A Service Reliability Model Predictive Control with Dynamic Safety Stocks and Actuators Health Monitoring for Drinking Water Networks

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Abstract—This paper presents a model predictive control strategy to assure reliability in drinking water networks given a customer service level and a forecasting demand. The underlying idea concerns a two-layer hierarchical control structure. The upper layer performs a local steady-state optimization to set up an inventory replenishment policy based on dynamic safety stocks for each tank in the network. At the same stage, actuators health is revised to set up their next maximum allowable degradation in order to efficiently distribute overall control effort and guarantee system availability. In the lower layer, a model predictive control algorithm is implemented to compute optimal control set-points to minimize a multi-objective cost function. Simulation results in the Barcelona drinking water network have shown the effectiveness of the dynamic safety stocks allocation and the actuators health monitoring to assure service reliability and optimizing network operational costs.

I. INTRODUCTION

Drinking Water Networks (DWNs) are large-scale multi-source/multi-node supply chain (SC) systems which must be reliable and resilient while being subject to constraints and continuously varying conditions with both deterministic and probabilistic nature [24]. In general, DWNs operate as pull interconnected systems driven by exogenous and endogenous demands. The overall objective of managers is to provide a reliable water supply in the most economical way, guaranteeing availability and continuity of the service with a certain probability and without delay under some operating conditions, specific environments and uncertain events.

Supply chain management (SCM) is a complex task and has become an increasingly research subject worldwide [2], with special attention to efficient handling of resources and planning against uncertainty of demand and/or supply. Strategic and tactical decisions in networks operation can be addressed by different methods (see, e.g., [14], [18]), but control and systems theory have shown to be suitable in SCM to handle the problem consisting of uncertainties, delays and lack of information (see, e.g., [12], [20]). A central coordinator, who controls the SC, can account for these issues and provide optimal operation. Hence, decision policies inspired from the model predictive control (MPC) framework, see [9], are used due to its flexibility to manage constraints and optimize multi-objective problems of complex dynamic systems. Relevant works apply MPC to DWNs and general SC in different schemes such as: centralized, decentralized, distributed, hierarchical, and robust, see, e.g., [11], [15], [16], [21], [25].

Reliability assurance of supply chains is mainly associated with demand forecasting and safety stocks allocation in storage units as a countermeasure to secure network performance against forecast inaccuracy, (see, e.g., [7], [6], [13], [19], [22]). Most of the results reported in the literature assume that demand forecast error is stationary and usually normally distributed while replenishment lead time (the time from the moment a supply requirement is placed to the moment it is received) is stationary and usually certain [7].

In practical operation of DWNs, the settling of the safety stock is typically determined by experience, estimating risk and assigning a fixed value (i.e., a proportion of the volume of the storage) for the entire planning horizon. This approach is too conservative and reduces the manoeuvrability space for economic optimization since the full tank excursion is limited and its capacity is not usable to save energetic costs in pumping actions [4]. On the other hand, lead time do vary over when capacity is limited or time varying, nevertheless, models that take into account non-stationary behaviours are not completely helpful if they use this information to just calculate safety stocks, especially when variations are caused by network supply components ageing. Demand behaviour is an exogenous disturbance in DWNs, but flows and actuators are manipulated and monitored elements, therefore reliability and control effort allocation in supply chains should be also assessed considering system health.

To the best of the authors’ knowledge, reliability degradation models (see, e.g., [5], [8], [10], [17]), have not been addressed for supply components in the framework of inventory control and supply chains optimization simultaneously with dynamic planned safety stocks in DWNs. In addition, inventory management for supply chains literature, even in multi-stage multi-echelon schemes, supposes a hierarchical and descendant flow of products, in a way that predicted safety stock changes are easily communicated backwards in order to support availability of quantities when they are needed [7], but this behaviour is not true in real large-scale supply networks (i.e., the Barcelona DWN), because a meshed topology with multi-directional flows between tanks and nodes prevails instead of spread tree configurations.

To circumvent the aforementioned difficulties, this paper presents a hierarchical control scheme based on two optimization layers and forecast modules. The main contribution
of this work is the enhancement of an MPC controller with capabilities to dynamically allocate the minimal safety volume in each storage unit to satisfy customer demand in the presence of uncertainties and operational constraints, avoiding stock-outs and distributing control efforts within actuators to extend their useful life and improve overall system service reliability.

