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CRITERIA FOR RAPID SLIDING. I. A REVIEW OF VAIONT CASE

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ABSTRACT:

Vaiont slide has been represented by a model of two interacting evolutive wedges. Mass conservation during the motion implies that the upper wedge transfers mass to the lower one through an internal shearing plane. The model respects available in situ observations. It was formulated in dynamics terms. Outcomes of the analysis are the determination of safety factors of the valley before dam impoundment, and the calculation of run-out distance once the motion starts. Rock strength degradation as motion develops has also been included. This degradation, even if it is intense, was unable to explain the very high estimated landslide velocities.

1 INTRODUCTION AND ORGANIZATION OF THE WORK

Vaiont landslide has attracted world wide attention into the causes and processes involved in the failure. Interest in Vaiont has never decreased within the technical community despite the 45 yrs elapsed since the accident. Papers analyzing the failure have been published at a maintained rate in Journals and Conferences. The landslide is one of the largest (in terms of volume of mobilized mass) in historic times. As stated by Hendron and Patton (1987), *"It is likely that more information has been published and more analyses have been made of the Vaiont data than for any other slide in the world"*. This paper is an additional contribution to this long list. Only the essentials of the landslide are reported here to justify the models developed. Vaiont has been described in many papers. A significant subset is given in the references of this paper.

One of the main reasons which explain this interest is the difficulty to explain the extremely high velocity of the moving mass. The implication of this lack of understanding is that the risk associated with other landslide occurrences of similar nature (natural slides affected in its toe by increasing water levels, a common situation in dam engineering) cannot be properly evaluated.

In this paper a review of Vaiont slide is presented. It focuses on the kinematics of the slide. Two representative sections have been analysed by formulating a dynamic equilibrium. The approach presented here gives an explanation to some inconsistencies that remains about the stability of the slide previous to the failure.

The acceleration of the slide into catastrophic velocities is out of the scope of this paper. The analysis for the rapid sliding is developed in the companion paper.

In the first part of this paper the main features of the slide and of the events happened previously to the failure are presented. This information is fundamental for the assumptions adopted in the subsequent analysis presented in this paper and the companion one.

2 FUNDAMENTAL INFORMATION ON VAIONT

At the end of 1960, once the dam was built and the reservoir partially impounded, a long continuous peripheral crack, 1 m wide and 2.5 km in length marked the contour of a huge mass, creeping towards the reservoir in the Northern direction (Figure 1). In the following three years the downward motion of the slide was monitored by means of surface markers. Some data provided by them is also plotted in Figure 2. In addition, water pressures in perforated pipes, located in four boreholes (location shown in Figure 2), were monitored starting in July 1961. However, all the investigation efforts provided limited information on some key aspects of the landslide such as the position and shape of the sliding surface and the pore water pressures acting on it. The measured rate of displacements of surface markers could be roughly correlated with the water level of the reservoir (Figure 3). After two cycles of reservoir elevation, which partially filled and emptied the reservoir in the period 1960-1962, the water level reached a maximum (absolute) elevation of 710 m, at the end of September 1963. At that time, the accumulated displacements of surface markers had reached values in excess of 2.50–3 m (Figure 3). The figure shows a good correlation between the increase in water level in the reservoir and the acceleration of landslide displacements. Surface velocities of 20–30 cm per day were registered in the days preceding the final rapid motion which took place on October 9, 1963. An estimated total volume of rock of $280 \times 10^6 \text{ m}^3$ became unstable, accelerated, and invaded the reservoir at an estimated speed of 30 m/s (around 110 km/hour).

2.1 Geological setting

The Vaiont river, which flows from east to west, cuts a large syncline structure which folds Jurassic and Cretaceous strata (Figure 4). The syncline created the “open chair” shape of the Jurassic strata of the left margin of the river, which can also be seen in the figure. The axis of the syncline plunges a few degrees towards the East (normal to the plane of the figure). The syncline shape eventually defined the geometry of the failure surface, which is always important information to understand the subsequent behaviour of the slide. E. Semenza, an engineering geologist son of the dam designer, made important geological contributions to understand the geology of the site. In his book “La Storia del Vaiont raccontata del geologo che ha scoperto la frana (“The story of Vaiont told by the geologist who has discovered the slide”, Semenza, 2001) he includes a tentative reconstruction of the past history of the slide in a series of representative cross-sections which are reproduced in Figure 5.

This reconstruction conveys a clear message from a geomechanical point of view: the failure surface, which was probably initiated several tens of thousands of years ago, has been subjected to an ever increasing story of accumulated relative displacements. The second important point is that the rock mass affected by the 1963 landslide had suffered a history of cracking and “damage” during recent geological times. The sliding surface is located in strata of the upper Malmö period (upper Jurassic). Clays and marls were found in these layers (see below the description of the failure surface). Above the sliding surface finely stratified layers of marl and limestone from the Malmö period were identified. Below the sliding surface, the Jurassic limestone banks of the Dogger period remained unaffected. In the upper part, limestone strata from the lower Cretaceous crowned the moving mass. In general, the folded layers of limestone and marl were strongly fractured (drilling water was often lost in the exploratory borings performed in 1960).

Two representative cross-sections of the slide, located upstream of the dam position at distances of 400 m and 600 m, respectively, are reproduced in Figure 6 (Sections 2 and 5; Hendron and Patton, 1985). The two cross-sections will later be used to analyze the stability conditions of the landslide.

2.2 The sliding surface

In his comprehensive report of 1985, Hendron and Patton (1985) describe the detailed investigation performed to identify the nature of the sliding surface. The conclusion is that thin (a few centimeters thick) continuous layers of high plasticity clay were consistently found in the position of the failure surface. Samples from these clay layers were tested by different laboratories and the results are described in Hendron and Patton (1985).

The clays were found highly plastic a result explained by their significant Ca-montmorillonite content. Liquid limits well in excess of 50% were often found. More recently Tika and Hutchinson (1999) reported the values $w_L = 50\%$ and $PI = 22\%$.

