^r(t)) = [F^r(t) \mid d^r(t) \mid \theta\Delta_\theta^{-1}KCT^{-1}(t)z^c(t) \\ \qquad\qquad \mid \theta\Delta_\theta^{-1}KCT^{-1}(t)\Delta(z^r(t)) \mid \theta\Delta_\theta^{-1}KV^r(t)] \\ F(t) = F^c(t) + F^r(t)[-1, +1]^n \\ d(t) = d^c(t) + d^r(t)[-1, +1]^n \\ V(t) = V^c(t) + V^r(t)[-1, +1]^s \end{cases} \tag{39}$$

is a HGIO for system (37) with the following properties:

$$z^r(t) \geq 0 \wedge z(t) \in z^c(t) \pm z^r(t) \subset \mathrm{C}^n, \forall t \in \mathrm{R}^+ \tag{40}$$

$$\hat{x}^r(t) \geq 0 \wedge \hat{x}(t) \in \hat{x}^c(t) \pm \hat{x}^r(t) \subset \mathrm{C}^n, \forall t \in \mathrm{R}^+ \tag{41}$$

$$\hat{x}^c(t) = T^{-1}(t)z^c(t), \quad \hat{x}^r(t) = T^{-1}(t) \diamond z^r(t). \tag{42}$$

□

Proof. The considered LTI system is a particular case of the class of nonlinear system (1) by defining

$$\phi(t, u(t), x(t)) = Bu(t) + F(t) \tag{43}$$

In this case the term, ϕ depends only on the inputs ($u(t)$ and $F(t)$) and not on the state $x(t)$. We might consider that the term ϕ can depend on the input and the states $x(t)$. In this case the structure of our LTI system will take the following form:

$$\phi(t, u(t), x(t)) = Bu(t) + F(t) + Dx(t) \tag{44}$$

In this case, rather than working with the constant matrix A, we will work with the constant matrix $A + D$ and the whole procedure of designing the observer will remain the same.

To prove Theorem 4.3, we follow the same procedure which was considered for the Theorem 4.1 with the following considerations:

$$\phi^c = Bu(t) + F^c(t), \phi^r = F^r(t)$$

The stability of the HGIO for the LTI systems derives from the stability of both the center and radius dynamics (38). As it was mentioned in the Section 4.2, the stability of the center dynamic ($z^c(t)$) can be easily deduced from the fact that the matrix M is Hurwitz stable and $\Psi^c(t)$ depends only on $u(t)$, $F^c(t)$, $d^c(t)$ and $V^c(t)$ which are continuous and bounded. The stability of the radius dynamics ($z^r(t)$) is ensured by satisfying Proposition 4.2 with $\alpha = 0$. Indeed, in the case of LTI systems we have:

$$\Psi^r(t, z^r(t)) = g_1(t) + f_1(t) + f_2(t) + f_3(t)$$

where:

$$\begin{aligned}
g_1(t) &= |T(t)[F^r(t) \mid d^r(t) \mid \theta\Delta_\theta^{-1}KCT^{-1}(t)z^c(t) \mid \theta\Delta_\theta^{-1}KV^r(t)]|\mathbf{1} \\
f_1(t) &= 0 \\
f_2(t) &= |T(t)\theta\Delta_\theta^{-1}KCT^{-1}(t)diag(z^{r,R}(t))|\mathbf{1} \\
f_3(t) &= |T(t)\theta\Delta_\theta^{-1}KCT^{-1}(t)\ i\ diag(z^{r,I}(t))|\mathbf{1}.
\end{aligned}$$

. Since the term $f_1(t) = 0$ in the case of the LTI system, we have $\alpha = 0$.

Note that as it is the case of the nonlinear class of systems (1) considered in the Theorem 4.1, the proposed HGIO satisfies the inclusion property given respectively by (40) in the new base ($z(t)$) and (41) in the original base ($\hat{x}(t)$).

5 Application examples

To prove the effectiveness of our proposed HGIO, we present in this section the application results in two examples which illustrate the design procedure for the considered classes of system in this paper.