The paper is organized as follows: Section II briefly describes a control-oriented model of DWNs. Section III concerns system service reliability. Section IV describes the case study where the effectiveness of the proposed approach is analysed via simulations. Section V highlights the concluding remarks that can be drawn from the results presented in this paper, as well as some ideas for future research.

II. DRINKING WATER NETWORK MANAGEMENT

A. Assumptions

(i) The DWN is modelled as the interconnection of storage tanks, actuators (pumps and valves), intersection nodes, sectors of consume (demands), and sources (superficial or underground). The relation between elements is only by material flows, but there are no dynamic couplings. (ii) Each tank and node can receive and place flows from and to multiple tanks/nodes and are modelled by a single first-principle mass balance equation. (iii) Each tank may face variability in the downstream demands or in the upstream lead times. (iv) Every source can supply its underlying demand. (v) Demands follow a periodic behaviour and are measured in an hourly base. (vi) Demands and lead times behave non-stationary. (vii) Each path between elements has a cost and transportation time. (viii) Network managers can estimate demands $N$ time instants ahead. (ix) All elements operate with a common review period $\Delta t$ and storage tanks are subject to the same replenishment MPC policy. (x) There is no delay in ordering.

B. Discrete-Time Dynamic Model

The control-oriented model of a water transport system is a simplified but representative model of the non-linear dynamic behaviour, which allows predicting the effect of control actions on the entire network. Modelling principles of DWNs have been reported in the literature, see [3], [11]. Considering the aforementioned references, the model of a DWN in discrete-time state space may be written as

\[
\begin{align*}
x_{k+1} &= Ax_k + Bu_k + B_p d_k, \\
E_1 u_k + E_2 d_k &= 0, \\
x_{\min} \leq x_k \leq x_{\max}, \\
u_{\min} \leq u_k \leq u_{\max},
\end{align*}
\]

where $x \in \mathbb{R}^n$ is the measurable state vector of water stock levels in $m^3$ corresponding to the $n$ buffer tanks at the current time instant $k \in \mathbb{N}_0$, subject to physical constraints $x_{\min}$ and $x_{\max}$; $u \in \mathbb{R}^m$ is the vector of manipulated flows in $m^3/s$ through the $m$ actuators subject to operational constraints $u_{\min}$ and $u_{\max}$; $d \in \mathbb{R}^p$ corresponds to the vector of the $p$ water demands (sectors of consume) in $m^3/s$; $A$, $B$, $B_p$, $E_1$ and $E_2$ are state-space system matrices of suitable dimensions dictated by the network topology.

C. DWN Operational Control

The main goal of the operational control of water transport networks is to satisfy demands at consumer sectors but optimizing, at the same time, management policies expressed as a multi-objective problem. Hence, MPC is a suitable technique to control a DWN due to its capability to deal efficiently with multi-variable dynamic constrained systems and predict the proper actions to achieve the optimal performance according to a user defined cost function. Specifically, the interest is to minimize the following objectives [15]:

\[
J = \min_{\Delta u_k} \left[ \sum_{i=0}^{H_p-1} f_{1,k+i|k} + \sum_{i=0}^{H_p} f_{2,k+i|k} + \sum_{i=0}^{H_p-1} f_{3,k+i|k} \right],
\]

where $H_p$ and $H_u$ are the prediction and control horizons, respectively; index $i$ represents the predicted time along the horizons; $f_{1,k} = (\alpha_1 + \alpha_2) u_k \Delta t | W_1$ minimizes the economic cost of network operation taking into account water production cost ($\alpha_1$) and water pumping electric cost ($\alpha_2$); $f_{2,k} = \| \varepsilon_k \|_2 W_2$ is a performance index which penalizes the amount of volume $\varepsilon$ that goes down from a safety volume value; $f_{3,k} = \| \Delta u_k \|_2 W_u$ minimizes control signal variations to extend actuators life and assure a smooth operation; $W_1$, $W_2$, and $W_u$ are diagonal weighting matrices included to prioritize the objectives.

With the above information, the MPC design follows a systematic procedure, which generates the control input signals to the plant by combining a prediction model and a receding-horizon control strategy [9]. The cost function (2) subject to (1), (10) and (11) represents the desired system performance over a future horizon. Once the minimization is performed, only the first computed control action is implemented and the system operates with this constant input until the next sampling instant. Then, the optimization is solved again with new feedback measurements to compensate for unmeasured disturbances and model inaccuracies. This scheme is repeated at each future sampling period.