Direct shear tests on remoulded specimens have also been reported by Hendron and Patton (1985). In some cases stress reversals were applied in order to find residual conditions. In fact, the past history of the landslide indicates that the residual friction angle was the relevant strength parameter along the failure surface. Measured average values of residual friction angle ranged between 8° and 10° . These values are consistent with existing correlations between residual friction angles and clay plasticity (Lupini *et al.* 1981). Tika and Hutchinson (1999) used the ring shear apparatus to find also the residual strength. This test, conducted on remoulded specimens, approximates better the large relative shear displacements experienced in nature by the actual sliding surface. They also measured a residual friction angle of 10° for a relative shear displacement in excess of 200 mm. Tika and Hutchinson (1999) also examined the effect of shearing rate. They found a further reduction in residual friction which reached low values (5°) for shearing rates of 0.1 m/s, a velocity which is still far lower than the estimated sliding velocities of the real failure. However, it is a common experience that increasing strain rate leads to an increase

in the strength of soils. More data on the effect of shearing rate on residual strength is probably needed before reaching definite conclusions on this issue.

Hendron and Patton (1985) estimated that some factors (areas of the sliding surface without clay, some localized shearing across strata, irregularities in the geometry of the sliding surface) could increase the average residual friction angle operating in the field and they estimate that $\phi'_{res} = 12^\circ$ is a good approximation for static conditions.

2.3 Monitoring data before the slide

The main purpose behind the limited instrumentation available was to relate the level of the reservoir with the measured vertical and horizontal displacements of a number of topographic marks distributed on the slide surface. Data on horizontal displacements, plotted as a function of position and time in several profiles following the South-North direction in Figure 1, suggest that the slide was essentially moving as a rigid body. The direction of the slide is also indicated in the figure by several arrows. Some of them (small arrows along the peripheral crack) indicate that the moving mass was actually detaching from the stable rock, implying no friction resistance along the eastern and western boundaries of the slide.

Seismic (volumetric P-wave) velocities were measured in central parts of the slide in December 1959 and again in December 1960. A drop in velocity from $v_p = 5-6$ km/s in 1959 to $v_p = 2.5-3$ km/s was recorded. This information may be interpreted as an indication of the progressive weakening of the rock mass due to the distortion induced by the creeping motion of the slide. The velocities initially recorded at the end of 1959 are very high and they correspond to a rock of good quality, Barton (2007). This is perhaps surprising in view of the prehistoric landslide motions described above. The strong drop in seismic velocity in just one year, which is a tiny fraction of time within the complex life of the landslide, seems exaggerated but it is pointing towards significant shear distortions within the rock mass, motivated by the first impoundment of the reservoir which implied a water level rise of 200 m. The associated increase in pore water pressures on the sliding surface is very large and it is unlikely that rainfall events in the past could have produced such a strong drop in effective stress, especially in the lower part of the slide.

It should be emphasized that these P-wave velocities are much higher than the velocities measured in soils, even if they are dense and compact. In other words, the strength which may be associated with the shearing of the rock mass above the sliding surface is orders of magnitude larger than the strength available at the clay-dominated thin layers at the base of the slide, being sheared along sedimentation planes of very high continuity.

2.4 Water pressures and rainfall

The position of piezometers (they were open perforated pipes) was indicated, in plan view, in Figure 1 and in cross-section in Figure 6. A perforated pipe only provides information on the

average water pressures crossed by the tube. Note too that the pipes did not reach the position of the sliding surface. Therefore, they did not provide direct information on the water pressures actually existing in the vicinity of the sliding surface, which is fundamental information to perform a drained stability analysis of the landslide.

In general, the water level recorded by the piezometers follow closely the changing level of the reservoir. The exception was Piezometer 2, at least during the initial part of the recording period. The initial lectures in this piezometer indicated water pressures significantly above (90 m of water column) the reservoir surface. This information has been interpreted as an indication of additional factors, other than the level in the reservoir, which may control the water pressure at the sliding surface. Since the cretaceous limestone, affected by karstic phenomena, is a rather pervious mass, rainfall water infiltrating at high elevations may result in artesian pore pressures against the impervious Malmö formations located at the base of the landslide. However, no further and direct evidence of this possibility was recorded. On the other hand, the simultaneous variation of piezometer and reservoir levels is a good indication of the high permeability of the rock mass above the sliding surface.

When water level in the reservoir is plotted against the recorded slide velocity (Figure 3), an interesting result is obtained. Increasing water level leads to an increase in sliding velocity. The relationship is highly nonlinear and it tends towards an asymptotic limit which is an indication of failure. The problem with Figure 3 is that this relationship is not unique, a result which is not expected if the slide motion is thought to be governed by the effective normal stresses acting on the sliding surface, which, in turn, is controlled by the reservoir level. In fact, the second reservoir filling led to a second asymptotic value for the water level in the reservoir.

This result was probably one important reason behind the decision to increase the water level for the third time in search of a higher (but safe) level in the reservoir which would allow the normal operation of the dam. The idea behind this decision, apparently put forward by L. Müller, is that the rock reacts in a different way when it is wetted for the first time, compared with its reaction when it has already been wet before. There is no fundamental mechanical basis for this proposition, however. The fact is that during the third attempt to raise the water level, displacement velocities increased continuously and the final attempts to reduce the velocity of the slide by lowering the level of the reservoir (Figure 3) did not work.

An explanation for the apparent inconsistency of results in Figure 3 could be found if reservoir water level and rainfall are combined in the spirit that the prevailing water pressures on the sliding surface, irrespective of their origin, should control the stability. Hendron and Patton (1985) found a reasonably good explanation if rainfall, averaged over the preceding 30 days, and water level are jointly considered to explain the landslide velocity. The actual failure occurred for a 30-day precipitation of 240 mm when the reservoir was at elevation 700 m. Leonards (1987) analyzed further the rainfall records and the history of reservoir elevation and could not find a satisfactory

explanation, free of inconsistencies, for the relationship between velocities of the slide, reservoir elevation and previous rainfall. The pore pressure regime prevailing at the sliding surface remains rather uncertain in the Vaiont landslide.