5.1 *First example*

The first considered example corresponds to an example of the system (1) with the following particular matrices and functions

$$A = \begin{bmatrix} -1 & -3 & -5 \\ 0 & 1 & -3 \\ 0 & 2 & -2 \end{bmatrix}, \; \phi(t, u(t), x(t) = \begin{bmatrix} 2u(t) + exp(-|x_1|) \\ u(t) + exp(-|x_2|) \\ -u(t) + exp(-|x_3|) \end{bmatrix}, \; C = [1\ 0\ 0]$$

The input is given by

$$u(t) = 3sin(t) - 2sin(3t) \tag{45}$$

while the input and output disturbances are given respectively by

$$d(t) = \begin{bmatrix} 0.1sin(t) \\ sin(0.1\pi t) \\ sin(2t) \end{bmatrix}, \; V(t) = 0.1sin(0.1\pi t) \tag{46}$$

According to the stability of the HGIO is achieved by selecting the observer gain vector equal to $K = 1e-6[1;\ 1e-3;\ 1e-6]$ and the parameter θ =1000 such that $(A - \theta\Delta_\theta^{-1}KC)$ is Hurwitz.

Let us now check that the assumptions considered in Theorem 4.1 are satisfied in this example:

- The system states are initialized inside the compact set defined by $\underline{x}(0) = x^c(0) - x^r(0)$ and $\overline{x}(0) = x^c(0) + x^r(0)$ with $x^c(0) = [0\ 3\ 4]^T$ and $x^r(0) = [0.09\ 0.2\ 0.6]^T$.

- The matrix $A - \theta\Delta_\theta^{-1}KC$ is

$$A - \theta\Delta_\theta^{-1}KC = \begin{bmatrix} -1.001 & -3 & 5 \\ -0.001 & 1 & -3 \\ -0.001 & 2 & -2 \end{bmatrix} \tag{47}$$

  such that additionally to be stable by design is C-diagonalizable with the following eigenvalues: -1.0025, -0.4992 + 1.9346i and -0.4992-1.9346i).

- Both the additive input and the output disturbances are bounded with the following bounds $V^c = 0$, $V^r = 0.1$, $d^c = [0\ 0\ 0]^T$ and $d^r = [0.1\ 1\ 1]^T$

Applying Theorem 4.1, the numerical simulations of HGIO were conducted in the time range from $t = 0$ to $t = 30s$ with the Euler algorithm with a discretisation step $h = 0.001$. The simulation results of are presented in Fig.1.

From the results presented in Fig.1, we can first conclude that the inclusion property is verified in both bases ($z(t)$ and $x(t)$). Moreover, since the matrix $[M+P, P; P, M+P]$ (where $P = \|\nu\theta\Delta_\theta^{-1}KC\nu^{-1}\| + |\alpha|$ with $\alpha = 0.01$) has all its eigenvalues with negative real part ($[-1.0035\ \ -0.9806\ \ -0.4541\ \ -0.5003\ \ -0.5003\ \ -0.5003]^T$), it can easily be checked that the sufficient condition introduced in Proposition 4.2 is verified. Thus, concerning the stability of the proposed observer, we can see also see that the upper and the lower bounds provided by designed high-gain interval observer are stable in spite of the presence of bounded additive and output disturbances. It should be mentioned that for this example, the considered assumption which have been introduced in the proof of Proposition 4.2 to complete is also verified. Indeed, for this example, we have

$$\phi^r(t, z^c(t), \Delta(z^r(t))) = exp(-|T^{-1}(z^c(t) + \Delta(z^r(t))\mathbf{1})|)$$

Using the fact that $\forall\beta, \exists\alpha > 0$ such that $exp(-|\beta|) \leq |\alpha\beta|$ and taking into account that the function $exp(-|\beta|)$ is Lipschitz with respect to β (with $\beta = T^{-1}(z^c(t) + \Delta(z^r(t))\mathbf{1})$ in this example), the following inequality is satisfied

$$|T\phi^r(t, z^c(t), \Delta(z^r(t)))|\mathbf{1} \leq |\alpha\overline{z}^c(t)| + |\Delta(z^r(t))|\mathbf{1}$$

where $\overline{z}^c(t)$ is the upper bound of $z^c(t)$. Thus, in this example, the considered assumption is not at all restrictive and can be easily satisfied since ϕ is Lipschitz w.r.t the state.

