A Kalman Filter Approach for the Estimation of Time Dependent OD Matrices Exploiting Bluetooth Traffic Data Collection

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ABSTRACT

Time-Dependent Origin-Destination (OD) matrices are a key input to Dynamic Traffic Models. Microscopic and mesoscopic traffic simulators are relevant examples of such models, traditionally used to assist in the design and evaluation of Traffic Management and Information Systems (ATMS/ATIS). Dynamic traffic models can also be used to support real-time traffic management decisions. The typical approaches to time-dependent OD estimation have been based either on Kalman-Filtering or on bi-level mathematical programming approaches that can be considered in most cases as ad hoc heuristics. The advent of the new Information and Communication Technologies (ICT) makes available new types of traffic data with higher quality and accuracy, allowing new modeling hypotheses which lead to more computationally efficient algorithms. This paper presents *Ad hoc* procedures, based on Kalman Filtering, that explicitly exploit traffic data available from Bluetooth sensors, and reports on the numerical results of the computational experiments for a network test site.

Keywords: Time-Dependent Origin Destination Matrices, Estimation, Prediction, Kalman Filter, ICT, ATIS, ATMS

INTRODUCTION

We draw attention in this paper to a main requirement of ATIS/ATMS: the estimation of time-dependent Origin to Destination (OD) matrices from measurements of traffic variables. We assume that the usual traffic data collected by inductive loop detectors (i.e. volumes, occupancies and speeds) are complemented by accurate measurements of travel times and speeds between two consecutive sensors based on new technologies able to capture the electronic signature of specific on-board devices, such as a Bluetooth device on board a vehicle. The sensor captures the public parts of the Bluetooth or Wi-Fi signals within its coverage radius, the most relevant being the MAC address, whose uniqueness makes it possible to use a matching algorithm to log the device when it becomes visible to the sensor. A vehicle equipped with a Bluetooth device traveling along the freeway is logged and time-stamped at time t_1 by the sensor at location 1. After traveling a certain distance it is logged and time-stamped again at time t_2 by the sensor at location 2 downstream. The difference in time stamps $\tau = t_2 - t_1$ measures the travel time of the vehicle equipped with that mobile device. The speed is also measured, assuming that the distance between both locations is known. Data captured by each sensor is sent to a central server by wireless telecommunications for processing.

Raw measured data cannot be used without pre-processing aimed at filtering outliers that could bias the sample. The quality of the input data is crucial for the applications, therefore it is a subject of intense research. A variety of nonlinear filters have been proposed recently by Van Lint and Hoogendoorn [1] and Treiber et al, [2]. A refined version of the Kalman Filter (Kalman [3], proposed in Barceló et al. [4]) has been used in this paper.

This clean data is the main input to a new Kalman Filter approach for estimating time-dependent OD matrices. The proposed approach, which exploits the explicit travel time measurements from Bluetooth detectors, is based on a reformulation of the Kalman Filter approach for freeways explored in Barceló *et al*, [4]. It also extends the approach to urban networks where *alternative paths are available and route choice is relevant*. This new approach exploits the measurements of travel times in order to reduce the number of state variables as well as to simplify the model. This paper is the follow-up to the preliminary work reported in Barceló *et al*, [5] and provides conclusions based on computational experiments in an urban network test.

DYNAMIC ESTIMATION OF OD MATRICES IN FREEWAYS, CORRIDORS AND NETWORKS

The space-state formulations based on Kalman Filtering have always been an appealing approach to the estimation of time dependent OD matrices. In this paper we propose a recursive **linear** Kalman-Filter for state variable estimation that combines and modifies the earlier work of Chang and Wu [6], Hu *et al* [7], Choi *et al*, [8] and Van Der Zijpp and Hamerslag [9], adapting their models to take advantage of ***travel times and traffic counts*** collected by tracking Bluetooth equipped vehicles and conventional detection technologies.

Chang and Wu [5] proposed a model for freeways that, for each OD pair that estimates time-varying travel times, uses time dependent traffic measures and implicit traffic flow models to account for flow propagation. The state variables are the time-varying OD proportions and the fractions of OD trips that arrive at each off-ramp *m* interval after their entrance from on-ramps at interval *k*. The observation variables are main section and off-ramp counts for each interval. An Extended Kalman-filter approach is proposed to deal with the nonlinear relationship between the state variables and the observations.

Hu *et al*. [6] proposed an Extended Kalman Filtering algorithm for the estimation of dynamic OD matrices in which time-varying model parameters are included as state variables in the model formulation. The approach takes into account temporal issues of traffic dispersion. Lin and Chang [10] proposed an extension of Chang and Wu [6] in order to deal with traffic dynamics, assuming that travel time information is available.

Work et al [11] proposed the use of an Ensemble Kalman Filtering approach as a data assimilation algorithm for a highway velocity model based on traffic data from active GPS mobile devices. Dixon et al. [12] used AVI data through the incorporation of previously observed OD information as measurements as well as link counts, which improved estimator accuracy in respect to an OD estimation based exclusively on link counts in a KF approach

Zhou and Mahmassani [13] proposed a nonlinear ordinary least-squares estimation model for dynamic OD estimation to extract valuable point-to-point split-fraction information from automatic vehicle identification (AVI) counts without estimating market-penetration rates and identification rates of AVI tags.

We assume flow counting detectors and ICT sensors located in a cordon and at each possible point for flow entry (centroids of the study area). ICT sensors are located at intersections in urban networks and cover access and links to/from the intersection. Flows and travel times are available from ICT sensors for any selected time interval length higher than 1 second. ***Trip travel times*** from origin entry points to sensor locations are measures provided by the detection layout. Therefore, they are no longer state variables but measurements, which simplify the model and makes it more reliable.