3 AN EVOLUTIVE TWO-WEDGE STABILITY MODEL

The two representative cross-sections 2 and 5 in Figure 6 have been represented in Figure 7 in a simplified version, which is, however, close to the original drawings. The two plots highlight that the failure surface could be described by two planes: a lower horizontal plane day lighting at the river canyon wall and an inclined planar surface. A rock wedge whose thickness decreases upwards rests on the inclined plane. The rock mass reaches its maximum thickness, 270 m, in the central lower part of the slide above the horizontal sliding plane.

A good proportion of reported stability analyses of Vaiont, specially in the years following the failure, has concentrated in the determination of the friction angle necessary for stability (Jaeger, 1965; Nonveiller, 1967; Mencl, 1966; Skempton, 1966; Kenney, 1967). Classic procedures for stability analysis in soil mechanics using limit equilibrium methods were used. The preceding account of the relevant information on Vaiont, namely the data presented by Hendron and Patton (1985) indicates, however, that the friction angle at the failure surface could hardly be larger than 12° .

Two main reasons support this statement: the fact that Vaiont was a case of landslide reactivation (which implies large previous shearing displacements at the sliding surface and hence a clear situation residual strength conditions) and the small residual friction angles (8° - 10°) measured in the highly plastic clays (Ca-montmorillonite rich) found in the clays associated with the sliding surface.

Therefore a relevant question is: Are the representative cross-sections in Figure 7 stable, given the value of the basal friction angle and the estimated conditions of pore water pressure, when the reservoir reached elevations in the range 650 to 700 m?

The cross-sections plotted in Figure 6 suggest that the slide may be defined as two interacting wedges: an upper one (wedge 1) sliding on a plane having a dip of 36° - 37° and a lower one (wedge 2) sliding on a horizontal plane. Since a (common) friction angle of 12° is acting at the basal sliding surfaces, the upper wedge is intrinsically unstable and will push the lower resisting wedge. The weights of the two wedges and the distribution of pore water pressures prevailing on the sliding plane will, as a first approximation, dictate the stability conditions. However, the interaction between the two wedges plays also a relevant role to explain the stability, as discussed below.

It is worth at this point to examine the kinematics of the slide. If the motion starts, one may imagine the slide as a train sliding downwards, an image which is brought to justify that the absolute

velocity in the upper and lower parts of the slide are essentially the same. Surveying data plotted in Figure 1 supports this simple hypothesis, which is to be expected in the reactivation of an old landslide. The difference in velocity (or displacement) when comparing the upper and lower parts of the slide lies obviously in the direction of these vectors: they will be parallel to the underlying failure surface. A conflict arises, however, at the kink or junction between the sliding two planes. It is hard to imagine that voids will develop in the layered sequence of marl and limestone at 270 m depth. The alternative is the bending and shearing of strata. In fact, a single shearing plane may be invoked to accommodate the sudden change direction of velocity at the kink. This is indicated in Figure 8a, where sliding velocity vectors v_1 (in the direction of the upper inclined surface) and v_2 (horizontal, parallel to the basal plane) are plotted with a common origin. This velocity diagram represents the conditions at the kink (point A), where the rock approaches A with velocity v_1 and leaves it with velocity v_2 . The relative motion of the two wedges (vector v_{12}) is directed in the direction of the bisector of the angle between the upper and lower sliding surfaces. Therefore, the change in the direction of the velocities of the two wedges may be accommodated by a relative shear in the direction of the bisector plane plotted in Figure 8.

The motion of the slide implies that (unstable) mass from the upper wedge becomes (stable) mass of the lower wedge. In this process the sliding resistance along the common plane separating the two wedges has to be overcome. If it is accepted, because of the preceding discussion, that the common plane of intense shear bounding the two wedges is the bisector plane, the evolution of the geometry of the sliding mass may be approximated by the successive cross-sections shown in Figure 8 for total slide displacements $s = 0$ m, $s = 100$ m and $s = 400$ m. Figure 8 is a graphical expression of the condition of mass conservation during the landslide motion. It will be used later to perform a dynamic analysis of the failure.

3.1 Internal shearing

Shearing across the common plane AB between the upper and lower wedges (Figure 8) has a direction approximately perpendicular to the sedimentation planes of the marls and limestones of the Malmö period overlying the failure surface. The shear resistance offered by plane AB is difficult to estimate because of the intricate geometry involved at several scales and the limited continuity of joints. Following Hoek (2007), the strength of rock masses may be approximated if some basic characteristics are determined (rock matrix unconfined strength; degree of jointing and state of the surfaces, lithology etc). Figure 9 shows the strength envelope in a Mohr stress plane for a rock mass which may approximate the Malmö layers above the sliding surface of Vaiont. Details of the defined rock mass are given in the caption of Figure 9. It may correspond to the Vaiont slide mass, which was described as follows by Müller (1987), after the failure:

“The part of the stratigraphic column exposed in the slide mass consists of beds of partially crystalline limestones, limestones with hard siliceous inclusions, marly limestones and marls. Many beds are strongly folded and show indications of slope tectonics. Its geological structure

but also its geological sequence has remained essentially unchanged. The entire rock mass remained intact and the sediment facies is nearly unchanged. Apart from some newly formed faults, the only other effects of the slide were the opening of existing joints and the development of new joints, resulting in an overall volume increase of 4-6% and an associated reduction of the mechanical coherence of the rock mass”

The strength envelope is nonlinear but a Mohr-Coulomb approximation is also shown in Figure 9 for a range of normal stresses centered at $\sigma'_n = 2$ MPa, a stress which may represent average conditions on the bisector plane AB (Figure 8). The Mohr-Coulomb strength parameters ($c' = 0.787$ MPa; $\varphi' = 38.5^\circ$) define the linear M-C approximation.