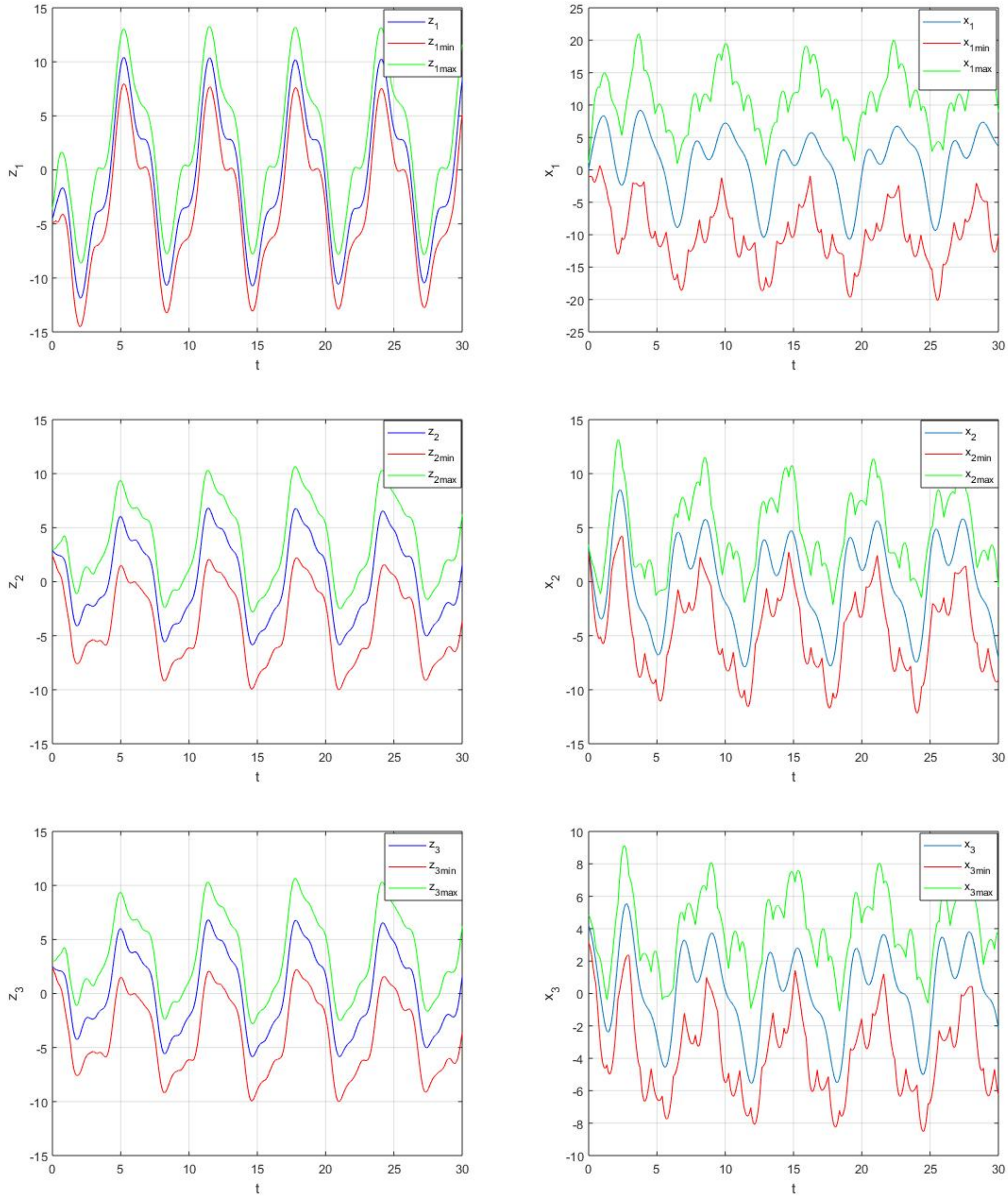

Figure 1. Simulation results for the HGIO applied to the first example. Left: States in the new base $(z(t))$. Right: states in the original base $(x(t))$. For the sake of clarity we choose to show the results of the numerical simulations of our proposed observer in both the new base $z(t)$ and the original base $x(t)$.