A basic hypothesis is that equipped and non-equipped vehicles follow common OD patterns. We assume that this holds true in what follows and that it requires a statistical contrast for practical applications. Expansion factors for everything from equipped vehicles to total vehicles, in a given interval, can be estimated by using the inverse of the proportion of ICT counts to total counts at centroids; expansion factors are assumed to be shared by all OD paths and pairs with a common origin centroid and initial interval.

We propose a linear formulation of the Kalman Filtering approach that uses deviations of OD path flows as state variables, as suggested by Ashok and Ben-Akiva [14,15], calculated in respect to DUE-based Historic OD path flows for equipped vehicles. But our approach differs in that we do **not require an assignment matrix**. We use instead the subset of the most likely OD path flows identified from a DUE assignment with Dynameq [16]. The DUE is conducted with the historic OD flows, and the number of paths to take into account is a design parameter (a maximum of 10 paths per OD pair is currently considered, those being the most important according to path flows across time-slices). DUE OD path proportions for selected paths and time-slices are not an input to the KF approach, only the description of the most likely OD paths. A list of paths going through the sensor is automatically built for each ICT sensor from the OD path description, ICT sensor location and the network topology. In this way, once an equipped car is detected by ICT sensor j, the travel time from its entry point to sensor j is available and it is used for updating *time varying model parameters* that affect OD paths (state variables) which are included in the list.

We model the *time-varying dependencies* between measurements (sensor counts of equipped vehicles) and state variables (deviates of equipped OD path flows), adapting an idea of Lin and Chang [10], for estimating discrete approximations to travel time distributions. The estimation of these distributions is made on the basis of flow models which induce nonlinear relationships that require extra state variables, leading to a non linear KF approach. Since our approach exploits the travel ICT time measurements from equipped vehicles, we can replace the nonlinear approximations by *estimates from a sample of vehicles.* This has advantages that constitute a major contribution of this paper:

- No extra state variables for modeling travel times and traffic dynamics are needed, since sampled travel times are used to estimate discrete travel time distribution (H bins are used for adaptive approximations) and non linear relationships involved in flow propagation can be avoided, making the Extended Kalman approach unnecessary.
- Travel times collected from ICT sensors are incorporated into the proposed model and it is not necessary that vehicles reach their destination, since at any intermediate sensor that they pass by the travel time measured from the entry point (centroid) to that sensor updates the discrete travel time approximation. Therefore, completed trips are not the only source for updating time-varying model parameters. No information about trajectories of equipped vehicles is used in this version. The use of sequential trajectory data from a given vehicle to define/identify paths and path travel time will be incorporated in the future.

In accordance with our experience, we agree with Antoniou *et al*. [16, 17] that formulations using deviates provide benefits in respect to those using OD path flows as state variables, because they incorporate more historical data as *a priori* structural information for the model.

The demand matrix for the period of study is divided into several time-slices, accounting for different proportions of the total number of trips in the time horizon.

The approach assumes an extended state variable for M+1 sequential time intervals of equal length Δt, M is the maximum number of time intervals required for vehicles to traverse the entire network in a congested scenario.

The solution provides estimations of the OD matrices for each time interval up to the *k*-th interval. State variables $\Delta g_{ijc}(k)$ are deviations of OD path flows $g_{ijc}(k)$ relative to historic OD path flows $\tilde{g}_{ijc}(k)$ for equipped vehicles. If historical OD matrices are not available, then the formulation reduces to the case where state variables are OD path flows. Total number of OD path flows $G_{ijc}(k)$ for all vehicles (equipped or not) are computed according to expansion factors $Q_i(k)/q_i(k)$ for each entry point and interval. OD path proportions are a post-processing result computed from the OD path flow deviations for equipped vehicles (state variables).

The total number of origin centroids is I, identified by index *i*, *i* = 1,..,I; the total number of destination centroids J, identified by index *j*, *j* = 1,…,J; the total number of ICT sensors is Q, identified by index *q*, *q* = 1,…Q, where Q = I+J+P, I ICT sensors located at origins, J, ICT sensors at destinations and P, ICT sensors located in the inner network; and the total number of *most likely* used paths between origins and destinations is K. The notation used in this paper is the following:

$\tilde{Q}_i(k)$, $\tilde{q}_i(k)$	: Historic total number of vehicles and equipped vehicles entering from centroid i at time interval k
$Q_i(k)$, $q_i(k)$	: Total number of vehicles and equipped vehicles entering from centroid i at time interval k.
$\tilde{y}_q(k)$, $y_q(k)$	: Historic and actual number of equipped vehicles crossing sensor q at time interval k
$G_{ijc}(k)$, $\tilde{G}_{ijc}(k)$, $g_{ijc}(k)$, $\tilde{g}_{ijc}(k)$	: Total number of current $G_{ijc}(k)$ and historic $\tilde{G}_{ijc}(k)$ vehicles as well as current $g_{ijc}(k)$ and historic $\tilde{g}_{ijc}(k)$ equipped vehicles entering the network from centroid i at time interval k headed towards destination j using path c.
$\Delta g_{ijc}(k)$	: ***State variables*** are deviates of equipped vehicles entering from centroid i during interval k headed towards centroid j using path c with respect to average historic flows $\Delta g_{ijc}(k) = g_{ijc}(k) - \tilde{g}_{ijc}$.
$\mathrm{z}(\mathrm{k})$, $\tilde{\mathrm{z}}(\mathrm{k})$	: The *current and average historic measurements* during interval k, a column vector of dimension J+P+I, whose structure is $z(k)^T = (y(k) \quad q(k))^T$
IJ, IJK	: Number of feasible OD pairs and *most likely* OD paths depending on the zoning system defined in the network.
$u_{iq}^h(k)$	: Fraction of vehicles that require h time intervals to reach sensor q at time interval k that entered the system from centroid i (during time interval $[(k-h-1)\Delta t \quad , \quad (k-h)\Delta t]$).
$u_{ijcq}^h(k)$	: Fraction of equipped vehicles detected at interval k whose trip from centroid i to sensor q might use OD path (i,j,c) lasting h time intervals of length Δt to arrive from centroid i to sensor q, where i = 1,…,I , j = 1,…, J , $h = 1 \ldots M$, $q = 1 \ldots Q$
$\bar{t}_{iq}(k)$	: Average measured travel time for equipped vehicles entering at centroid i and crossing sensor q during interval k