The relevant point is that the shear plane AB may offer a substantial resistance to be sheared and this resistance has probably a significant role in stability. Shearing across a rock mass is typically associated with the release of energy. In fact, in the years preceding the failure, when three attempts to fill the reservoir were made, seismic events were recorded on the slide surface. Their location has been plotted in Figure 1. They approximately span, in plan view, the position of the shear plane AB plotted in Figure 8. Nonveiller (1987), quoting a report on these shocks mentions that “...the shocks generated in the zone of the slide signify dilation of the material in a zone of sagging of the rock.”

These events had an increasing frequency in periods of slide acceleration, when the reservoir level increased. This is shown in Figure 2, where seismic events have been plotted as small marks in the time axis (lower part of the figure).

It was also reported that the rock experienced a global degradation, reflected in a substantial drop of p-wave velocities, as a result of the slide motion during the period December 1959-December 1960. All this evidence supports the conclusion that a rock mass around the position of the ideal shear plane AB was subjected to intense shearing during the cycles of filling and emptying the reservoir in the years previous to the failure.

A loss of strength (reduction of mechanical coherence in Müller words) was certainly a consequence of this straining. Typically cohesion is first lost but friction tends to remain without much change. This drop of cohesion as a result of straining along plane AB has also been shown in Figure 8. In the model described below, the apparent cohesion in the shear plane AB will be reduced as the slide moves forward. Going back again to Figure 8, as slide displacement increases, “new” planes of rock cross the shearing position AB, which remains fixed at the position of the bisector plane, which is independent of the slide motion. The consequence is that the shear strength along this plane will not decrease in a sudden and intense manner. Certainly the motion of the slide will have some weakening effect, which is difficult to quantify.

3.2 Motion equations

A model based on the interaction of two wedges will now be developed. The main assumptions are: The upper and lower wedges change their geometry during sliding, as shown in Figure 8. The upper wedge loses volume, which is added to the lower one. During this process the common plane AB reduces in length. Shearing across AB (or, more generally, AB') is described by a Mohr-Coulomb strength criterion ($\tau = c'_r + \sigma' \tan \phi'_r$). In addition, the cohesive intercept, c'_r , is made dependent on the slide displacement, s . This is a simplified procedure to introduce strength degradation of the rock mass during the slide motion. The friction angle is maintained constant. The lower sliding surface is assumed to be in residual conditions with strength parameters ($c'_b = 0$; $\phi'_b = 12^\circ$). Pore water pressures are given by a horizontal phreatic level.

Equilibrium conditions are formulated in dynamic terms. In this way it will be possible to analyze the effect of strength degradation of the shearing plane AB' on slide motion. Static conditions of equilibrium are a particular case of the dynamic case. Only inertia terms are considered. No viscous effects are introduced.

Equilibrium conditions will be written for the upper and lower wedge and a common interaction force across plane AB will be enforced.

Upper Wedge (1). Consider the wedge geometry and external forces in Figure 10. Equilibrium parallel to the motion (displacement s ; velocity $v = ds/dt$) reads:

$$W_1 \sin \alpha - T_1 - N'_{\text{int}} \cos(\alpha/2) - Q_{\text{int}} \sin(\alpha/2) - U_{\text{int}} \cos(\alpha/2) = M_1 \frac{dv}{dt} \quad (1)$$

where M_1 is the mass of wedge 1, ($W_1 = M_1 g$; g : gravity acceleration). Equilibrium in normal direction to the basal sliding plane:

$$W_1 \cos \alpha + N'_{\text{int}} \sin(\alpha/2) + U_{\text{int}} \sin(\alpha/2) - Q_{\text{int}} \cos(\alpha/2) = N'_1 + U_1 \quad (2)$$

where the interaction effective forces Q_{int} and N'_{int} are related through

$$Q_{\text{int}} = c'_r AB' + N'_{\text{int}} \tan(\phi'_r) \quad (3)$$

In addition, the shear resistance on the base of the wedge is given by

$$T_1 = N'_1 \tan(\phi'_b) \quad (4)$$

The motion equation (1), in view of (2)-(4) becomes:

$$M_1 \frac{dv}{dt} = W_1 s_1 - N'_{\text{int}} s_2 + c'_r AB' s_3 - U_{\text{int}} s_4 + U_1 \tan(\phi'_b) \quad (5)$$

where s_i are trigonometric constants, given by:

$$\begin{aligned}
s_1 &= \sin \alpha - \tan \varphi'_b \cos \alpha \\
s_2 &= \tan \varphi'_b \sin(\alpha / 2) - \cos(\alpha / 2) \tan(\varphi_r) \tan(\varphi'_b) + \\
&\quad + \cos(\alpha / 2) + \sin(\alpha / 2) \tan(\varphi_r) \\
s_3 &= \tan \varphi'_b \cos(\alpha / 2) - \sin(\alpha / 2) \\
s_4 &= \tan \varphi_b \sin(\alpha / 2) + \cos(\alpha / 2)
\end{aligned} \tag{6 a,b,c,d}$$

For static equilibrium ($M_1 \frac{dv}{dt} = 0$), equation (5) provides the normal interaction force between the two wedges:

$$N'_{\text{int}} = \frac{W_1 s_1 + c'_r AB' s_3 - U_{\text{int}} s_4 + U_1 \tan(\varphi'_b)}{s_2} \tag{7}$$

When the wedge slides a distance s along the basal plane, the length of the shear plane reduces from AB to AB' (Figure 9). Since triangles AVB and $AV'B'$ are similar, it is easy to find:

$$AB' = \frac{L_0 / \cos \alpha - s}{L_0 / \cos \alpha} \frac{H_1}{\cos(\alpha / 2)} \tag{8}$$

where H_1 is the thickness of the lower wedge over the sliding plane. The volume of rock crossing the bisector plane when the wedge moves forward a distance s is given by the trapeze $ACDB$ where $AC = DB' = s$. This incremental volume is given by the expression,

$$V_{\text{incr}} = \frac{H_1 s}{2} \left(1 + \frac{L_0 / \cos \alpha - s}{L_0 / \cos \alpha} \right) \tag{9}$$

Therefore the weights of the two wedges will change, when s increases, as follows:

$$\begin{aligned}
W_1 &= W_{10} - V_{\text{incr}} \gamma_r \\
W_2 &= W_{20} + V_{\text{incr}} \gamma_r
\end{aligned} \tag{10 a,b}$$

Lower wedge (2). The wedge geometry and external forces are given in Figure 11. The wedge is shown displaced forward a distance s .