5.2 *Second example*

In this second example, an LTI system with an additive disturbances and unknown but bounded input term $f(t)$ described by (37) in considered with the following particular values for matrices

$$A = \begin{bmatrix} -1 & -3 & 5 \\ 0 & 1 & -3 \\ 0 & 2 & -2 \end{bmatrix}, \quad B = \begin{bmatrix} 2 \\ 1 \\ -1 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}$$

On the other hand, the inputs $u(t)$ and $F(t)$, the additive disturbance term $d(t)$, and the output disturbance term $V(t)$ are

$$u(t) = 6sin(\pi t), \; F(t) = 0.2 \begin{bmatrix} cos(t) \\ 2sin(t) \\ 3cos(t) \end{bmatrix}, \quad d(t) = \begin{bmatrix} 0.09sin(0.1t) \\ 0.01cos(0.3\pi t) \\ 0.04sin(0.2t) \end{bmatrix}, \quad V(t) = 0.07sin(0.2\pi t)$$

To ensure the stability of the matrix $(A - \theta\Delta_\theta^{-1}KC)$, the vector of the observer gain was chosen equal to $K = 1e - 6[8;\; 4e - 3;\; 2e - 6]$ and the parameter θ is 1000.

Let us not check that the considered assumptions in Theorem 4.3 are satisfied in this numerical example. For this purpose we have

- The initial system states are bounded with known bounds : $\underline{x}(0) = x^c(0) - x^r(0)$ and $\overline{x}(0) = x^c(0) + x^r(0)$ where $x^c(0) = [0 \; 3 \; 4]^T$ and $x^r(0) = [0.09 \; 0.2 \; 0.6]^T$.
- The matrix $A - \theta\Delta_\theta^{-1}KC$ is stable and C-diagonalizable.

$$A - \theta\Delta_\theta^{-1}KC = \begin{bmatrix} -1.008 & -3 & 5 \\ -0.004 & 1 & -3 \\ -0.002 & 2 & -2 \end{bmatrix} \tag{48}$$

with

- The additive disturbance and the output disturbance terms are bounded i.e we have $V^c = 0$, $V^r = 0.07$, $d^c = [0 \; 0 \; 0]^T$ and $d^r = [0.09 \;\; 0.01 \;\; 0.04]^T$

The numerical simulations were conducted between $t = 0$ and $t = 10s$ with a step size $h = 0.001$. The simulation results of the proposed HGIO for LTI systems described by equations (38)-(39) are reported in Fig.2.

Looking at the simulation results presented in Fig.2, we can conclude that the inclusion property is verified in the both bases ($z(t)$ and $x(t)$). Moreover, the matrix $[M+P, P; P, M+P]$ (where $P = \|\nu\theta\Delta_\theta^{-1}KC\nu^{-1}\|$) has all its eigenvalues with negative