The model also assumes that:

- For equipped vehicles, the entry points to the network and the time entering the network is known and thus $q_i(k)$ is also known. This accounts for trips traversing the scenario, trips starting outside and ending inside the scenario, starting inside and ending outside, or starting and ending at inner centroids. In these cases they are allocated to the centroid representing the zone where its associated ICT device has first been detected and counts have to be expanded by a factor that depends on the penetration of the ICT applications unless other counts were available.
- Conservation equations from entry points (centroids) are explicitly considered.
- Without $Q_i(k)$, a generic expansion factor has to be applied to $g_{ijc}(k)$ to get $G_{ijc}(k)$.

Let $\Delta \mathrm{g}(\mathrm{k})$ be a column vector of dimension IJK containing the state variables $\Delta g_{ijc}(k)$ for each time interval k for all *most likely* OD paths (i,j,c). The state variables $\Delta g_{ijc}(k)$ are assumed to be stochastic in nature, and OD path flow deviates at current time k are related to the OD path flow deviates of previous time intervals by an autoregressive model of order r <<M; the state equations are:

$$\Delta \mathrm{g}(\mathrm{k}+1) = \sum_{w=1}^{r} \mathrm{D}(\mathrm{w}) \Delta \mathrm{g}(\mathrm{k}-\mathrm{w}+1) + \mathrm{w}(\mathrm{k}) \qquad \textbf{(1)}$$

Where the $w_{ijc}(k)$'s are zero mean and covariance matrix $\mathbf{W_k}$, and $\mathbf{D}(\mathbf{w})$ are IJKxIJK transition matrices which describe the effects of previous OD path flow deviates $\Delta g_{ijc}(k-w+1)$ on current flows $\Delta g_{ijc}(k+1)$ for $w = 1,\ldots,r$. In this paper we assume simple *random walks to provide the most flexible framework for state variables, since no convergence problems are detected*. Thus r=1 and matrix $\mathbf{D}(\mathbf{w})$ is the identity matrix.

Two structures for the $\mathbf{W_k}$ varianc**e**-covariance matrix of state variables have been considered: a multinomial structure and a diagonal. Multinomial at OD pair level means that for a given time interval k and OD pair, the covariance of the OD path flow behaves as counts in multinomial variables. When an OD path proportion increases, then the rest of the OD path flows (and thus proportions) for that OD pair decreases; i.e. OD path flows belonging to the same OD pair are related. Diagonal $\mathbf{W_k}$ var-cov structures, the most used in the reported applications in literature, means that state variables are considered statistically independent.

The structural constraints to be satisfied by the state variables are:

$$\begin{aligned} &\Delta g_{ijc}(k) \geq -\tilde{g}_{ijc}(k) \qquad i=1...I, \quad j=1....J \quad c=1....K_{ij}^{\max} \\ &q_i(k)=\sum_{j=1}^{J}\sum_{c=1}^{K_{ij}^{\max}} g_{ijc}(k) \rightarrow \sum_{j=1}^{J}\sum_{c=1}^{K_{ij}^{\max}} \Delta g_{ijc}(k) = q_i(k)-\tilde{q}_i(k) \qquad i=1...I \end{aligned} \tag{2}$$

Equality constraints (2) are explicitly considered as observation equations through the definition of dummy sums at centroids; measurement errors are allowed. The observation equations are counts at sensors of the number of equipped vehicles entering the network for each interval *k*. The relationship between the state variables and the observations involves time-varying model parameters (congestion–dependent, since they are updated from sample travel times provided by equipped vehicles) in a linear transformation that considers:

- The number of equipped vehicles entering from each entry centroid during time intervals k, k-1, k-M, $q_i(k)$.
- H<M *time*-varying *model parameters* in form of *fraction matrices,* $\left[U_{ijcq}^{h}(k)\right]$.

Structural constraints should also be satisfied by the *time-varying model parameters* $u_{iq}^{h}(k)$, reflecting temporal traffic dispersion and accounting for congestion, but are not explicitly considered as observation equations:

$$\begin{aligned} &u_{iq}^{h}(k) \geq 0 \qquad i=1\ldots I, \quad q=1\ldots Q, h=1\ldots H \\ &\sum_{h=1}^{H} u_{iq}^{h}(k)=1 \qquad i=1\ldots I, \quad ,q=1\ldots Q, \end{aligned} \tag{3}$$

Since time-varying model parameters are being taken into account to model congestion, they are inferred from the H adaptive fractions that approximate u_{iq}^{h}. Measures provided by ICT sensors are direct ***samples of travel times*** that allow the updating of discrete approximations of travel time distributions, making it unnecessary to incorporate models for traffic dynamics. This model simplification, due to the availability of the new ICT, is another major novelty in our proposed formulation. However, we must be aware that the final destination *j* is unknown. Time-varying model parameters u_{iq}^{h} to account for temporal traffic dispersion in affected paths u_{ijcq}^{h}, have to satisfy structural constraints, where H<M:

$$
\begin{aligned}
& u_{ijcq}^{h}(k) \geq 0 \quad i=1....I, \quad j=1....J, \quad c=1....K_{ij}^{\max}, \quad q=1,...Q, \quad h=1...H \\
& \sum_{h=1}^{H} u_{ijcq}^{h}(k)=1 \quad i=1....I, \quad j=1....J, \quad c=1....K_{ij}^{\max}, \quad q=1,...Q
\end{aligned}
\tag{4}
$$