III. HIERARCHICAL MPC STRATEGY FOR SERVICE RELIABILITY IN DWN

A. Control System Structure

The control strategy addressed in this paper is based on a multilayer (hierarchical) control system structure enhanced with forecasting demand and actuators health estimation modules (see Fig. 1). The hierarchical architecture has been widely used in process control with satisfactory results, optimizing economic profits when disturbances are slowly varying, see [23]. In a DWN, these disturbances follow a pattern in a daily basis and can be well predicted for an hourly sampling time, which makes the hierarchical structure suitable to optimize targets for the policies of the operation level.
The proposed control system is a two-layer structure combining an Economic Optimization Layer (EOL) and an Optimal Feedback-Control Layer (OFCL). The EOL is a strategic layer that deals with the adjustment of targets, bounds and weights for the control problem taking into account economic cost functions. The OFCL is a tactical layer which executes a dynamic optimization within an MPC algorithm to translate strategic policies into desired control actions for the operational control level by solving (2).

B. Safety Stocks Allocation Policy

There is the need to guarantee a safety water stock in each tank of the network in order to decrease the probability of shortages (when a tank or a node has insufficient water to satisfy external demands or the transfer request coming from other tank/node) due uncertain events. To determine the amount of safety water stocks, an inventory planning strategy is addressed here to enrich previous control approaches (see [11], [15]), with replenishment policies.

The goal is to dynamically allocate the minimal volume \( x_s \) in each tank to avoid stock-outs. To do so, the EOL first estimates future flows for a short-term prediction horizon, as follows:

\[
\hat{\mathbf{u}}_{s,k} = \arg \min_{\mathbf{u}} \sum_{i=0}^{N-1} f_{1,k+i|k},
\]

subject to

\[
\begin{align*}
\mathbf{x}_{k+i+1|k} &= \mathbf{A}\mathbf{x}_{k+i|k} + \mathbf{B}\mathbf{u}_{k+i|k} + \mathbf{B}_p\mathbf{d}_{k+i|k}, \\
\mathbf{E}_1\mathbf{u}_{k+i|k} + \mathbf{E}_2\mathbf{d}_{k+i|k} &= \mathbf{0}, \\
\mathbf{x}_{\min} \leq \mathbf{x}_{k+i|k} & \leq \mathbf{x}_{\max}, \\
\mathbf{u}_{\min} \leq \mathbf{u}_{k+i|k} & \leq \mathbf{u}_{\max},
\end{align*}
\]

where \( \hat{\mathbf{u}}_{s,k} \equiv [\mathbf{u}_{s,k|k}^T, \ldots, \mathbf{u}_{s,k+N-1|k}^T]^T \).

The previous linear programming problem (LPP) uses the same model structure as in (1) but including a stochastic forecasting demand. The resultant sequence of estimated flows \( \hat{\mathbf{u}}_{s,k} \) allows to virtually decouple tanks interconnections and estimate their net demand \( \mathbf{d}_{\text{net}} \), and mean forecasted demand \( \mathbf{d}_{\text{avg}} \), for a short horizon \( N \in \mathbb{N}_+ \). Then, it is possible to compute \( n \) individual safety volumes \( (x_{s,k})_{j=1:n} \) based on forecasting error deviation and a given service level, as follows:

\[
\begin{align*}
\mathbf{d}_{\text{net},k+i} &= \mathbf{B}_{\text{out}}\mathbf{u}_{s,k+i} + \mathbf{B}_p\mathbf{d}_{k+i}, \quad \forall i \in \mathbb{N}_0^{N-1}, \\
\mathbf{d}_{\text{avg},k} &= \sum_{i=0}^{N-1} \mathbf{d}_{\text{net},k+i}/N-1, \\
\sigma_{j,k} &= \sqrt{\langle \sigma_{d,k}^2 \rangle_j} + (\langle \sigma_{\tau,k}^2 \rangle_j)(\hat{\mathbf{d}}_{\text{avg},k}), \\
\sigma_k &= [\sigma_{1,k}, \ldots, \sigma_{n,k}]^T, \\
s_k &= \Phi^{-1}(\gamma)\sigma_k, \\
\mathbf{x}_{s,k} &= (\tau_k)\mathbf{d}_{\text{avg},k} + (s_k), \quad \forall j \in \mathbb{J}_1^n
\end{align*}
\]