Equilibrium parallel to the direction of motion at a velocity $v = ds/dt$ reads:

$$M_2 \frac{dv}{dt} = N'_{\text{int}} \cos(\alpha / 2) - Q_{\text{int}} \sin(\alpha / 2) - T_2 \tag{11}$$

Where M_2 is the mass of Wedge 2, ($W_2 = M_2 g$; g : gravity acceleration). Note that the horizontal components of the water pressure forces U_{int} and U_{fy} are equal and opposite in sign. Under limiting conditions, the base resistance is given by

$$T_2 = N'_2 \tan(\varphi_b) \tag{12}$$

and taking equation (3) into account,

$$M_2 \frac{dv}{dt} = N'_{\text{int}} \cos(\alpha/2) - N'_2 \tan \phi_b - \sin(\alpha/2)(c'_r AB' + N'_{\text{int}} \tan \phi'_r) \quad (13)$$

Equilibrium in normal direction to the horizontal sliding plane:

$$W_2 + N'_{\text{int}} \sin(\alpha/2) + U_{\text{int}} \sin(\alpha/2) + Q_{\text{int}} \cos(\alpha/2) + U_{fy} = N'_2 + U_2 \quad (14)$$

where U_{fy} is the vertical component of the water pressure force acting on the slope surface. Equation (2) provides an expression for N'_2 which is then introduced in (13). The following expression is then found for the equation of motion in the direction of sliding:

$$M_2 \frac{dv}{dt} = N'_{\text{int}} s_5 - c'_r AB' s_6 - U_{\text{int}} s_7 - (U_{fy} - U_2 + W_2) \tan(\phi'_b) \quad (15)$$

Where s_i are trigonometric constants, given by:

$$\begin{aligned} s_5 &= \cos(\alpha/2) - \tan \phi'_b \sin(\alpha/2) - \cos(\alpha/2) \tan(\phi'_r) \tan(\phi'_b) - \\ &\quad - \sin(\alpha/2) \tan(\phi'_r) \\ s_6 &= \tan \phi'_b \cos(\alpha/2) + \sin(\alpha/2) \\ s_7 &= \tan \phi'_b \sin(\alpha/2) \end{aligned} \quad (16 \text{ a,b,c})$$

The effective interaction force between the two wedges is now found from (15):

$$N'_{\text{int}} = \frac{M_2 \frac{dv}{dt} + c'_r AB' s_6 + U_{\text{int}} s_7 + (U_{fy} - U_2 + W_2) \tan(\phi'_b)}{s_5} \quad (17)$$

A single motion equation may now be found if N'_{int} , given by (17), is replaced in the motion equation for the upper wedge (5). Rearranging terms, the following equation of motion is derived:

$$M^* \frac{dv}{dt} = W_1 s_1 - t_1 W_2^* + c'_r AB' t_2 - U_{\text{int}} t_3 + U_1 \tan(\phi'_b) \quad (18)$$

where M^* may be interpreted as the combined “effective” mass of the two wedges:

$$M^* = M_1 + \frac{s_2}{s_5} M_2 \quad (19)$$

and

$$W_2^* = W_2 + U_{fy} - U_2 \quad (20)$$

The remaining coefficients in equation (18) are trigonometric expressions:

$$\begin{aligned}
t_1 &= \frac{s_2 \tan(\phi'_b)}{s_5} \\
t_2 &= s_3 - \frac{s_2 s_6}{s_5} \\
t_3 &= \frac{s_2 s_7}{s_5} + s_4
\end{aligned}
\tag{21 a,b,c}$$

4 STATIC EQUILIBRIUM AT FAILURE AND SAFETY FACTORS

Under strict static equilibrium conditions, ($\frac{dV}{dt} = 0$), Equation (18) provides the value of the apparent effective cohesion along the shearing plane AB in terms of the friction angle on AB , ϕ'_r , the wedge weights, the pore pressure forces on their boundaries and the geometrical factors:

$$c'_r = \frac{t_1 W_2^* + U_{\text{int}} t_3 - W_1 s_1 - U_1 \tan(\phi'_b)}{AB' t_2} \tag{22}$$

The resultants of water pressure forces entering the above equations are easily found as follows:

$$\begin{aligned}
U_{fy} &= \frac{h_w^2 \gamma_w}{2 \tan \delta} \\
U_2 &= (L_1 + L_2 + s) h_w \gamma_w \\
U_1 &= \frac{h_w^2 \gamma_w}{2 \sin \alpha} \\
U_{\text{int}} &= \frac{h_w^2 \gamma_w}{2 \cos(\alpha/2)}
\end{aligned}
\tag{23 a,b,c,d}$$

Initial ($s=0$) wedge volumes, in view of Figures 9 and 10, are given by

$$\begin{aligned}
V_{10} &= \frac{L_0 H_1}{2 \cos \alpha} \\
V_{20} &= \frac{L_1 + L_2 + L_3}{2} H_1
\end{aligned}
\tag{24 a,b}$$

which allow the calculation of wedge weights.