As in Lin and Chang [10], we assume that the travel time T_{iq} from origin *i* to sensor *q* is a random variable that depends on the evolution of traffic conditions, with a probability density function $t_{iq}^{(k)}(t)$ at interval k that can be approximated by the discrete travel time distribution for vehicles reaching sensor *q* at time interval *k* from centroid *I*, *i.e.*, in terms of $u_{iq}^{h}(k)$ updated from the (assumed random) sample of on-line travel time data of equipped vehicles (see Figure 1). A rolling horizon of H bins is kept updated, thus the mass probability is always divided into H consecutive interval bins of length Δt; but the first bin proportion belongs to a duration of h_0 intervals, the second to duration of h_0+1, … and the last one to duration h_0+H-1 time intervals, all of length Δt. Initialization to h_0 corresponding to free flow travel time is set and mass probability $u_{iq}^{h_0}(0)=1$ and $u_{iq}^{h_0+h}(0)=0 \quad h=2,\ldots,H-1$. As congestion increases the approximations to travel time distributions adapt according to sampled data collected from equipped vehicles. Thus, h_0 increases and the updated bin proportions are obtained by weighting former proportions and current sampled average travel time. This is one of the modeling contributions of this paper: the substitution of hypotheses on the dynamics of traffic flows with actual measures of travel times provided by ICT sensors.

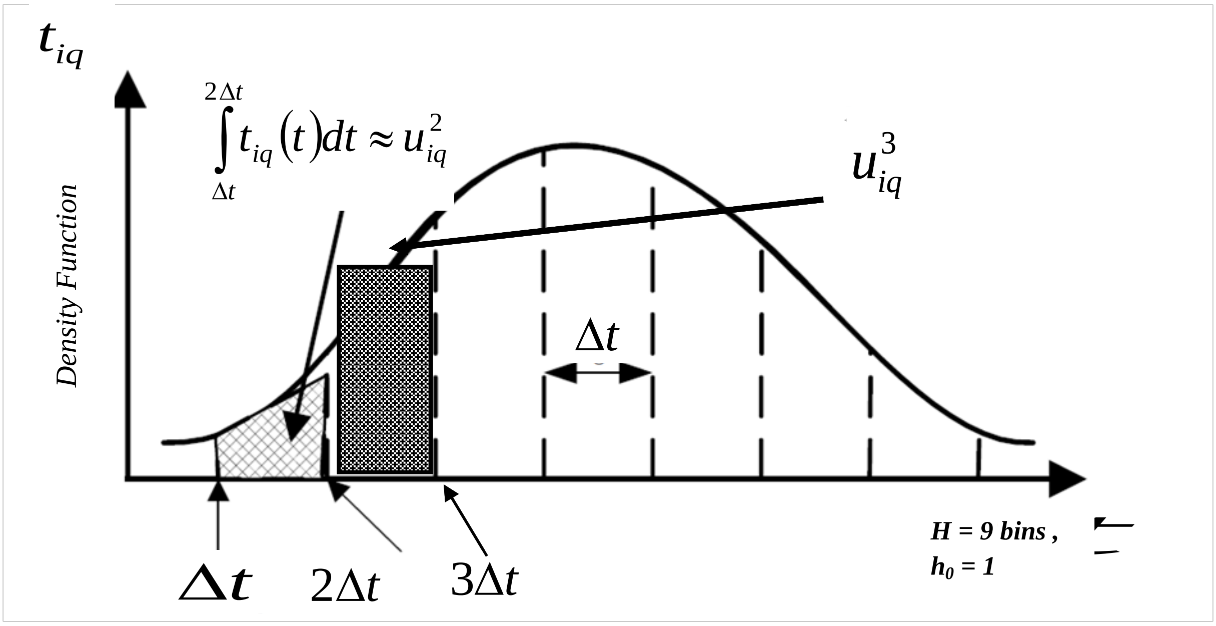


FIGURE 1. Discrete Approximation To Travel Time Distribution At Interval k.

To complete the model formulation we define the auxiliary matrices:

E(*k*)	:	Row matrix of dimension I containing 0 for columns related to state variables in time intervals $k-1,\ldots,k-M$ and defining conservation of flows (sum of OD path flows for each entry) at *k*.
U(*k*)	:	Consisting of diagonal matrices $U(k),\ldots,U(k-M)$ containing $u_{ijcq}^{h}(k)$. For $U(k-h)$ is a matrix with the estimated proportion of equipped vehicles whose

		travel time from the access point to the network takes *h* intervals and goes through the *q* sensor at interval *k*.
A	:	Matrix that adds up sensor traffic flows from any possible entry, given time-varying model parameters at interval *k*.

At time interval *k*, the values of the observations are determined by those of the state variables at time intervals *k, k-1, …k-M*.

$$\Delta z(k) = \begin{pmatrix} \mathbf{AU(k)}^{T} \\ \mathbf{E(k)} \end{pmatrix} \Delta\mathbf{g(k)} + \begin{pmatrix} v_1(k) \\ v_2(k) \end{pmatrix} = \mathbf{F(k)}\Delta\mathbf{g(k)} + \mathbf{v(k)} \qquad \mathbf{(5)}$$

Where v_i, i = 1,2 are, respectively, white Gaussian noises with covariance matrices R_i. **F(k)** maps the state vector $\Delta\mathbf{g(k)}$ onto the current blocks of measurements at time interval *k*: counts of equipped vehicles by sensors and entries at centroids, accounting for time lags and congestion effects. Deviate counts at *k* mean the observed counts minus the historical demand $\tilde{g}_{ijc}(k)$ counts, given the current traffic conditions (*time-varying model parameters*).