where \( \mathbf{B}_{\text{out}}\mathbf{u}_{s,k+i} \) is the outflow of the tanks caused by water requirements from neighbouring tanks or nodes, and \( \mathbf{B}_p\mathbf{d}_{k+i} \) is the estimated exogenous demand for a given time instant; \( \sigma_k \in \mathbb{R}^n \) is the vector of total forecast deviations, where for each \( j \)-th tank at every time instant \( k \), the deviation is given by \( \sigma_{j,k} \in \mathbb{R} \). This individual total deviation, takes into account the sample standard deviation \( \sigma_{d,k} \in \mathbb{R}^n \) of the individual net demand forecast error, and the sample standard deviation \( \sigma_{\tau,k} \in \mathbb{R}^n \) of the individual lead-time error. Moreover, \( s_k \in \mathbb{R}^n \) is the vector of safety stocks in \( \mathbb{J}^3 \); \( \Phi^{-1}(\gamma) \) is the inverse cumulative normal distribution; \( \gamma \in (0, 100\%) \) is the desired customer service level (percentage of customers that do not experience a stock-out); and \( (x_{s,k})_{j} \) is the individual safety stock for each \( j \)-th tank.

This way, the vector of water base-stocks defined as \( \mathbf{x}_{s,k} \equiv [(x_{s,k})_1, \ldots, (x_{s,k})_n]^T \in \mathbb{R}^n \), is introduced in the MPC design of Section II-C as a soft constraint to lead the tank volumes to be greater than such stocks (when possible) and let the system employ safety volumes \( s \) to face uncertainties (when needed) but penalizing the used amount of safety. This soft constraint is expressed as follows:

\[
\mathbf{x}_k \geq \mathbf{x}_{s,k} - \mathbf{e}_k \geq \mathbf{0}.
\]
which relies on actuators availability to guarantee a customer service level.

C. Actuators Health Degradation Policy

Unless some damage mitigating policy is adopted to ensure the availability of actuators for a maintenance horizon, the inherent degradation of them could compromise the overall service reliability. Therefore, system safety can be enhanced by taking into account the health of the components in the controller design.

Several models have been proposed in literature to describe reliability and actuators ageing under nominal operation (see, e.g., [5], [10]). In control systems, where conditions usually change, it is also required to include the impact of the exerted control effort into the model [8]. For the sake of simplicity, the linear proportional ware model presented in [17] and its uniform rationing heuristic are adopted in this paper to estimate and manage the health of the actuators, as follows:

\[ z_{k+1} = z_k + \Gamma |u_k| + \Psi |\Delta u_k|, \] (11a)

\[ z_k \leq z_{\text{thresh}}, \] (11b)

\[ z_{k+H_p} \leq z_{\text{max},k}, \] (11c)

\[ z_{\text{max},k} = z_k + H_p \frac{z_{\text{thresh}} - z_k}{k_M + H_p - k}, \] (11d)

where \( z \in \mathbb{R}^m \) is the vector of actuators health degradation, \( \Gamma = \text{diag}(\gamma_1, \gamma_2, \ldots, \gamma_m) \) and \( \Psi = \text{diag}(\psi_1, \psi_2, \ldots, \psi_m) \) are diagonal matrices of constant degradation coefficients associated with the \( m \) elements of \( u_k \) and \( \Delta u_k \), respectively; \( z_{\text{thresh}} \in \mathbb{R}^m \) and \( z_{\text{max},k} \in \mathbb{R}^m \) are vectors of safe thresholds, and \( k_M \) is a maintenance horizon over which is desired to keep actuators in reliable conditions.

This approach controls the maximum allowable degradation of actuators health by adding (11c) as a terminal constraint for the accumulated degradation in the operational control problem of Section II-C when solving (2).