Cross-sections 2 and 5 (Figure 4) are characterized by the geometrical parameters given in Table 1. The upper wedges of Sections 1 and 2 have a similar volume. However, the lower wedge of Section 2 has a significantly lower volume than Section 5. Therefore, Section 5 is more stable than Section 2, for a common set of strength parameters. Conditions for static equilibrium of these two sections will be first examined with the help of the set of relationships derived in the previous section. Since it has been argued that the residual friction at the basal sliding surface is a parameter known with sufficient certainty, the condition of stability may be used only to determine the strength parameters on the shear plane AB . In fact, only combinations of the pair (c'_r ; ϕ'_r) may

be found since only one condition is available: the condition of static equilibrium at the initiation of failure (Equation 22).

Table 1. Geometrical parameters of cross-sections 2 and 5

	H_0 (m)	H_1 (m)	L_0 (m)	L_1 (m)	L_2 (m)	α (°)	δ (°)	V_1 (m ³ /m)	V_2 (m ³ /m)
Section 2	580	245	750	190	260	37.7	43.3	116142	68149
Section 5	510	260	700	240	320	36	39.1	112590	93000

This is a nonlinear equation relating c'_r and φ'_r , which has been plotted in Figure 12 for Sections 2 and 5, assuming φ'_b equal to 12° and a rock specific weight of 23.5 kN/m³.

Forces U (Equation 23), which provides the effect of water pressures on both wedges, should correspond to failure conditions. Since a horizontal water level has been assumed and the preceding rain was shown to have a non-negligible effect, all the water pressure influence will be associated with the water level height above the lower horizontal sliding surface, h_w . Data given by Hendron and Patton (1985) provides the estimation of the equivalent value of h_w , i.e.: the reservoir water level, in the absence of rain in the preceding 30-day period, which explains the failure. This height correspond to the elevation 710 m approximately and therefore, in Section 5 it implies a value $h_w = 120$ m. This reservoir elevation corresponds, in Section 2, to water height of $h_w = 90$ m. (the failure surface daylights at a higher elevation at section 2; see figure 4). The $(c'_r; \varphi'_r)$ values plotted in Figure 12 correspond to these two water elevations over the lower horizontal sliding plane.

Section 2 is “more demanding” in terms of required rock strength simply because of the relative weight of upper and lower wedges. This situation is reflected in the higher strength values required for equilibrium calculated for Section 2 (Figure 12). It is interesting to check that the $(c'_r; \varphi'_r)$ combinations in Figure 11 are in fairly good agreement with the strength expected in the rock sheared across bedding planes, discussed before. Since the variability of φ'_r values is small compared with the expected variation of cohesive intercepts (c'_r), a band of expected $(c'_r; \varphi'_r)$ pairs, centred around $\varphi'_r = 38^\circ\text{-}40^\circ$ has been plotted in Figure 11 as a reasonable estimation of the rock strength along the shear plane AB .

If Section 5 is taken as a representative cross-section of the slide, the following combinations lead to strict equilibrium of Vaiont slide: $(c'_r = 768.35 \text{ kPa}; \varphi'_r = 38^\circ)$; $(c'_r = 561.3 \text{ kPa}; \varphi'_r = 40^\circ)$.

The model of two interacting wedges developed before includes two failure surfaces: the “basal” surface which bounds the landslide and an internal shear surface (AB) which makes it kinematically possible. The nature of both surfaces is quite different: the former is located in a high plasticity clay in residual conditions whereas the internal shear surface crosses sedimentary planes, distorts a competent rock and exhibits a significant strength (however, it is quite possible that shear displacements will decrease to some extent the shear strength of this shear plane). For a particular situation of the slide (for instance, under natural conditions before dam construction) the two shearing surfaces will most probably not mobilize their shear strength in equal proportions. Likewise, if a change in external conditions takes place (reservoir impoundment, or rainfall) the available strength will not be mobilized at the same time among the two surfaces because the shear stiffness of the shearing surfaces and, indeed, of the whole rock mass will also play a significant role.

Since the problem is complicated, let us accept, to initiate the discussion, that two different safety factors, F_b and F_r , are appropriate for the two surfaces. Then, the mobilized strength parameters will be defined as follows:

$$\tan \varphi'_{b\text{mob}} = \frac{\tan \varphi'_b}{F_b}; \tan \varphi'_{r\text{mob}} = \frac{\tan \varphi'_r}{F_r}; c'_{r\text{mob}} = \frac{c'_r}{F_r} \quad (25 \text{ a,b,c})$$

A relevant issue is to ask for the safety factor, F_r , of the Vaiont slide at the beginning of impoundment (i.e.: $h_w=0$) in the hypothesis that the mobilized stress at the basal sliding surface remained at the residual value, $\varphi'_b=12^\circ$, (i.e.: $F_b=1$). It is also of interest to know how would F_r change, still under $F_b=1$, if the slide moves forward following the mechanism described in Figure 8.

Alternatively, one may wish to maintain the classic approach and to find a unique and global safety factor, F , for the two situations mentioned, ($F = F_b = F_r$). The two possibilities will be examined here.

For cross-section 5, it was found that the following set of strength parameters: $\varphi_b = 12^\circ$; $c'_r = 768.35$ kPa; $\varphi'_r = 38^\circ$ leads to failure when $h_w = 120$ m. If these parameters are accepted as true strength parameters, then the equilibrium equations derived before are also valid, for conditions other than failure, if the reduced strength parameters (25a,b,c) are used instead of the true strength values (which now are assumed to be known). In other words, equilibrium conditions are now satisfied for the mobilized stresses prevailing at the shear surfaces. In fact, mobilized shear stresses are defined as those which satisfy equilibrium conditions. Therefore, in view of Equation 25, the overall equilibrium equation can be used to find the safety factor. However, the equilibrium equation will now be a function of F_b and F_r and therefore only one safety factor may be determined - either F if it is accepted that $F = F_b = F_r$, or F_r if F_b is fixed, for instance at $F_b = 1$, or any other alternative -.