APPLICATION TO AMARA TEST SITE

Amara District is an urban network with 232 links, 142 nodes and 85 OD pairs, with a rich structure of alternative paths between OD pairs, totaling 358 paths according to the DUE with Dynameq [16] for the selected demand matrix. Figure 2 (a) displays a snapshot of the microsimulation model used to emulate Bluetooth data collection. Highlighted in red dots, figure 3(b) displays the detection layout of the 48 detectors.

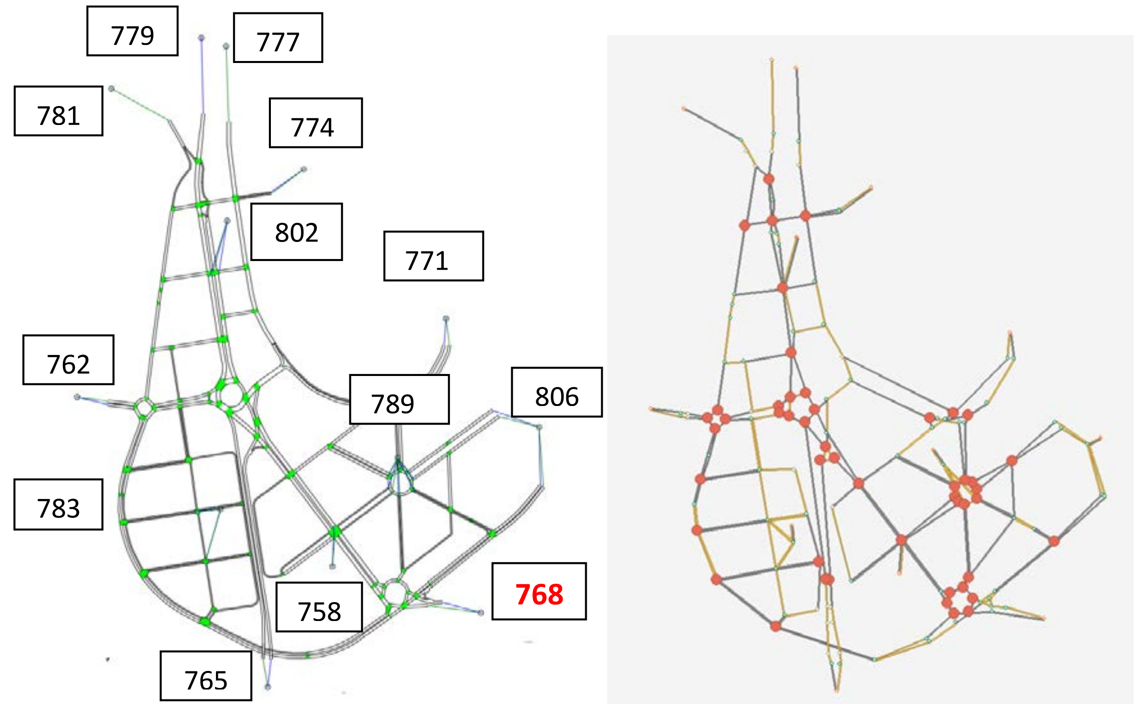


FIGURE 2: (a) Microsimulation Model: AMARA Test Site (b) Detection Layout

The detection layout raises some methodological concerns. As we have specified, it consists of two components: the cordon component encircling the network with sensors at input-output gates (currently available in most of the urban pricing systems), and the detection layout at the interior of the encircled area. However, when dealing with sensors capturing the electronic signature, such as the detectors of Bluetooth devices on board vehicles, the detector location requires a new approach based on a node covering formulation of the detection layout problem that has been developed and tested in Barceló *et al.* [19]. This has been the method of determining Amara's layout, depicted in Figure 2(b).

Once a vehicle is generated in the simulation model, it is randomly identified as an equipped vehicle according to the design proportions of technological penetration in computational tests. The simulation emulates the logging and time stamping of this random sample of equipped vehicles. In the preliminary computational experiments conducted to test the KF approach, we focused our attention on the quality of the results and not on computational efficiency. A prototype has been implemented in MATLAB (Identified as KFX in what follows) and tested computationally. The length of the time interval has been set at 90 seconds and the simulation period to 1 hour.

DESIGN OF COMPUTATIONAL EXPERIMENTS

The simulation experiments for testing the proposed approach have been conducted in the Amara network using a dynamic OD matrix with four time slices as input, corresponding to a rise in congested conditions. This OD, considered the true historical OD matrix, has been determined through simulation by building the Macro Fundamental Diagram (MFD) for the network (Daganzo and Geroliminis [20]). Therefore, congestion is not a design factor in these experiments. This OD is sliced into four 15-minute slices --each one accounting for 15%, 25%, 35% and 25% of the total number of trips-- to emulate demand variability. The simulation experiments with Aimsun use the described OD matrix, emulating traffic detection, and provide the inputs to the KFX model in terms of flow counts for ordinary and ICT sensors and travel times from entry cordon i to q ICT sensor.

Figure 3 depicts the methodological framework for the simulation experiments. The assumed OD matrix is the result of applying some perturbation to the true historical matrix.