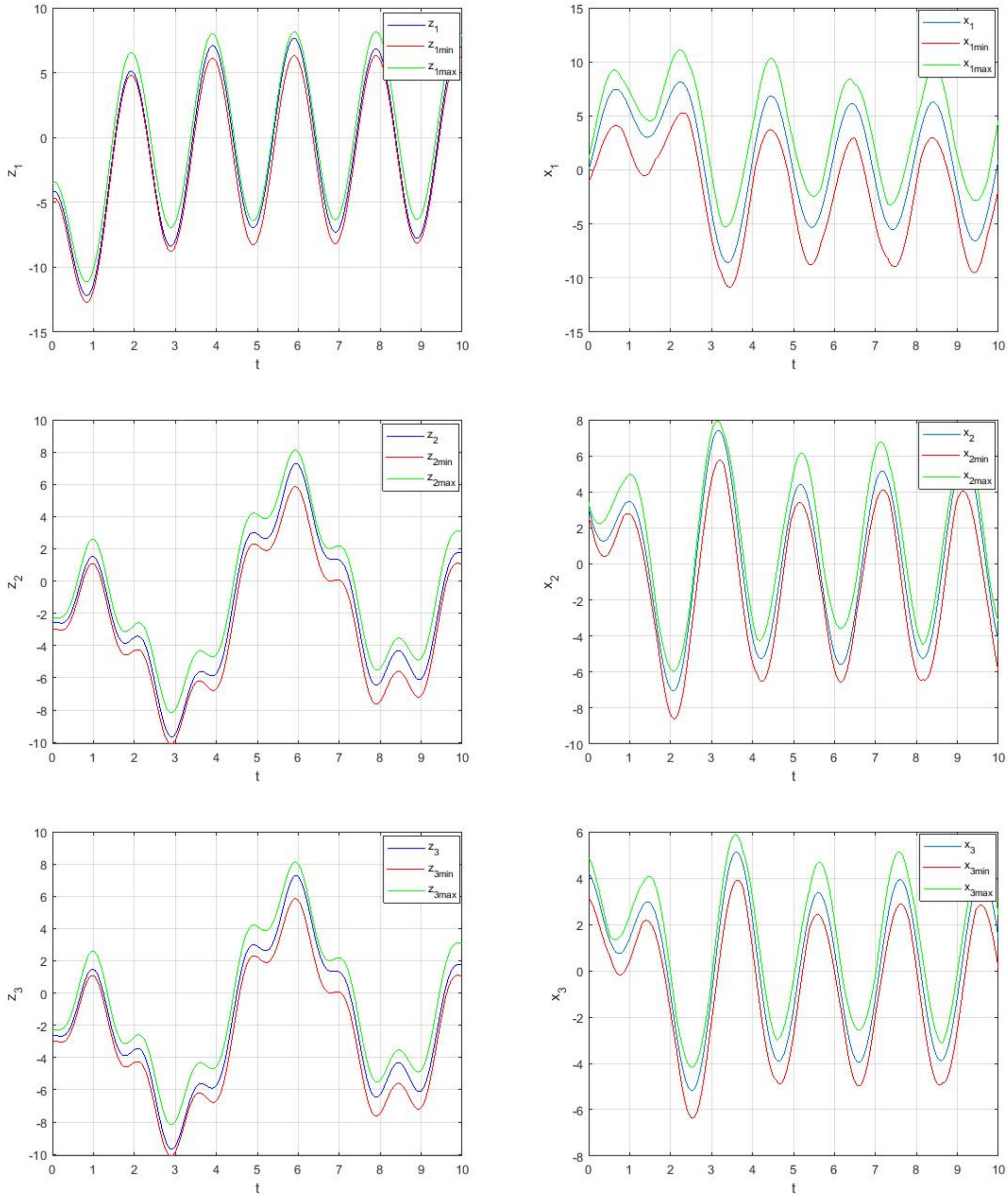

Figure 2. Simulation results for the HGIO applied to the second example. Left: States in the new base ($z(t)$). Right: states in the original base ($x(t)$).

real part ($[-1.0155 \quad -0.9873 \quad -0.4840 \quad -0.4978 \quad -0.4978 \quad -0.4978]^T$). Hence, the sufficient conditions given in Proposition 4.2 (with $\alpha = 0$ for LTI systems) are verified. Concerning the stability of the proposed observer, we can see also see that the upper and the lower dynamics of our observer are stable in spite of the presence of bounded additive and output disturbances.

6 Conclusions

This paper has proposed a high-gain interval observer for a class of partially linear system affected by unknown but bounded additive disturbances term and measurements noise. The proposed observer is based upon a classical high-gain structure from which an interval observer for the system is designed. The proposed interval observer is designed based on suitable change of coordinates which ensure the cooperativity of the system. To prove the effectiveness of the proposed approach, numerical simulations are performed and presented in the paper. As a future works, the proposed approach will be further extended to provide a method to tune the high-gain observer parameters in an optimal manner establishing a trade-off between the input and output uncertainties. To well characterize and capture more information of the disturbances than using just the bounds, the estimation problem of the disturbances will be also addressed.

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