If the mobilized strength parameters (Equation 25) are substituted into the equilibrium Equation 22, the following expression is obtained:

$$\frac{c'_r}{F_r} = \frac{t_1(F_r, F_b)W_2^* + U_{\text{int}}t_3(F_r, F_b) - W_1s_1(F_b) - U_1 \frac{\tan(\phi'_b)}{F_b}}{AB't_2(F_r, F_b)} \quad (26)$$

where the dependence of the t_i and s_i expressions on the safety factors has been explicitly indicated. If Equation 26 is developed it turns out to be a second order algebraic equation for F_r , which may be solved, if F_b is assumed to be known.

If F_b is known, Equation 26 is a second order algebraic equation which can be solved explicitly for the root F_r .

Safety factors F_r of Section 5 of Vaiont slide were obtained for:

- Water pressure conditions prior to failure. As discussed before, pore water pressure effects are integrated into the variable h_w , the reservoir level over the lower horizontal sliding plane
- The changing geometry, as the slide moves forward and the water level maintains the maximum elevation, $h_w = 120$ m. This is a purely static analysis performed on different geometries of the slide as it moves forward. The dynamics of the motion will be introduced in the next section and it will be discussed in more detail in the companion paper.

The effect of h_w on safety factor F_r , when $F_b = 1$, is plotted in Figure 13. The calculated value for $h_w = 0$ ($F_r = 1.2$) is not particularly high and it indicates that the mobilized strength in the rock mass before any impounding was quite substantial in order to maintain the slope in equilibrium.

The analysis of the changing geometry, sketched in Figure 8, leads to the safety factors F_r plotted in Figure 14. The increase of F_r , again for $F_b = 1$, becomes more pronounced as the slide displacement increases. The high values calculated for $s = 150$ m ($F_r = 5$), indicate that the mobilized resistance across the shear plane AB is no longer necessary to maintain equilibrium. In fact, beyond $s = 179$ m, the residual friction angle at the main sliding surface is able to maintain the slope in equilibrium without any contribution from the sheared rock mass across the shear plane AB .

Let us consider now the determination of a unique global safety factor F . The condition $F = F_b = F_r$ has to be introduced in Equation 26. The equilibrium Equation 26 now becomes a fourth order polynomial for the unknown F . Calculated global safety factors have been plotted also in Figures 13 and 14. Computed values of F are now significantly lower than the previously reported values of F_r .

The global safety factors calculated for changing water levels within a very large range (0 to 120 m of water column) (Figure 12) look particularly low (F decreases from $F = 1.07$ for $h_w = 0$ m to $F = 1$

for $h_w = 120$ m). This is a consequence of the very large size of the landslide but it also points out that the presence of the reservoir implied a relatively minor change in the safety of the slope, always within the perspective of risk associated with the classical definition of a global safety factor. Moreover, this result is also an indirect indication that in very large landslides, feasible remedial measures are expected to lead to relatively low increments of safety factor.

Figure 13 shows that the motion of the slide results in geometries with increasing global safety factor. Given the preceding comments, changes are far from being negligible. In fact, displacements of 40, 100 and 150 m imply F values of 1.08, 1.22 and 1.36 respectively.

5 LANDSLIDE RUNOUT

Equilibrium conditions, when inertia terms are included, results in the motion equation (15). This equation has the following form:

$$a = \frac{dv}{dt} = f(s) = f\left(\int_0^t v dt\right) \quad (27)$$

At any given time of the motion, slide acceleration ($a = \frac{dv}{dt}$) is a function of slide displacement, s .

Function f includes also information on geometry, specific weights, water pressures and strength parameters. Finding a close form solution for $v(t)$ is a hard task but the structure of (27) allows to develop a simple explicit numerical algorithm of integration. In view of the nature of the problem and the simplicity of the underlying mechanical model it is probably not justified in this case to look for more sophisticated integration procedures.

It was argued, when developing the model of two interacting wedges, that the effective rock cohesive intercept, c'_r , would be degraded during shear along the plane AB. Since relative shear displacements along AB are controlled by displacement s , a simple degradation model is to make c'_r dependent on s . For instance,

$$c'_r = c'_{r0} \exp(-\Gamma s) \quad (28)$$

where Γ is a constant (units: length^{-1}) which controls the rate of rock degradation and c'_{r0} is the initial cohesion intercept ($c'_{r0} = 768.35$ kPa for cross-section 5, if $\varphi'_r = 38^\circ$, accepting $\varphi_b = 12^\circ$). Expression (28) was also included in the motion equation in order to explore the effect of loss of shear strength on the dynamics of the motion. It is not reasonable, however, to expect a strong degradation of cohesion along AB' and the reason is that the rock mass “crosses” the plane AB' during the motion and therefore new –more or less undisturbed- rock is continuously sheared across AB .

Consider the following scenario: in a situation of strict equilibrium (reservoir elevation at $h_w = 120$ m in cross-section 5) the water level is increased by a small amount (say $h_w = 121$ m) and it is maintained constant thereafter. It is desired to find the motion of the slide until a new situation of equilibrium is reached. Since the slide improves its static stability conditions as s increases it should be expected that after some displacement, the slide will come to rest.

The solution to this problem (which is the solution of Equation 15) plotted as a relationship between the run out (s) and the velocity on the moving mass (v) is shown in Figure 14 for no degradation of the rock strength ($\Gamma = 0$). The result shows that the slide stops after a displacement of 0.56 m and reaches a maximum velocity of 1.8 cm/s. If the water level is increased to $h_w = 124$ m and to $h_w = 130$ m, maximum displacements and velocities increase as shown also in Figure 15 but the calculated values are very far from the actual behaviour of the landslide, which reached velocities estimated in 30 m/s, more than two orders of magnitude higher than the maximum values found in this calculation.