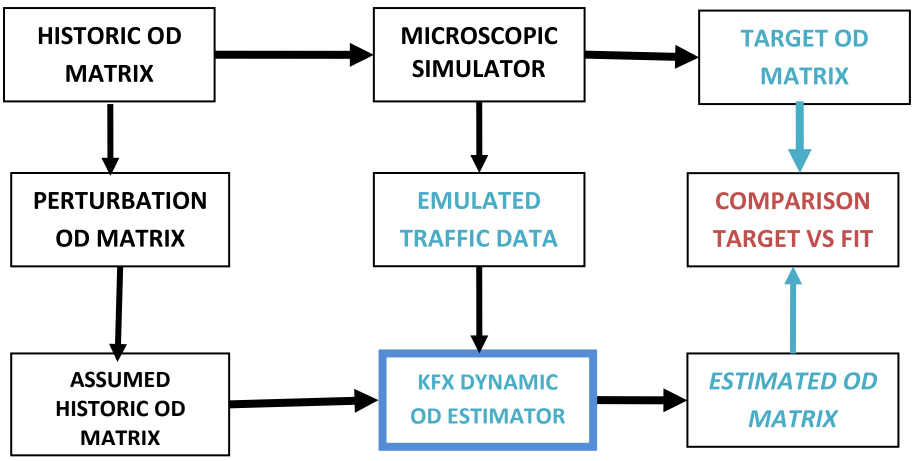


Figure 3. Methodological Design Of The Computational Experiment By Simulation

Tuning parameters (not considered as design factors) are:

- *Deviate tuning parameter (v_1)*: scalar value affecting the historic OD flows for generating the *assumed historic* OD flows for deviate setting. Its value is set to 1 if the historic OD flows are reliable, otherwise increase/decrease.

- *Interval length Δt.*

Design factors considered in computational experiments are:

Factor 1. The initialization of the state variables $\Delta g_{ijc}(0)$ is set to 0. That is, OD path flows are equal to the *assumed historical OD path flows for all time intervals*, and the OD paths are the most likely used paths from a DUE assignment. The levels of Factor 1 depend on the generation of the assumed historical OD according to whether:

a) It is generated by a perturbation of the historical OD by the υ_1 tuning parameter and the OD pattern is preserved. The path proportions for each OD pair are taken as constant and equal.
b) It is generated by a perturbation of the historical OD by the υ_1 tuning parameter but the OD pattern is not preserved. A subset C of origin centroids is selected in terms of the total demand, partitioning the set $\Im$ of OD pairs as $\Im=\Im_C \cup \{\Im \backslash \Im_C\}$, where $\Im_C$ is the set of OD pairs associated with C. Path proportions for OD pairs in $\{\Im \backslash \Im_C\}$ follow the original OD pattern as in (a), while path proportions for OD pairs in $\Im_C$ are those in (a) multiplied by a factor that depends on $|\Im_{D|i}|$, the number of destinations that can be reached from origin $i \in C$. The level of perturbation is determined by $|C|=\{0,1,2,3....I\}$, where $C=\emptyset$ corresponds to case (a).

Factor 2. The variable percentage of ICT equipped vehicles, with levels of 20%, 30%, 50% and 100%.

Factor 3. Diagonal and Multinomial structures for var-cov matrices of state variables. Only the diagonal structure applies for var-cov measures.

Factor 4. Number H of bins used for the adaptive discrete approximations of travel time distributions (time-varying model parameters). H=3 and 5 have been tested.

Factor 5. The dimension IJKx(M+1) of the extended state vector. The values M=5, 10 have been considered.

Factor 6. Tolerance to conservation of entry flows at origins has been explicitly considered as measure equations with near-zero variance if tolerance is set to 0.

Critical factors have proved to be Factors 1, 2 and 3; mainly the first two.

Collected performance indicators

Target OD flows per interval are compared with estimated OD flows (filtered OD flows) per interval at OD pair level by means of Theil's coefficient. Theil's inequality coefficient is a measure on how close two time series are; bounded between 0 and 1, U=0 can be interpreted as a perfect fitting between the two series, while U=1 represents an unacceptable discrepancy. Values of U>0.2 recommend rejecting the fit. Theil's coefficient (U) can be computed for each OD pair or for a subset of OD pairs (i.e., defined by the quantiles of hourly historical OD flows). If all feasible OD pairs are considered, then a global Theil measure of fit, GU, is computed as:

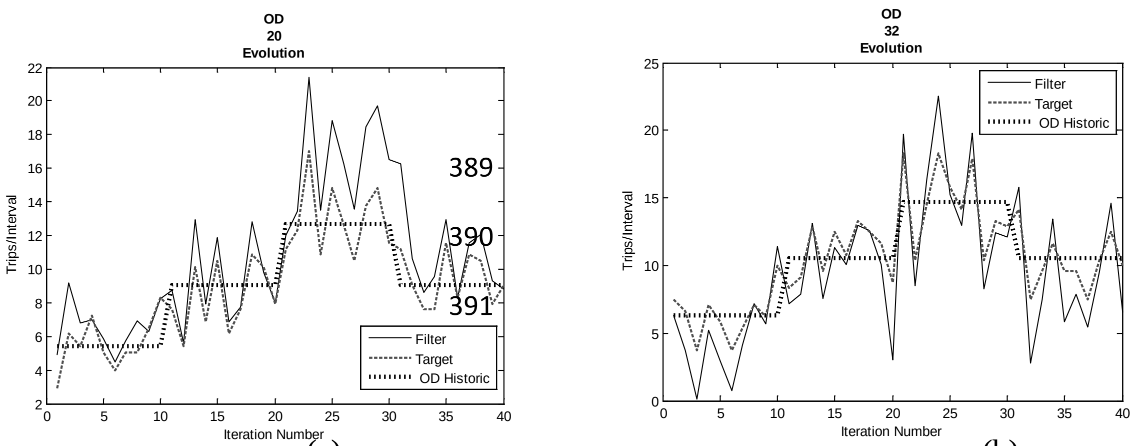
$$GU = \frac{\sqrt{\frac{1}{IJ \cdot G}\sum_{k=1:G}\sum_{od=1:IJ}\left(y_{od,k} - \hat{y}_{od,k}\right)^2}}{\sqrt{\frac{1}{IJ \cdot G}\sum_{k=1:G}\sum_{od=1:IJ}\hat{y}_{od,g}{}^2} + \sqrt{\sum_{k=1:G}\sum_{od=1:IJ}y_{od,k}{}^2}} \qquad (6)$$