A more realistic ageing process is usually represented by non-linear models. Hence, the estimation of \( z_k \) and setting of \( z_{\text{max},k} \) can be improved in the EOL using a modified 2-parameters Weibull distribution function presented in [8]. The cumulative probability function of failure rate for the \( i \)th actuator can be written as follows:

\[ F_{i,k} = 1 - \exp \left( -\left( \frac{k \Delta t \exp(rms(u_{i,0,k}))}{\alpha_i} \right)^\beta_i \right) = z_{i,k}, \] (12)

where \( \text{rms}(u_{i,0,k}) = \frac{\|u_{i,0,k}\|_2}{\sqrt{k}} \) is the root-mean-square of the historical control inputs applied during \( k \) time instants, \( \alpha_i \) is the scale parameter representing the time to take total cumulative failure under nominal conditions, and \( \beta_i < 1 \) is the shape parameter that reveals the trend of component failure in the test time.

D. Demand Forecasting

This module focuses on the problem of water demand forecasting for real-time operation of the DWN. An hourly consumption data analysis is used for training an artificial neural network multilayer perceptron (MLP) with Bayesian regulation back-propagation. The inputs to the forecasting model are chosen based on literature review [2], and correlation analysis, considering consumption historical data and meteorological variables such as temperature and air relative humidity. Principal component analysis (PCA) preprocessing is applied to the training patterns. See, [4] for more details.

IV. SIMULATION RESULTS

A. Case Study

The proposed approach has been implemented taking the DWN of Barcelona as case study, which consists of 63 tanks, 114 manipulated actuators, 88 measured demands and 17 pipes intersection nodes. Figure 2 shows the meshed topology of the network and the interconnections between elements. Simulations have been carried out using Matlab® R2011b (64 bits) with the TOMLAB® 7.6 optimization package and the Neural Network Toolbox™. The computer used to run the simulations is a PC Intel® Core™ E8600 running both cores at 3.33GHz with 8GB of RAM. The problem has been modelled and solved for a four days operation (96 hours) with a prediction horizon of one day (24 hours) and an hourly update of demands and states. Results are compared with a baseline strategy developed in [11]. The controllers are:

- **MPCo**: baseline approach of MPC with fixed prediction and control horizons (24h), constant safety water stocks and constant tuning weights for the prioritization of management objectives.
- **MPCsr**: current approach of a two-layer MPC, which implements analytically the dynamic optimization of safety stocks and takes into account actuators health. It considers fixed horizons (24h) and fixed tuning weights.

Table 1 shows the value of specific key performance indicators for the aforementioned controllers, where the economic indicator is expressed in economic units (e.u.) rather than in real values (Euro) due to confidentiality reasons.

### TABLE I
**KEY PERFORMANCE INDICATORS FOR THE DIFFERENT APPROACHES**

<table>
<thead>
<tr>
<th>Controller</th>
<th>Economic Indicator (×10^3 e.u.)</th>
<th>Safety Stocks ( (\times 10^{10} \text{ m}^3) )</th>
<th>Smoothness ( ((\text{m}^3/\text{h})^2) )</th>
<th>Time ( (\text{s}) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>MPCo</td>
<td>234.53</td>
<td>2.14</td>
<td>80.82</td>
<td>202.89</td>
</tr>
<tr>
<td>MPCsr</td>
<td>219.92</td>
<td>3.47</td>
<td>68.28</td>
<td>288.94</td>
</tr>
</tbody>
</table>

B. Service Reliability Control

It is important to notice how the control strategy proposed in this work outperforms the baseline approach, where safety stocks were fixed for all the horizon and the actuators health was not considered. Simulations show that fixed parameters such as safety stock, tuning weights and horizons are a drawback for the management of complex systems.
V. CONCLUDING REMARKS

This paper has shown, through a real case study, the effectiveness of the proposed hierarchical MPC strategy enhanced with forecasting demand, dynamic planning of safety stocks and actuators health monitoring, to assure reliability in the water supply given a customer service level and to minimize operational costs. The EOL allows to efficiently solve the non-linear problems and the tuning of strategic targets such as minimum tank volumes and maximum degradation of actuators, before the MPC algorithm executes; this simplifies the inherent optimization process by maintaining the dynamic model and constraints in the linear domain.

The core of the approach relies on the quality of the forecasting demand. An increment in the forecasting error leads to require greater amounts of safety stocks, causing a reduction of the available capacity in the tank to perform optimal excursions, which increases operational costs.

Future avenues for research in this area include: multi-period analysis with different replenishment cycle for each tank, distributed control of the network with pooling risk.
analysis and actuators ageing models enriched with the effect of maintenance quality, fault tolerant control and reachability analysis.

REFERENCES