The situation changes if some rock strength degradation is introduced into the analysis. Figure 16 is a plot of Equation 28 for a few values of the degradation parameter Γ . It will be used as a reference for the results of run-out calculations. Now the scenario is to start the slide motion by increasing the water level (to $h_w = 121$ m) and to accept a certain degradation of the rock during the motion. The calculated response of the slide, again in terms of velocity vs. displacement, is shown in Figures 17 and 18. A moderate degradation of the effective strength parameter of the rock ($\Gamma = 0.01$, Figure 17) has a limited effect on the maximum sliding velocity and on the travelled distance. However, if the degradation of rock effective cohesion is more rapid ($\Gamma = 0.1$ and $\Gamma = 1$; Figure 17), the slide is able to travel long distances (100-120 m) although the maximum velocity does not increase beyond 4.2 m/s (16.2 km/h) even if a very rapid and complete destruction of the rock effective cohesion is imposed (for $\Gamma = 1$, see Figure 14). Even if the reservoir level is suddenly increased in 5 m (to $h_w = 125$ m), and a value $\Gamma = 1$ is maintained, the maximum velocity does not reach 4.5 m/s. Under the more realistic assumption of moderate rock degradation, $\Gamma \leq 1$, the maximum slide velocity is quite small. In all the cases analyzed, the mechanism leading to stop the landslide motion is the change in geometry of the slide as it moves downwards.

The dynamic analysis developed here maintains unanswered the key question of the extremely high velocities reached by the slide. However, it indicates that a loss of internal rock strength, associated to the slide motion itself, is a potential mechanism to accelerate the slide.

6 DISCUSSION

The investigations on the past history of the landslide by Semenza (2001), synthesized in Figure 5, and the work of Hendron and Patton (1987) highlight two fundamental aspects: Vaiont was a case of a slide reactivation and the sliding surface was located in fairly continuous layers of high

plasticity clay. Taken together, the implication is that the basal sliding surface could not offer, against a new reactivation of the slide (essentially induced by an increase in pore water pressures in the lower massive passive wedge of the slide), an effective friction angle larger than, say, 10° - 12° . A good proportion of published back-analyses of Vaiont, which use conventional methods of limit equilibrium in order to find the actual friction angle prevailing at the sliding surface at the time of failure, lead to an inconsistent situation. In fact, published back-analysis leads to friction angles in the range 18° - 28° . Vaiont exhibits a safety factor significantly lower than one if a friction angle of 10° - 12° (and zero effective cohesion) is used in any of the currently available methods of slices.

In order to address this inconsistency, Hendron and Patton (1987) argue that the side friction on the Eastern edge of the slide provided the necessary resisting force to ensure equilibrium (however, some limited information on the direction of displacements on this border- plotted in Figure 1-, tends to indicate that the moving mass was detaching from the stable rock massif). The alternative explanation developed here is that the kinematics of the motion, even in a two-dimensional cross-section, requires the relative shearing between the two large rock wedges defining the slide. Leonards (1987) also pointed out that the motion of the slide required such a rock shearing between the upper and the lower sliding blocks. The estimated shearing strength parameters across the common plane are in a reasonable accordance with the expected mass strength of cretaceous marls and limestones of Vaiont.

The acceleration of the motion during the catastrophic failure escapes the capabilities of the models presented here. A loss of strength is expected when rock masses are sheared, due to its inherent brittleness and the complex development of strains within the moving mass. The end result is a loss of the cohesive components of strength. Such a loss, when imposed on the strength available on the interacting shearing plane between the upper and lower wedges, results in an acceleration of the slide, which is, however, unable to explain the high velocities reached by the landslide, even if a rapid and complete loss of rock cohesion is imposed. If the mechanism of side friction proposed by Hendron and Patton (1985) is accepted as additional resisting phenomena, the need for a convincing mechanism for strength loss is even more pronounced.

7 CONCLUSIONS

Some fundamentals aspects of Vaiont slide are invoked to propose a consistent, yet simple, kinematic model for the slide. They are:

- The basal sliding surface was most probably in residual conditions. This is explained by the known geologic history of the left bank of Vaiont River. Vaiont was a case of reactivation of an ancient slide which experienced several large scale motions in the past.
- Residual shear tests on Malmö clays found on the sliding surface indicate that the basal operative friction angle was close to 12° .

- The geometry of the cross-section of the slide requires that during the sliding motion shearing of rock strata normal to bedding planes take place. The strength of this rock mass has been estimated on the basis of available descriptions. Other field data, namely the recorded seismic events, support also a progressive shearing of rock strata in the years previous to the slide.
- The rock overlying the failure surface was essentially pervious. Water pressures were essentially controlled by reservoir elevation although there is evidence which suggests also a contribution of the previous rainfall regime.

A simple evolutive two-wedge model was developed to accommodate these observations. Mass is being transferred from an upper unstable wedge to a lower stable one during the motion. A common shearing plane bounds the two wedges.

Dynamic equilibrium equations are formulated. They incorporate the conditions of mass conservation. Internal shearing is approximated by a Mohr-Coulomb failure criterion. The partial motion equations combine into a unique motion equation for the entire landslide which can be integrated.

In a first series of analyses, static conditions were investigated. Safety factors have been defined and found for conditions of the slide previous to reservoir water elevation. A low global safety factor (SF=1.07) was calculated for an empty reservoir. It was also found, as expected, that the increasing slide displacement, once the motion was started, leads to an increase in safety factor (the weight transfer from the upper to the lower wedge explains this result).

In attempt to explain the increasing velocity of the slide, the possibility of internal rock strength degradation was introduced. Under this scenario the strength of the internal shearing plane is made dependent on the slide displacement. Only cohesion intercepts are degraded. It was found that if no degradation of the rock strength is considered, an initial unbalance of forces (say by increasing in one meter the reservoir elevation over the situation for strict equilibrium) leads to small displacements (56 cm) and very small maximum velocities (1.8 cm/s) before reaching a new equilibrium state. These figures change as degradation is assumed to increase. However, a full loss of cohesive strength of the rock leads to maximum velocities not exceeding 4.5 m/s (against the estimated value of 30 m/s). Full degradation of strength is very unlikely, however, because the internal shearing, as the slide displaces, is affecting “new” rock masses in their downhill motion. It is concluded that the internal rock strength degradation is a contributing factor to the acceleration of the slide but it fails to explain the high velocity reached by the slide.

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