Other performance indicators used as an OD pair or set of OD pair levels are the normalized root mean square error (RMSEN), a weighted indicator for subsets of OD pairs (usually subset of OD pairs whose hourly flow is in 25% of higher flows) and a weighted global indicator for the whole set of OD pairs (GRMSEN, sum of squared differences between target and estimated path flows per interval, relative to total target flows during the simulation horizon):

$$RMSEN = \frac{\sqrt{G\sum\left(y_k - \hat{y}_k\right)^2}}{\sum y_k} \qquad GRMSEN = \frac{\sqrt{IJ \cdot G \sum_{k=1:G} \sum_{od=1:IJ}\left(y_{od,k} - \hat{y}_{od,k}\right)^2}}{\sum_{k=1:G}\sum_{od=1:IJ} y_{od,k}} \quad (7)$$

Computational Results for AMARA TEST SITE

Microscopic simulation with AIMSUN induces variability in the historical inputs in a realistic way and produces target OD flows per interval (related to the true historical OD matrix). Figure 4 visualizes the results for the two most relevant OD flows when the *base initialization* is selected for Factor 1, i.e., the assumed historical OD matrix is reliable. The concordance is numerically quantified in terms of RMSEN and U for OD flows from origins 765 and 768 to all destinations in the network in Table 1. The low values of RMSEN and U prove good behavior of KFX for OD pairs with large demand (those in the 4th quartile of OD flows).



(a) (b)

FIGURE 4 (100% Equipped Vehicles): Filtered OD Flows, Target OD Flows And Historic OD Flows For OD-Pair 20.765-777 (a) and OD-pair 32.768-783 (b).

Theil's coefficients are 0.11 and 0.10, respectively, for the selected OD pairs, and GU is 0.15. Small OD flows show worse behavior, due to few observations per interval and, thus, design parameter Δt should be increased to properly deal with the sparse reality. Although not shown in Table 1, we studied the effect of decreasing the penetration rates of BT technology (Factor 2) in the selected OD pairs, showing an increase in indicators (U, RMSEN) and, thus, a decrease in the quality of the results as equipped rate decreases (after controlling Factor 1 and 3 to 6). So, the decrease in BT penetration means fewer observations and larger Δt needed for practical purposes.

The aggregated hourly fit between estimated KFX versus Target OD flows shows a regression R squared higher to 90%. In 100% of equipped vehicles, the results show that the joint effects of factors 4, 5 and 6 are irrelevant: more than H=3 bins (Factor 4) are not necessary. Results do not improve substantially if the number of time intervals M to define the *extended state vector* is increased from 5 to M=10 (Factor 5) intervals, since M=5 accounts for more than 90% of the OD travel times and dimensions of the state vector is divided by 2. There are some OD pairs with low demand in Table 1 that exhibit a complete lack of convergence according to Theil's coefficient U, since few observations per interval are present (OD pairs in the 1st and 2nd quantile, according to flow magnitudes).

		OD pairs from selected centroids to all – 4 Time Slices											
		758	**762**	**765**	**768**	**771**	**774**	**777**	**781**	**783**	**789**	**802**	**806**
765	**Average Historic OD (veh/90s)**	0.30	1.00	0.00	4.83	0.00	0.13	**8.98**	0.05	0.18	1.13	0.53	7.73
	RMSEN	0.83	0.56	-	0.52	-	2.28	**0.26**	0.56	0.47	0.20	0.45	0.22
	U	0.41	0.34		0.31		0.56	**0.11**	0.36	0.27	0.10	0.20	0.09
768	**Average Historic OD (veh/90s)**	1.73	0.33	4.83	0.00	0.70	0.13	0.58	0.00	**10.20**	5.30	0.48	0.28
	RMSEN	0.32	0.80	0.34	-	0.41	3.29	0.47	-	**0.21**	0.50	1.15	0.22
	U	0.14	0.50	0.17		0.23	0.69	0.25		**0.10**	0.20	0.72	0.10

TABLE 1. OD Pairs From Selected Centroids To All – 4 Time Slices For The *Base Initialization* And 100% Of Equipped Vehicles

Since OD path flows are not restricted to being positive, negative OD path flows might occur for low demand OD pairs. However, in the aggregation process of obtaining OD pair flows, no negative values have been encountered. Considering the 4th quantile OD flows, we tested how the *assumed Historical OD path flows* affect KFX convergence in 100% of equipped vehicles (Factor 2) and a diagonal structure for var-cov matrices (Factor 3) (see Table 2). For practical purposes, Factor 1 is critical. When controlling Factor 2 to 100% equipped (not realistic), the historical matrix reliability does not seriously affect the global results (GU and GRMSEN): even in the most uninformative assumed historical OD flows, GRMSEN stays around 0.3, considering 25% of the most important OD pairs, and is reduced to 0.16 and 0.2 for pairs 20 and 32 (the largest ones). In terms of serial matching for the overall 4^{th} quantile OD flows, the effect of a bad input is minor: GU never increases to 0.20 but, for the most important pairs 20 and 32, U values range from 0.07 to 0.17 (excellent coefficients). The behavior is very clear: OD pairs with large flows converge to correct values but, when sparse observations are available, no correction for *a priori* assumed flows is possible and, thus, no convergence is achieved (for OD pairs in the 1^{st} and 2^{nd} quantile of OD flows, GU for them ranges from 0.28 to 0.74). Thus, OD pairs with large volumes are not seriously affected by the quality of the *historical matrix*, which exhibits good convergence properties that are needed for practical applications. In Table 2, in the columns related to 100% BT penetration, aggregated results for the whole set of OD pairs and for 25% of the most important OD pairs are detailed. Comparison are based on GRMSEN, GU and R^2 indicators. Thus, even with the worst input, the overall GU is 0.42 and GRMSEN is 502%. But 25% of the larger OD pairs exhibit values of 0.16 and 31%, respectively.

Factor 1 – Non-informative OD entry pattern	Factor 1 –Quality of assumed historic OD flows vs Factor 2 -- Percentage of ICT equipped vehicles. **OD pairs with large volumes; i.e. 4th Quantile OD flows in parenthesis**											
	20%			**30%**			**50%**			**100%**		
	GU	**GRMSEN**	**R^2**	**GU**	**GRMSEN**	**R^2**	**GU**	**GRMSEN**	**R^2**	**GU**	**GRMSEN**	**R^2**
0-none	0.34 (0.23)	1.04 (0.53)	88.9%	0.31 (0.19)	0.88 (0.43)	92%	0.30 (0.17)	0.74 (0.37)	93.4%	0.27 (0.15)	0.63 (0.31)	93.9%
1	0.36 (0.23)	1.44 (0.52)	87.2%	0.34 (0.19)	1.29 (0.42)	91.1%	0.32 (0.17)	1.21 (0.36)	92.7%	0.25 (0.15)	0.93 (0.30)	92.9%
2	0.36 (0.23)	1.47 (0.51)	87.2%	0.34 (0.19)	1.31 (0.42)	91%	0.32 (0.17)	1.20 (0.36)	92.5%	0.26 (0.15)	0.95 (0.30)	92.9%
3	0.38 (0.23)	2.17 (0.51)	84.9%	0.37 (0.19)	2.04 (0.42)	88.9%	0.35 (0.17)	1.94 (0.36)	90.5%	0.28 (0.15)	1.68 (0.31)	90.7%
all	0.45	5.21	76.9%	0.43	4.97	81.8%	0.42	4.99	82.9%	0.42	5.02	81.7%

	(0.24)	(0.53)			(0.20)	(0.42)			(0.18)	(0.37)			(0.16)	(0.31)

TABLE 2. GU, GRMSEN, R^2 According To Design Factors 1 And 2 (Quality Of The Assumed Historical OD Matrix And Percentage Of ICT Equipped Vehicles)

When facing Factor 2, i.e., BT penetration rates, the experiments show that when the proportion of equipped vehicles decreases, *fewer observations* of vehicles through sensors are available for feeding KFX and global indicators GU, GRMSEN and R^2 increase. Therefore, the approach to dynamic estimation of OD matrices gets worse. This behavior is shown in Table 2 by the effect of Factor 2 (percentage of ICT equipped vehicles). In the case of a reliable OD matrix (Factor 1 – level 0), for 25% of the most important OD pairs, GU increases from 0.15 (100% penetration) to 0.23 (20% penetration) when all other design factors are kept constant. But in the case of a non-reliable assumed matrix (Factor 1 level -all, shown in the last row of Table 2), GU increases from 0.16 (100% penetration) to 0.24 (20% penetration) for the 25% most important OD pairs. This is nearly non-sensible in terms of bad quality regarding the assumed OD matrix. But for the global set of OD pairs, GU ranges from 0.42-0.45, regardless of the percentage of equipped vehicles (again, shown in the last row of Table 2). Meanwhile, within a good assumed matrix (first row Table 2), the global GU for the whole set of OD pairs is 0.27 in 100% of equipped vehicles, but 0.34 in under 20% of equipped vehicles. So, important OD pairs are affected by the percentage of equipped vehicles much more than by the quality of the assumed matrix.

The applicability to a real test-site is clear: since the length of the time interval is critical, it should be the minimum length of time that is able to capture the actual percentage of equipped vehicles in the most important OD pair relations. This happens when non-null registers of observations for equipped vehicles per interval are available. The trade-off interval length and equipped counts has to be carefully addressed.

A few comments on Factor 3: the structure of the variance-covariance matrix for the extended state variables (OD path flows in M+1 time intervals) provides a very different behavior in KFX performance. KFX tuning parameters are highly dependent on variance-covariance structure. In the absence of reliable OD historical patterns, a diagonal var-cov proposal is recommended, since it adapts more easily and obtains faster convergence. But performance indicators are globally more stable across Factor 2 levels, when a multinomial structure is selected. Further work is needed on Factor 3.

CONCLUSIONS

The computational experiments show that the proposed linear KF approach to dynamic OD matrix estimation provides good estimates of target values in the synthetic simulation tests in network and freeway sites. ICT data simplifies the dynamic estimation of OD matrices by a Kalman-Filtering approach because it is a linear filter and reduces the computational burden when compared to well-known formulations in the literature that use Extended Kalman Filter. The strategy of collecting the DUE *most likely used paths* --and, thus, defining KFX state variables-- seems promising, since time-dependent path proportion shares and assignment matrices are not employed in the formulation. The formulation takes into account network topology, multiple paths between OD pairs and *most likely used paths* according to Dynamic User Equilibrium models. The horizon of study is divided in the time intervals of length Δt, usually 1.5 to 5 minutes, depending on the network size (and travel times involved) and BT penetration rates. A Δt variable across the study horizon will be considered in the future, depending on congestion, since longer intervals seem convenient at low congestion. But as congestion increases, the time interval length should be decreased.

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